

Distributions for Parameters

Nancy Reid
University of Toronto

WIML Unconference

July 13 2020



Those pesky p -values



David Spiegelhalter

@d_spiegel



This paper motivates the call for the end of significance. A 25% mortality reduction, but because $P=0.06$ (two-sided), they declare it 'did not reduce' mortality. Appalling.

jamanetwork.com/journals/jama/...

Research

JAMA | **Original Investigation** | **CARING FOR THE CRITICALLY ILL PATIENT**

Effect of a Resuscitation Strategy Targeting Peripheral Perfusion Status vs Serum Lactate Levels on 28-Day Mortality Among Patients With Septic Shock

The ANDROMEDA-SHOCK Randomized Clinical Trial

Glenn Hernández, MD, PhD; Gustavo A. Ospina-Tascón, MD, PhD; Lucas Petri Damiani, MSc; Elisa Estenssoro, MD; Arnaldo Dubin, MD, PhD; Javier Hurtado, MD; Gilberto Friedman, MD, PhD; Ricardo Castro, MD, MPH; Leyla Alegría, RN, MSc; Jean-Louis Teboul, MD, PhD; Maurizio Cecconi, MD, FFICM; Giorgio Ferri, MD; Manuel Jibaja, MD; Ronald Pairumani, MD; Paula Fernández, MD; Diego Barahona, MD; Vladimir Granda-Luna, MD, PhD; Alexandre Biasi Cavalcanti, MD, PhD; Jan Bakker, MD, PhD; for the ANDROMEDA-SHOCK Investigators and the Latin America Intensive Care Network (LIVEN)

- comparing two treatments for septic shock
- randomized clinical trial
- estimated hazard ratio 0.75 [0.55, 1.02] after adjusting for confounders
- 2-sided p-value 0.06 34.9% vs 43.4% unadjusted
- Discussion: “a peripheral perfusion-targeted resuscitation strategy did not result in a significantly lower 28-day mortality when compared with a lactate level-targeted strategy”
- Abstract: “Among patients with septic shock, a resuscitation strategy targeting normalization of capillary refill time, compared with a strategy targeting serum lactate levels, did not reduce all-cause 28-day mortality.”

- Clinical characteristics and predictors of outcomes of hospitalized patients with COVID-19 in a London NHS trust: a retrospective cohort study
- “all patients hospitalised with laboratory-confirmed SARS-CoV-2 infection at Imperial College Healthcare NHS Trust between February 25 and April 5, 2020
outcomes were recorded as of April 19, 2020
- “**logistic regression models**, survival analyses and cumulative competing risk analyses ... to evaluate factors associated with COVID-19 hospital mortality
- “**ethnic minority groups were over-represented in our cohort**
- “compared to whites, people of black ethnicity **may be at increased odds of mortality**
- “Further research is urgently needed to investigate these associations ”
several other conclusions around co-morbidities and various blood-test indicators

- “the crude OR of death of black compared to white patients was not significant”
odds ratio: 1.14 95%CI 0.69-1.88 p=0.62
- “adjusting for age and comorbidity showed a trend towards significance”
adj. odds ratio: 1.72 95%CI 0.98-3.02 p=0.06
- “further accounting for admission severity ... was significant”
adj. odds ratio: 1.83 95%CI 1.02-3.30 p=0.04
- “when adjusting for age, Elixhauser comorbidity score and severity of disease on admission, we observed higher odds of death for those from black ethnic background compared with whites
- “this finding merits further investigation **given its borderline statistical significance**”

my emphasis

29 April 2020

Imperial College COVID-19 response team

Table 3: Clinical characteristics by ethnicity and logistic regression of odds of death

*Adjusted logistic regression for age, Elixhauser and EWS differences of ethnic groups

**Additional logistic regression models adjusted for individual comorbidities that had statistically significant variation across ethnic groups, but were inferior predictors compared to the selected model with Elixhauser score. CKD, cirrhotic liver disease and HIV/AIDS were not used in logistic regression models, as they had low n values for ethnic groups

