Risk and Insurance Studies Centre

Student Research Competition

RISC SRC 2021

RISC SRC Committee

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Submissions should be sent by October 15, 2021, to src@riscinternational.org from a university e-mail address, be in one file and in pdf, be prepared in LaTeX, have a cover page with the name, university, year of study, and the university e-mail address of the submitter.

Fairly complete solutions – after presentations via video conference call – will receive RISC SRC 2021 Diplomas and, when appropriate, endorsements to apply to our graduate programs.

Undergraduate and Master's students are especially encouraged to participate.

Problems

Problem 1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. For any constant $p > 0$ and non-negative random variable $X \geq 0$ with $\mathbb{E}(X^p) < \infty$, prove the equation

$$\mathbb{E}(X^p) = \int_{[0,\infty)} \mathbb{P}(X > x) p x^{p-1} \mathrm{d}x.$$

Will the equation remain true if we replace $\mathbb{P}(X > x)$ by $\mathbb{P}(X \geq x)$ on the right-hand side? Now, instead of the function $h(x) = x^p$, consider an arbitrary absolutely-continuous function h. Does a similar representation hold for $\mathbb{E}(h(X))$?

Problem 2. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be an atomless probability space and $B(\Omega)$ be the space of all bounded random variables on Ω. Let $\rho : B(\Omega) \to \mathbb{R}$ be a mapping such that the following four conditions hold:

1. $\rho(X) \geq 0$ if $X \geq 0$
2. $\rho(\alpha X + \beta Y) = \alpha\rho(X) + \beta\rho(Y)$ for any $X, Y \in B(\Omega)$ and $\alpha, \beta \in \mathbb{R}$
3. $\rho(X) = \rho(Y)$ if $X, Y \in B(\Omega)$ have the same distribution
4. $\rho(\mathbf{1}_\Omega) = 1$, where $\mathbf{1}_\Omega$ is the indicator of the set Ω

Show that

$$\rho(X) = \mathbb{E}(X) \quad \forall X \in B(\Omega).$$

Problem 3. Let $X : \Omega \to \mathbb{R}$ be an absolutely-continuous random variable, that is, its distribution $\mathbb{P} \circ X^{-1}$ has a density f_X with respect to the Lebesque measure. Furthermore, let[1]

$$Y = (X - d)^+,$$

where $d \in \mathbb{R}$ is a constant and x^+ denotes the positive part of $x \in \mathbb{R}$, that is, $x^+ = x$ when $x > 0$ and $x^+ = 0$ when $x \leq 0$. Determine whether or not the random variable Y has a density with respect to the Lebesgue measure, and if/when it has, express the density f_Y of Y in terms of the density f_X.

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[1] The random variable $Y = (X - d)^+$ shows up prominently in financial and insurance risk measurement and management, where it is called "shortfall", with d carrying – especially in insurance – the connotation of "deductible."