Some thoughts on model-based priors

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Thursday Nov 17 2005
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Some thoughts on model-based priors

Model-based prior? Sort of a contradiction

A | affiliation of statisticians...

B | Prior

C | Model

D | Inference

E | B-f interplay

F | B-f exclusiveness

G | Wrap up

A Large affiliation of statisticians...

Persuasion.... "<u>model-based priors</u>"

<u>Name</u>: invariant, default, flat, non-subjective, objective ?

Huge 'industry', committed, power, ...

Believe in 'Cause'; spin; turf

Dare frequentists ignore ? ... Not an idle question !

A Large affiliation of statisticians...

Persuasion.... <u>model-based</u> priors

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"They" think frequentists are <u>asleep</u>: "lost their way!"

Belief
Mystique
Pontification
Statistical version of ID ... this isnt Kansas !

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Persuasion.... model-based priors

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$$\left| \begin{array}{ll} f(y;\theta) & \pi(\theta) \\ y^0 & \end{array} \right\}$$ Isn't this just "frequentist"?

Welch & Peers 1963
Re-designing "frequentist"

B Priors

Classify:

1) Objective
2) Subjective
3) Model-based

B Priors

Classify:

1) Objective → There really is a $\pi(\theta)$ distribution

Fisher 1956 SMSI .... examples

... and many before

Just prob. modeling & conditional prob. !

Hadn't been addressed as such !

Brad Efron Nov 9 at U of T

"50 years of empirical Bayes"

All examples: an objective distribution!

Bayesian ? How so ?

Why "new" ?

B Priors

Classify:

1) Objective

2) <u>Subjective</u> Jimmy Savage 1954 ... but

3) Model-based

B Priors

Classify:

1) Objective

2) Subjective

3) Model-based

Bayes 1763
Laplace 1812
Welch & Peers 1963
Neo-Bayesian Bernardo 1979 DSZ 1973

Name?? Evolutionary....

Invariant
Default
Non-subjective
Objective Call "black white"
Obfuscation
Pre-empt....

Just use $L^{\circ}(\theta)$ and weight it F AMS 1972

but then... frequentists neglected $L^{\circ}(\theta)$
Stanford 1962

B Priors

Classify:

1) Objective

2) Subjective

3) Model-based Neo-Bayesian Just use $L^0(\theta)$ and weight it

Method:

$y ; \theta$ $\quad f(y;\theta)$

y^0

θ $\quad \pi(\theta)$

frenquentist ?

B Priors

Classify:

1) Objective
2) Subjective
3) <u>Model-based</u> <u>Neo-Bayesian</u> Just use $L^{\circ}(\theta)$ and weight it

<u>Method:</u>

$y;\theta$ $\quad f(y;\theta)$

y°

θ $\quad \pi(\theta)$

frenquentist?

<u>Credo:</u>

Prior of convenience....

Use algorithm

and how the algorithm processes information

- Information processing

Eschew ..... <u>any</u> <u>other</u> <u>model</u> <u>structure</u> ! More than "Ety..." shy !

Belief : <u>Suppress</u> things you <u>know</u> Neo-science ID

Laugh at "f" afraid to use $L^{\circ}(\theta)$

C <u>Models</u>

Assume: $f(y;\theta)$ .... in <u>all</u> Bayesian analysis

in <u>some</u> frequentist "

scalar fn on $\{y\} \times \{\theta\}$

How much model info is <u>used</u> / <u>allowed</u> ? .... in Bayesian context ?

Plot $f(\cdot;\cdot)$ over $S \times \Omega$ $\rightarrow$

C Models

Assume: $f(y;\theta)$ .... in all Bayesian analysis

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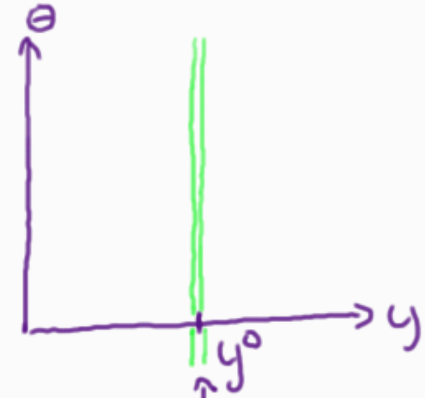
scalar fn on $\{y\} \times \{\theta\}$

How much model info is used / allowed ..... Bayesian context?

Plot $f(\cdot;\cdot)$ over $S \times \Omega$ →



Bayesian uses →

L° .... y°-section of model

plus info-processing .... IP Zellner

ie How model processes $\pi(\theta) \to \pi(\theta|y)$

$$\pi(\cdot) \longrightarrow \boxed{} \longrightarrow \pi(\cdot|y)$$

K-L distance

Only model structure to be used ???