Abbreviations: COPD, chronic obstructive pulmonary disease; DVT/PE, deep vein thrombosis / pulmonary embolism; HDU/ITU, high dependency unit / intensive treatment unit; HIV/AIDS, human immunodeficiency virus / acquired immunodeficiency syndrome; IVS, invasive ventilation support; SD, standard deviation

	White (n = 196)	Black (n = 116)	Asian (n = 78)	Other (n = 15)	NA (n = 115)	p-value
Male, n (%)	119 (61%)	66 (57%)	51 (65%)	10 (67%)	76 (66%)	0.60
Mean age (SD)	69.94 (15.6)	62.8 (19.53)	66.05 (15.45)	55.07 (16.01)	59.67 (17.91)	<0.01
Mean days to admission (SD)	6.78 (5.76)	7.3 (6.12)	5.87 (4.84)	7.47 (6.8)	8.34 (5.09)	0.28
Mean Elixhauser score (SD)	4.89 (6.81)	3.63 (5.84)	4.96 (6.66)	2.47 (4.53)	2.38 (4.72)	<0.01
Mean EWS score (SD)	5.01 (3.26)	4.98 (3.11)	4.53 (3.29)	5.47 (2.75)	5.53 (2.76)	0.15
Outcomes						
FI02 >=60%, n (%)	88 (45%)	54 (47%)	38 (49%)	5 (33%)	67 (58%)	0.14
Admitted HDU/ITU, n (%)	22 (11%)	18 (16%)	12 (15%)	2 (13%)	26 (23%)	0.12
Received IVS, n (%)	21 (11%)	16 (14%)	11 (14%)	2 (13%)	23 (20%)	0.27
Died in hospital, n (%)	54 (28%)	35 (30%)	23 (29%)	3 (20%)	29 (25%)	0.86
Discharged alive, n (%)	111 (57%)	67 (58%)	43 (55%)	12 (80%)	69 (60%)	0.46
Pending outcome, n (%)	31 (16%)	14 (12%)	12 (15%)	0 (0%)	17 (15%)	0.49
Comorbidities						
Ischaemic heart disease, n (%)	18 (9%)	9 (8%)	10 (13%)	0 (0%)	6 (5%)	0.27
Chronic heart failure, n (%)	10 (5%)	6 (5%)	0 (0%)	0 (0%)	5 (4%)	0.30
Hypertension, n (%)	68 (35%)	45 (39%)	27 (35%)	5 (33%)	42 (37%)	0.96
Hyperlipidaemia, n (%)	32 (16%)	13 (11%)	18 (23%)	1 (7%)	18 (16%)	0.20
Diabetes**, n (%)	36 (18%)	39 (34%)	31 (40%)	3 (20%)	29 (25%)	<0.01
Chronic kidney disease**, n (%)	31 (16%)	12 (10%)	19 (24%)	0 (0%)	8 (7%)	<0.01
Peripheral vascular disease, n (%)	7 (4%)	2 (2%)	0 (0%)	0 (0%)	2 (2%)	0.38
Stroke, n (%)	17 (9%)	9 (8%)	6 (8%)	0 (0%)	2 (2%)	0.12
Atrial fibrillation**, n (%)	33 (17%)	10 (9%)	9 (12%)	2 (13%)	7 (6%)	<0.05
DVT/PE history, n (%)	4 (2%)	2 (2%)	0 (0%)	0 (0%)	1 (1%)	0.68
Hemiplegia, n (%)	3 (2%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0.29
Dementia**, n (%)	23 (12%)	5 (4%)	4 (5%)	0 (0%)	3 (3%)	<0.01
Asthma, n (%)	9 (5%)	11 (9%)	5 (6%)	3 (20%)	9 (8%)	0.15
COPD**, n (%)	15 (8%)	2 (2%)	2 (3%)	0 (0%)	1 (1%)	<0.01
Chondrocyte tissue disease, n (%)	1 (1%)	1 (1%)	0 (0%)	0 (0%)	1 (1%)	0.93
Peptic ulcer, n (%)	5 (3%)	2 (2%)	1 (1%)	0 (0%)	2 (2%)	0.92
Liver (non-cirrhotic), n (%)	12 (6%)	9 (8%)	7 (9%)	3 (20%)	5 (4%)	0.20
Liver [cirrhotic]**, n (%)	2 (1%)	2 (2%)	5 (6%)	0 (0%)	1 (1%)	<0.05

