Continuity
and Statistical Methodology

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Statistics
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2nd Princeton Day of Statistics
Princeton
New Jersey

October 22 2010

http://www.utstat.toronto.edu/dfraser/documents/pu2010.pdf
... / xxx.pdf

1. Politics, Census, Accuracy 19/20

2. fermilab LHC Accuracy

3. Continuity: ... Scalar

4. ... Scalar & vectors

5. What does it do for you?

6. Example

A ⇒ Conditioning on data information

B ⇒ Default priors : Nec. & Suff conditions

C ⇒ Bayes & parameter linearity

10. Curvature & Directions

1. Politics Census Accuracy 19/20

Politics:

Statistics Canada.... "Compulsory Long Form"

downgraded to a survey!

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Surprising: Extreme: concern for Accuracy

that 19/20 should mean something!

that ·000013 should mean something!

That reported results --- not be just casual "exploring"!

2. fermilab LHC Accuracy

1994 Evidence for top quark HEP ... Discussion by Sineruo

Poisson (θ) $\theta \geq \theta_0$ representing background radiation

"New particle" $\longleftrightarrow$ $\theta > \theta_0$

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Why? $\dfrac{\text{Accuracy}}{\text{Quantiles}}$ Want Discover / Verify New particle

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3. Continuity:

Case: Scalar

Distn fn Continuous

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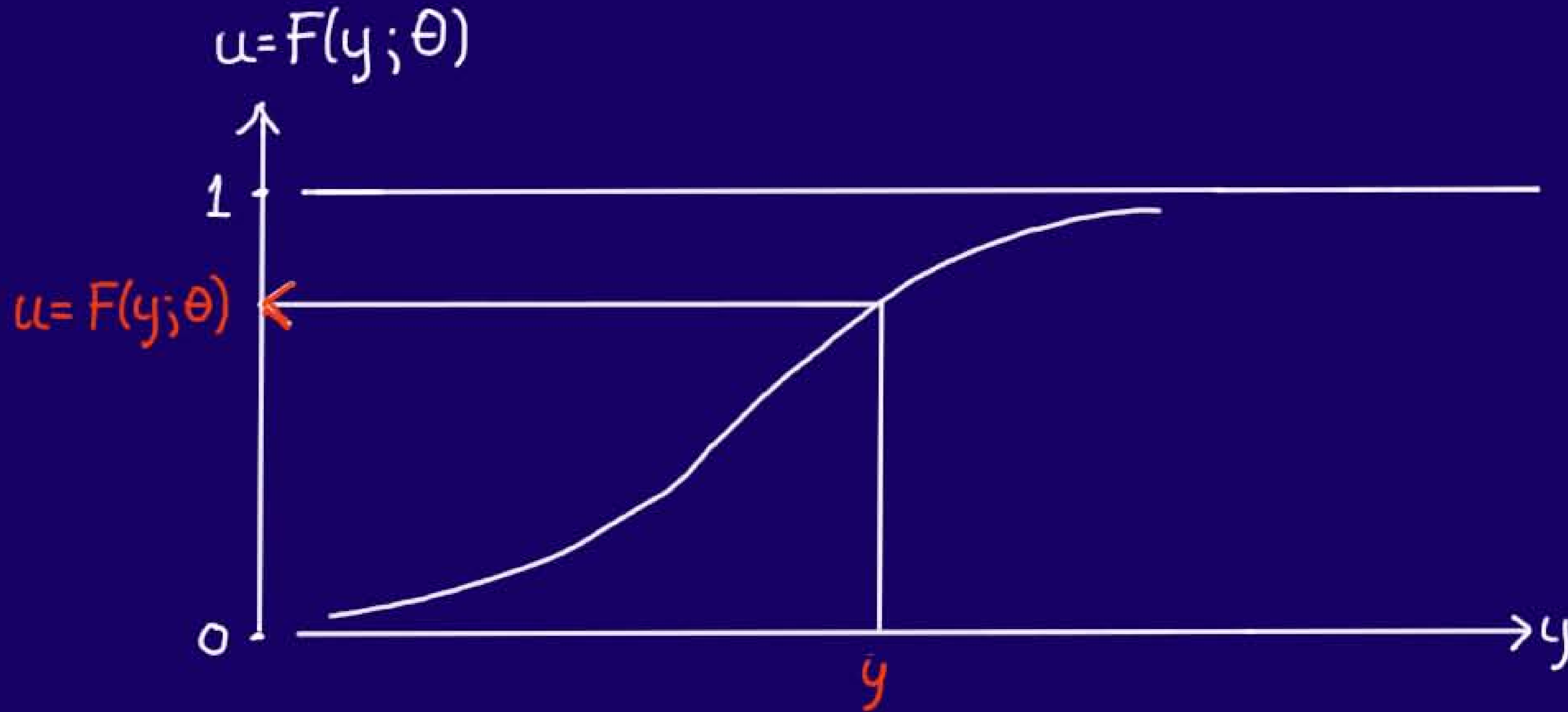
Distn fn Continuous