D Inference

$f(y;\theta) + y^o \Rightarrow$?

Case: Scalar y & θ ... Where Things should start!

Obvious: $f^o = f(y^o;\theta)$ $\quad L^o(\theta)$ $\quad$ Prob at

$F^o = F(y^o;\theta)$ $\quad p^o(\theta)$ $\quad$ Prob left

Is there anything more?

D Inference

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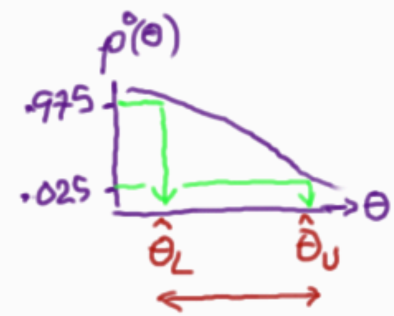
Is there anything more ?

$u = F(y;\theta)$ is a pivot

Invert; get all confidence bounds, conf. int's

$95\%\ CI = \{p^{-1}(.975),\ p^{-1}(.025)\}$



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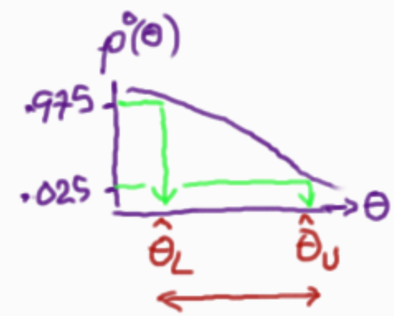

you better use it or you'll lose it

There is an end run in play !! $\rightarrow L^o(\cdot)$

...think bigger

D Inference

$f(y;\theta) \;+\; y^\circ \;\Rightarrow\; ?$

Case: Scalar y & θ ... Where Things should start!

obvious: $f^\circ = f(y^\circ;\theta)$ $\quad L^\circ(\theta)$ $\quad$ Prob at

$\qquad\qquad F^\circ = F(y^\circ;\theta)$ $\quad p^\circ(\theta)$ $\quad$ Prob left

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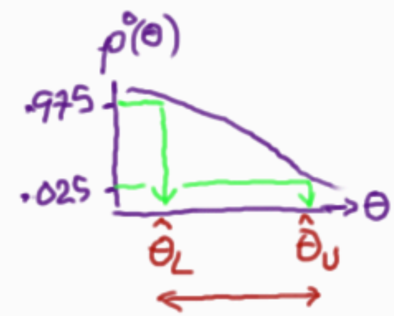
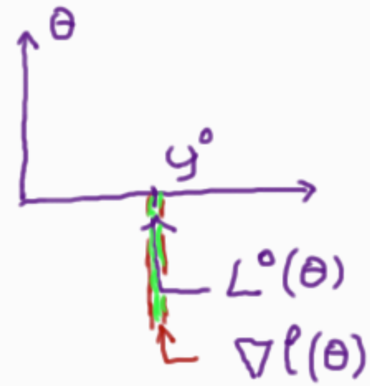


you better use it or you'll lose it

There is an end run in play!! $\rightarrow L^\circ(\cdot)$

Can do in general;

Min. assumptions!



$\ell(\theta)$

$\varphi(\theta)$

you can answer questions
the Bayesians can't even get near!

E B-f interplay

1958 Lindley

Case: y and θ scalar

When is Confidence distribution = Posterior distribution ?

$$p_\theta(\theta) = L(\theta)\,\pi(\theta)\,/\,m(y)$$

$$-F_{;\theta}(y;\theta) = F_y(y;\theta)\,\pi(\theta)\,/\,m(y)$$

Fear
Stigma
real

elim.
nuis.

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$$-\frac{F_{;\theta}}{F_y} = \frac{dy}{d\theta} = \frac{\pi(\theta)}{m(y)}$$

Diff thro' pivot

$$m(y)\,dy = \pi(\theta)\,d\theta$$

$$\tilde{y} = \tilde{\theta} + z \qquad \text{... ie Location model}$$

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Lindley: Carping at upstart Fisher ('30) .... "Conf. is wrong"

Real message: Global Bayes is wrong

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Just examine near data point!