29 April 2020

Imperial College COVID-19 response team

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Logistic regression of odds of death by ethnicity						
Unadjusted OR (95%CI)	Intercept	1.14 (0.69, 1.88)	1.10 (0.62, 1.96)	0.66 (0.18, 2.42)	0.89 (0.52, 1.50)	---
Adjusted OR (95%CI) *	Intercept	1.86 (1.03, 3.35)	1.74 (0.90, 3.36)	1.72 (0.42, 7.01)	1.73 (0.94, 3.18)	---

... Example 2

Perez-Guzman et al. 2020

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adj. odds ratio: 1.83

p=0.04

odds ratio: 1.14

p=0.62

Examples 1 and 2

unadjusted for confounders:

Example 1: ANDROMEDA trial

	Died	Lived	
New	74 34%	138	212
Old	92 43%	120	212
Total	166	258	424

randomized controlled trial
sepsis very difficult to treat
 $p = 0.06$

Example 2: IC Report 17

	Died*	did not**	
White	54 28%	142	196
Black	35 30%	81	116
Total	89	223	312

*in hospital; **unresolved or discharged alive

observational study
large number of potential confounders
 $p = 0.67$ (0.04)

“ From now on, BASP is banning the NHSTP” [Null Hypothesis Significance Testing]

p -values, confidence intervals, ...



The screenshot shows the homepage of the journal 'Basic and Applied Social Psychology'. On the left is the journal cover, which features the title 'BASIC AND APPLIED SOCIAL PSYCHOLOGY' and a silhouette of a person's head. A red circular badge on the cover indicates a '2018 Impact Factor 1.047'. To the right of the cover, the journal title 'Basic and Applied Social Psychology' is displayed in large, bold, black font. Below the title is an orange button with a lock icon and the text 'Publish open access in this journal'. A search bar with the placeholder text 'Enter keywords, authors, DOI, ORCID etc' is positioned below the title. To the right of the search bar is a dropdown menu labeled 'This Journal' and a blue search button with a magnifying glass icon. Below the search bar and dropdown menu is a link to 'Advanced search'. At the bottom of the page are five buttons: 'Submit an article' (green), 'New content alerts' (blue with an envelope icon), 'RSS' (blue with an RSS icon), 'Subscribe' (green), and 'Citation search' (blue with a quote icon).



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AMERICAN STATISTICAL ASSOCIATION RELEASES STATEMENT ON STATISTICAL SIGNIFICANCE AND P -VALUES

*Provides Principles to Improve the Conduct and Interpretation of Quantitative
Science*

March 7, 2016

nature > nature human behaviour > comment > article



nature
human behaviour

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Comment | Published: 01 September 2017

Redefine statistical significance

Daniel J. Benjamin , James O. Berger, [...] Valen E. Johnson

Nature Human Behaviour 2, 6–10(2018) | [Cite this article](#)

43k Accesses | **561** Citations | **887** Altmetric | [Metrics](#)

We propose to change the default P-value threshold for statistical significance from 0.05 to 0.005 for claims of new discoveries.



P-Values on Trial: Selective Reporting of (Best Practice Guides Against) Selective Reporting

by Deborah Mayo

outlines a 2018 Supreme Court case appealing a conviction for wire fraud,
based on misleading investors

Harkonen v. United States 13-180

the fraud centered on p -hacking the results of a Phase III trial of a drug

marketed by Harkonen

in the appeal “his defenders argued that the **ASA guide provides** compelling new
evidence that the scientific theory upon which petitioner’s conviction was based [that of
statistical significance testing] is demonstrably false”

What to do?

