

On direction in statistics theory

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Statistical Sciences

U Toronto

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XC

Munk School of Global Affairs

www.utstat.toronto.edu/~dfraser/documents/ut2015.pdf

Anecdote

Why interested?

Where are we now?

What can we do?

Use Tangent Exponential Model

Consequences

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Big vs. Big Scientist, ego,
direction in statistics, in science

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Spreading centers Science 2015-02-06

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3) Bayes: Efron in Science 2013 pp1177-8
Laplace "troublesome"

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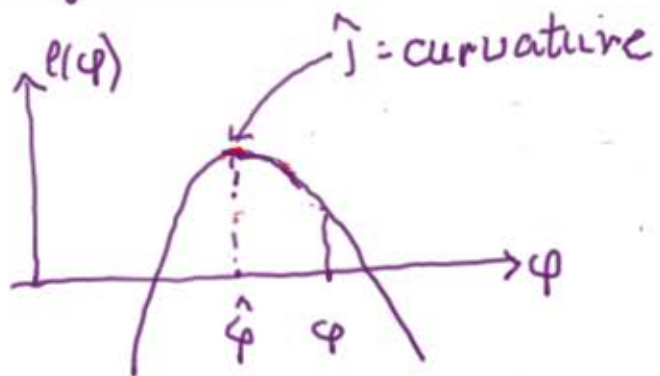
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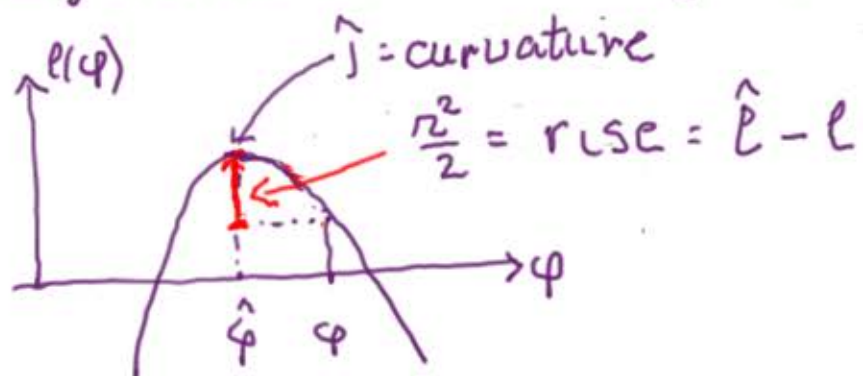
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