

HW Question Week 10

STA2101F 2021

Due December 2 2021 11.59 pm

Homework to be submitted through Quercus

- (a) *Adapted from ELM Ex. 4.4 (1st edition, Ex.6.4 2nd edition)* The dataset `death` in the library `faraway` cross-classifies death penalty sentences according to the race of the defendant and the race of the victim.
- (i) Construct a summary table showing the association of race of defendant with the sentence (death-penalty yes or no), and compute the χ^2 test of independence.
 - (ii) Do this separately for the two tables formed by considering the race of the victim.
 - (iii) Compare the results to those obtained from a binomial regression with sentence (death-penalty yes or no) as the response and the explanatory variables race of defendant and race of victim.
- (b) *SM Ex. 8.2.9* Consider a normal linear model $y = X\beta + \epsilon$, where $\text{var}(\epsilon) = \sigma^2 W^{-1}$ and W is a known positive definite symmetric matrix.
- (i) Re-express the least squares problem in terms of $y_1 = W^{1/2}y$, $X_1 = W^{1/2}X$, and $\epsilon_1 = W^{1/2}\epsilon$.
 - (ii) Show that $\text{var}(\epsilon_1) = \sigma^2 I_n$.
 - (iii) Find the least squares estimates, hat matrix, and residual sum of squares for the re-expressed regression, in terms of the original y , X , and W .
- (c) *Bonus/Required for PhD* Suppose that W depends on an unknown scalar parameter ρ . Find the profile log-likelihood function for ρ : $\ell_p(\rho) = \max_{\beta, \sigma^2} \ell(\beta, \sigma^2, \rho)$, and outline how to use a least squares algorithm to provide a confidence interval for ρ .