- report actual p -value, not “*”, $p < 0.05$, etc. to sensible number of decimal points
- supplement p -value with sample size, estimated power, etc.
- clarify ‘exploratory’ and ‘confirmatory’ p -values Spiegelhalter 2017
- report effect sizes and estimated standard errors
- report confidence intervals
- pre-register trials, specifying primary and secondary outcomes
- pre-specify data analysis NEJM
- provide a p -value function significance function
- or some analogous distribution Bayes posterior

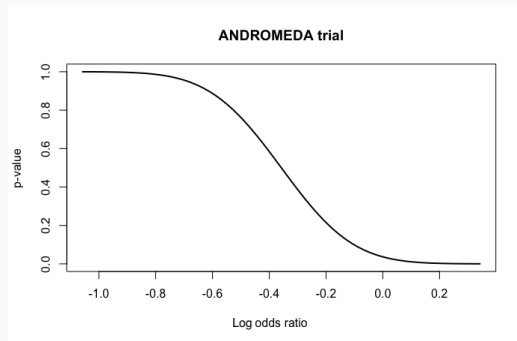
Distributions for parameters

ANDROMEDA trial

	Died	Lived	
New	74	138	212
Old	92	120	212
Total	166	258	424

2-sided p -value = 0.07

likelihood ratio test
no adjustment for covariates

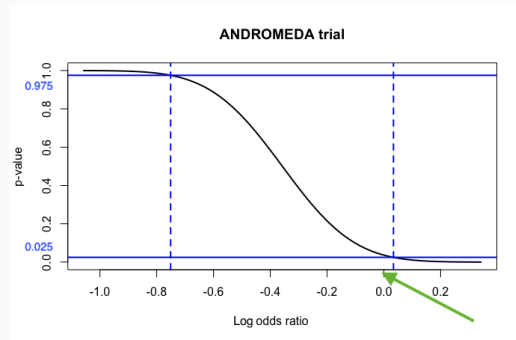


ANDROMEDA trial

	Died	Lived	
New	74	138	212
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Total	166	258	424

2-sided p -value = 0.07

likelihood ratio test
no adjustment for covariates



90% confidence interval: $[-0.688, -0.030]$

95% confidence interval: $[-0.751, 0.034]$

99% confidence interval: $[-0.825, 0.107]$

Distributions for parameters

- significance function

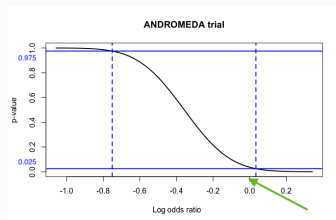
$$p(\theta) = \Pr(y \geq y^o \mid \theta)$$

- confidence distribution

set of all confidence intervals

- fiducial probability

inversion of data-generating equation



- structural probability

a re-formulation of fiducial probability for transformation models

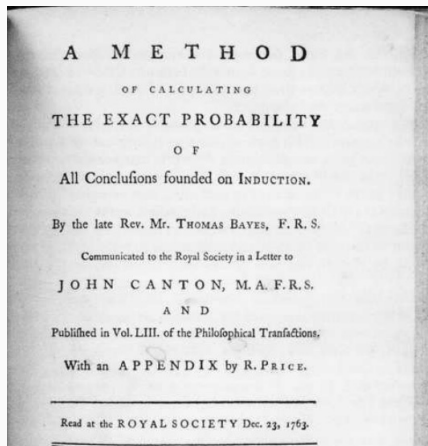
Fraser 1966

- belief functions

upper and lower probabilities

Dempster '66; Schafer '76

In spite of the naming, these are not 'real' probability distributions



$$\pi(\theta | y^o) = f(y^o; \theta)\pi(\theta)/m(y^o)$$

a 'real' probability distribution for θ

y^o is fixed

probability comes from $\pi(\theta)$

$$\Pr(\Theta \in A | y^o) = \int_A \pi(\theta | y^o) d\theta$$

Stigler 2013

Why do we want distributions on parameters?

- inference is intuitive
- combines easily with decision theory
- de-emphasizes point estimation and arbitrary cut-offs
- “it’s tempting to conclude that μ is more likely to be near the middle of this interval, and if outside, not very far outside”

Cox 2006

- “assigns probability 0.05 to θ lying between the upper endpoints of the 0.90 and 0.95 confidence intervals, etc.”