Answer: Data-dependent priors

- Box and Cox
- David C here at Waterloo
- Wasserman ... and many more

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Locally (near data)

- Model is location
- f & B agree ! ... local flat prior

Can't use just $\ell(\theta)$

Also need $\varphi(\theta) = \nabla \ell(\theta; y^0)$

$\{\ell(\theta), \varphi(\theta)\}$

F B-f exclusiveness

Bayesian : Always use $\mathcal{L}^o(\theta) = f^o(\theta)$ - - - - - - - - - - and only $\mathcal{L}^o(\theta)$

and conditional probability - - - - - - - - - - plus MCMC (Nuclear power)

frequentist : Ignores $\mathcal{L}^o(\theta)$

- global probabilities - - - easy !
- novel - - - "theory" bankrupt

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Two solitudes !
... in fundamental principles
... just playing 'their own game' fair enough

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but Is what you know from an investigation a game ?

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Two solitudes ! ... in fundamental principles
... just playing 'their own game' fair enough

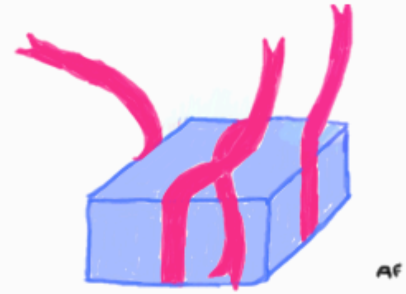
but... Is what you know from an investigation "a game" ?

- I use ONLY "likelihood" .... fully condition
- I use "global" probabilities".... never condition

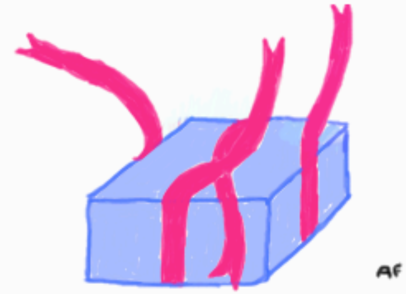
Sand box "TIFF"

... Let's play ... !

G Wrap Up:



G Wrap Up: What are options?



On,... High Principle

- Use only the observed likelihood $L^{\circ}(\theta)$ ---- Bayesian

or

- Use only "novel" & global probability assessments ---- frequentist

or

- Use likelihood $L^{\circ}(\theta)$ and recalibrate parameter $\varphi(\theta)$ and more!

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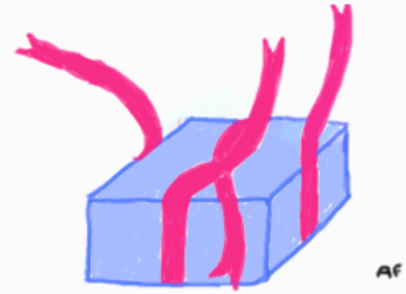

go for it … on High Principle

- use only the observed likelihood $L^\circ(\theta)$ ---- Bayesian

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- use only novel & global probability assessment ---- frequentist

or

- use likelihood $L^\circ(\theta)$ and calibrate parameter $\varphi(\theta)$ ---

Why exclusiveness? … Dumbing down? Turf?

Structure on parameter … $\varphi(\theta)$ | … go from 1st to 3rd & Deconstruct the model

An example: Q since 1929!

<u>Example</u> Behrens-Fisher 1929 1934 Ghosh Kim 2001

Example Behrens-Fisher 1929 1934 Ghosh Kim 2001

$y_{11}, \dots, y_{1n}$ N μ_1 σ_1^2

$y_{21}, \dots, y_{2n}$ N μ_2 σ_2^2

Interest $\psi = \mu_1 - \mu_2$

GK: $\pi = \sigma_1^{-3} \sigma_2^{-3} \left(\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m} \right)$ vs Jeffreys(right): $\sigma_1^{-1} \sigma_2^{-1}$ familiar?

Example Behrens-Fisher 1929 1934 Ghosh Kim 2001

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Some simulation numbers: Case

n	m	σ_1^2	σ_2^2	ψ
2	2	2	1	2

Nominal 5% 95% points by various methods including flat prior (data dependent!)

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Kim Ghosh	1.7%	97.9%
Lik ratio	13.2%	86.9%

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$p(\psi), s(\psi)$	4.23%	95.8%
Sim 95% limits	(4.86, 5.14)	(94.9, 95.14)

flat prior, data dep., $O(n^{-3/2})$, $B \equiv f$

N=100,000

Recall

1) Priors: Objective, Subjective, Model-based

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2) Implications: Use model-based <u>locally</u>: B & f agree to $O(n^{-3/2})$

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I only use 'novel', global prob's
I fully condition
I dont condition
!

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8) Model-context: at minimum, calculate $\ell(\theta)$, $\varphi(\theta)$.
   - Get 3rd order $p(\psi)$ for any nice $\psi(\theta)$ scalar
   - Get " " $L(\psi)$ " " " " scalar or vector
   - Get 2nd " $p(\psi)$ " " " " vector
   - Tell the neo-B's how they can do it right

H9

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Thank you ....