STA 2212S

Jan 29 /2021

1. deviance in R
2. summary re testing (+ article)
3. NP Lemma + proof

$\boxed{\text{Wed. g-o-fit, multiple testing}}$

?glm ... Value

dev. full $\quad -2\ell(\hat{\theta}; \underset{\sim}{x}) + c$ — deviance "fitted" model e.g. full

dev. red. $\quad -2\ell(\hat{\theta}_0; \underset{\sim}{x}) + c$ — deviance "restr" $\theta_1 = \theta_2 = \theta_3 = 0$

dev. red − dev. full =

$$-2\,\ell(\hat{\theta}_0; x) + 2(\ell(\hat{\theta}; \underset{\sim}{x}))$$

$$= 2\{\ell(\hat{\theta}) - \ell(\hat{\theta}_0)\}$$ G. Lik Ratio Stat.

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p-values : tell us $\Pr\{T > t^{obs}; \theta_0\}$ $\triangleq p$ for H_0

→ "$p < 0.05$" statistically significant ←

Testing : $N \quad \chi^2 \quad t \quad F$

: in many parametric models, likelihood tests are used

1. $U(\theta_0)\, I_n^{-1/2}(\hat{\theta}) \;\dot\sim\; N(0, I)$ ~~Rao~~ score test

coef
~~R~~ 2. $(\hat{\theta} - \theta)\, I_n^{1/2}(\hat{\theta}) \;\dot\sim\; N(0, I)$ Wald test

anova
R 3. $2\{\ell(\hat{\theta}) - \ell(\theta)\} \;\dot\sim\; \chi^2_k$ $k = \dim \theta$

"analysis of deviance"

$H_0: \underset{\sim}{\theta} = \underset{\sim}{\theta}_0$ simple null

$H_0: \theta \in \Theta_0$ compos.

versions based on profile log. lik.

fitdistribution :: fitdistr

$H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$ $N(\mu, \sigma^2)$

composite H_0

$$T = \frac{\sqrt{n}\,(\bar{X} - \mu_0)}{s \;\; \cancel{\sigma}} \qquad \text{where } s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

under normality $\sim$ t_{n-1} d.f.

$T^2 \sim F_{1, n-1}$ d.f. $\leftarrow$ can be gen'd (to $F_{p,q}$)

$f(\bar{x}, s)$ from $f(\underset{\sim}{x})$ [$\bar{x}, s^2$ are ind't]

? $f(t)$ from $\int f(\bar{x}, s)\, 1\{t = t\}\, d\ldots$?

T- & F- tests are quite robust to ass= of normality (work reasonably well when X_i iid $f(\cdot)$)

recommend 1) $p^{obs} = P\left(\frac{\sqrt{n}(\bar{X}-\mu_0)}{S} > \cancel{t_{\alpha/2}}\ t^{obs}\right)$ Wald

$\hat{\theta}$

$\hat{se}$

2) report $(\bar{X}, S)$

3) construct C.I. $\bar{X} \pm \frac{S}{\sqrt{n}} t_{n-1}^{\alpha/2}$ $1-\alpha$ level C.I.

if σ known $\bar{X} \pm \frac{\sigma}{\sqrt{n}} z^{\alpha/2}$

If Wald test has $p < 0.05$
the 95% C.I. based on Wald stat will not include θ_0

Andrew Gelman ← "garden of forking paths"

NP Lemma: $\underset{\sim}{X} \sim f(\underline{x};\theta)$

$H_0: \theta = \theta_0 \qquad H_1: \theta = \theta_1$

MP test of size α has rejection region
critical

$$R = \left\{ \underline{x} : \frac{f(\underline{x};\theta_1)}{f(\underline{x};\theta_0)} > k \right\} \qquad k > 0$$

If k is chosen so $Pr(\underset{\sim}{X} \in R; \theta_0) = \alpha$.

Proof:

$$R : \int_R f(\underline{x};\theta_0)\,d\underline{x} = \alpha$$

competing test $\tilde{R}$: $$\int_{\tilde{R}} f(\underline{x};\theta_0)\,d\underline{x} \leq \alpha$$

$$\int_R f(\underline{x};\theta_0)\,dx \geq \int_{\tilde{R}} f(x;\theta_0)\,dx$$

$$\geq \int_{R-(R\cap\tilde{R})} f(\underline{x};\theta_0)\,dx + \cancel{\int_{R\cap\tilde{R}} f(\underline{x};\theta_0)\,dx}$$

$$\geq \int_{\tilde{R}-(R\cap\tilde{R})} f(x;\theta_0)\,dx + \cancel{\int_{R\cap\tilde{R}} f(x;\theta_0)\,dx}$$

in R $\quad \dfrac{f(x;\theta_1)}{f(x;\theta_0)} > c_\alpha > 0 \quad f(x;\theta_0) < \dfrac{f(x;\theta_1)}{c_\alpha}$

not in R $\quad \dfrac{f(x;\theta_1)}{f(x;\theta_0)} < c_\alpha \quad f(x;\theta_1) < c_\alpha f$

$$\cancel{\frac{1}{c_\alpha}} \int_{R-(R\cap\tilde{R})} f(x;\theta_1) \geq \cdots \geq \int_{\tilde{R}-(R\cap\tilde{R})} \frac{f(x;\theta_1)}{\cancel{c_\alpha}}\,dx$$

$$\int_{R-(R\cap\tilde{R})} f(x;\theta_1)\,dx \geq \int_{\tilde{R}-(R\cap\tilde{R})} f(x;\theta_1)\,dx$$

$$\int_{R-(\ )} f(x;\theta_1) + \int_{(R\cap\tilde{R})} \geq \int_{R\cap\tilde{R}} + \int$$

$$\int_R f(x;\theta_1)\,dx \geq \int_{\tilde{R}} f(x;\theta_1)\,dx$$

power of the test $R = \left\{ x : \dfrac{f(x;\theta_1)}{f(x;\theta_0)} > c_\alpha \right\}$

is c_α chosen $\overset{\text{size}}{=} \alpha$

is $>$ power of a test with any other rej. region

rejection $\longrightarrow$ critical (get away from "reject")