Efron 1993

- all inference statements are **seem to be** probability statements about unknowns



The First Workshop on BFF Inference and Statistical Foundations (BFF 2014)

November 10 – November 14, 2014

7th Bayes, Fiducial and Frequentist Statistics Conference

Methodological, Computational, and Ethical Principles for Data Science

May 6 - 8, 2020, [The Fields Institute](#)

Location: Fields Institute, Room 230



Objective Bayes

Objective Bayes

- there are many proposals for priors meant to be non-informative
- examples include reference, default, matching, vague, ... priors
- a popular choice is Jeffreys' prior $\pi(\theta) \propto |i(\theta)|^{1/2}$ expected Fisher information
- some versions may not be correctly calibrated e.g. Jeffreys'
- requires checking in each example
- calibrated versions must be targetted on the parameter of interest Fraser 2011
- only in very special cases can calibration be achieved for more than one parameter in the model, from the same prior
- the simplicity of a fully Bayesian approach to inference is lost

- the simplicity of a fully Bayesian approach to inference is lost
- for example

Gelman 2008

$$\pi(\psi | y) = \int_{\psi(\theta)=\psi} \pi(\theta | y) d\theta, \quad \text{for any } \psi : \Theta \rightarrow \Psi$$

lower dimension

- the prior can have unexpected influence on the posterior
- even if they are seemingly noninformative

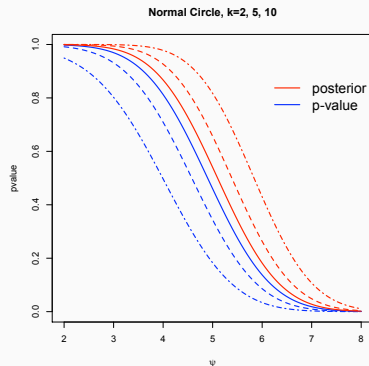
objective Bayes fails

- Stein's example:

$$\begin{aligned} y_i &\sim N(\theta_i, 1/n), \quad i = 1, \dots, k \\ \pi(\theta_i) &\propto 1 \\ \pi(\theta | y) &\propto N(y, I_k/n) \end{aligned}$$

- $y_i \sim N(\theta_i, 1/n)$, $i = 1, \dots, k$; $\pi(\theta_i) \propto 1$
- posterior distribution of $a^T \theta$ is well-calibrated
- marginal posterior distribution of $\psi = \sum \theta_j^2$ is not
- discrepancy is a function of $\frac{k-1}{\psi \sqrt{n}}$
- $p(\psi) = \Pr\{\chi_k^2(n\psi^2) \geq n\|y\|^2\}$

$$s(\psi) = \Pr\{\chi_k^2(n\|y\|^2) \geq n\psi^2\}$$



True prob* 0.001 0.01 0.025 0.05

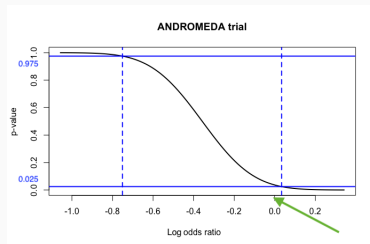
Bayes prob* 10^{-6} 10^{-4} 0.001 0.003

* left tail

Targetting on parameter

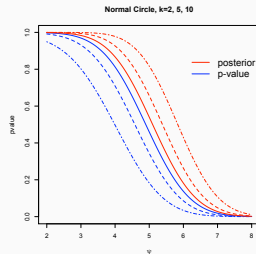
Example 1: 2×2 table

Based on conditional distribution of odds-ratio,
given marginal totals



Example 3: $y_i \sim N(\theta_i, 1/n)$

Based on marginal distribution of Σy_j^2



High-dimensional inference and model selection

- $Y_i \sim N(\beta_i, 1), \quad i = 1, \dots, n$

$$y_i \sim N(\theta_i, 1/n), \quad i = 1, \dots, k$$

- prototype for nonparametric modelling
- sparsity assumption needed on $\beta = (\beta_1, \dots, \beta_n)$
- special case of high-dimensional regression

$$y = X\beta + \epsilon, \quad \epsilon \sim N_n(\mathbf{0}, I), \quad \beta \in \mathbb{R}^p, \quad p \gg n$$

- can we find an objective Bayes prior?
- or at least a prior leading to frequentist coverage of posterior credible intervals

- $Y = X\beta + \epsilon$, $\epsilon \sim N_n(\mathbf{0}, I)$, $\beta \in \mathbb{R}^p$, $p \gg n$
- assumption of **sparsity** – many components of β are 0

Lasso

- prior

$$\pi(\beta, S) \propto \pi_p(|S|) \frac{1}{\binom{p}{|S|}} g_S(\beta_S) \delta_0(\beta_{S^c})$$

- prior specification first on **dimension**:

$$\pi_p(|S|) \propto (cp^a)^{-|S|}$$

- then on **subset** $S \subset \{1, \dots, p\}$, given dimension: $\frac{1}{\binom{p}{|S|}}$

- finally on $\beta_S = \{\beta_j, j \in S\}$, given S :

$$\beta_j \sim \text{i.i.d. Laplace}(\lambda), \quad j \in S$$

- under conditions on design matrix X

$$\inf \left\{ \frac{\|X\beta\|_2}{\|X\| \|\beta\|_2} : |S_\beta| \leq s \right\} > 0$$

- and on the scale parameter in the Laplace prior

$$\frac{\|X\|}{p} \leq \lambda \leq 2\|X\|(\log p)^{1/2}$$

- obtain various consistency results on posterior estimates of $|S|$, S and β

- in particular, Bayesian credible sets for β are well-calibrated

- special case $n = p, X = I$: “sequence model” $Y_i \sim N(\beta_i, 1), \quad i = 1, \dots, n$

Stein's example

- **Lasso posterior not useful for inference**

Lasso point estimate is posterior mode (from Laplace prior)

- $y_i \sim N(\beta_i, 1)$, $i = 1, \dots, n$; β sparse

nonparametric Bayes

- prior specification first on dimension s , then on subset $S \subset \{1, \dots, p\}$ with $|S| = s$, finally on β_S

$$\pi(\theta, S) \propto \pi_p(|S|) \frac{1}{\binom{p}{|S|}} g_S(\beta_S) \delta_0(\beta_{S^c})$$

- as above

$$\pi_p(|S|) \propto (cp^a)^{-s}, \quad \text{but } \beta_S \sim \text{Laplace} \rightarrow N(Y_S, \sigma^2 \tau^{-1} I_{|S|})$$

- and tempered likelihood

generalized Bayes

$$\pi(\beta, S | y) \propto \{L_n(\beta_S; y)\}^\alpha \pi(\beta, S)$$

- posterior coverage for linear functions of β

- van der Pas et al 2018: **horseshoe prior** $\theta_i \sim N(0, \nu_i^2 \tau^2)$, $\nu_i \sim C^+(0, 1)$
 τ hyperparameter

- Bhadra et al 2016: **horseshoe plus** with hyperparameters e.g. $\nu_i \sim C^+(0, \tau \eta_i)$
 nonlinear marginals

- Miller & Dunson 2017: c-posterior based on tempered likelihood a a hyper-parameter

$$\pi^c(\theta | y) \propto \{L(\theta; y)\}^{a/(a+n)} \pi(\theta)$$

- Grünwald and van Ommen 2018: ‘generalized posterior’ large ML literature

$$\pi(\theta | y) \propto \{L(\theta; y)\}^{\eta_n} \pi(\theta), \quad \eta < 1$$

- Giordano et al. 2018: ‘local robustness’ and mean-field variational Bayes
 a link to frequentist properties of Bayes estimates; Efron 2015

Summary

- dichotomizing conclusions based on p -values is not a good idea
- statistical science is more nuanced than that
- science rarely advances on the basis of a single study
- posterior distributions need to be treated with care
- they can depend heavily on the prior, even when it seems uninformative
- calibrated inference in high-dimensional models still a w.i.p.
- considerable scope for increased interactions between ML and Stat

Thank you!