p-value dist'n fn

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Case: Scalar

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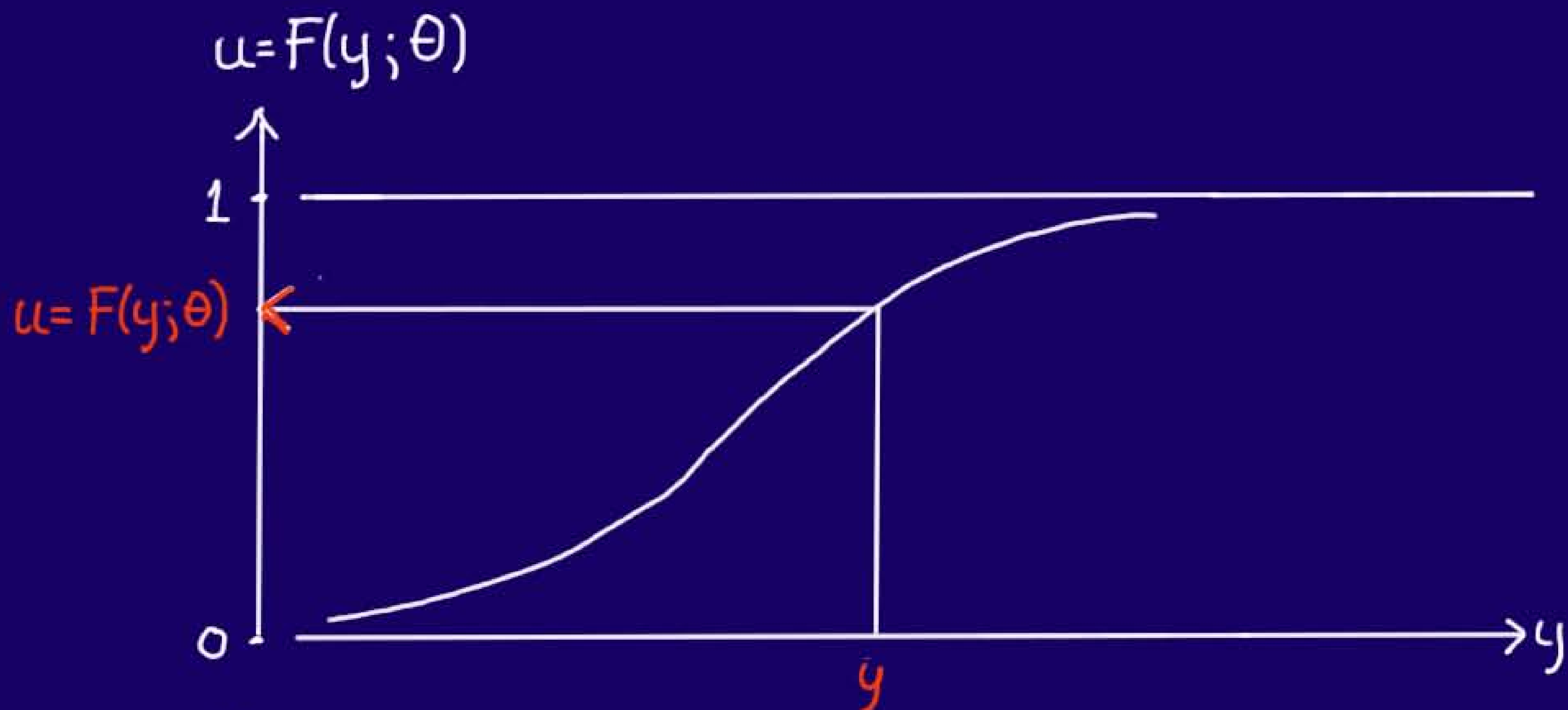
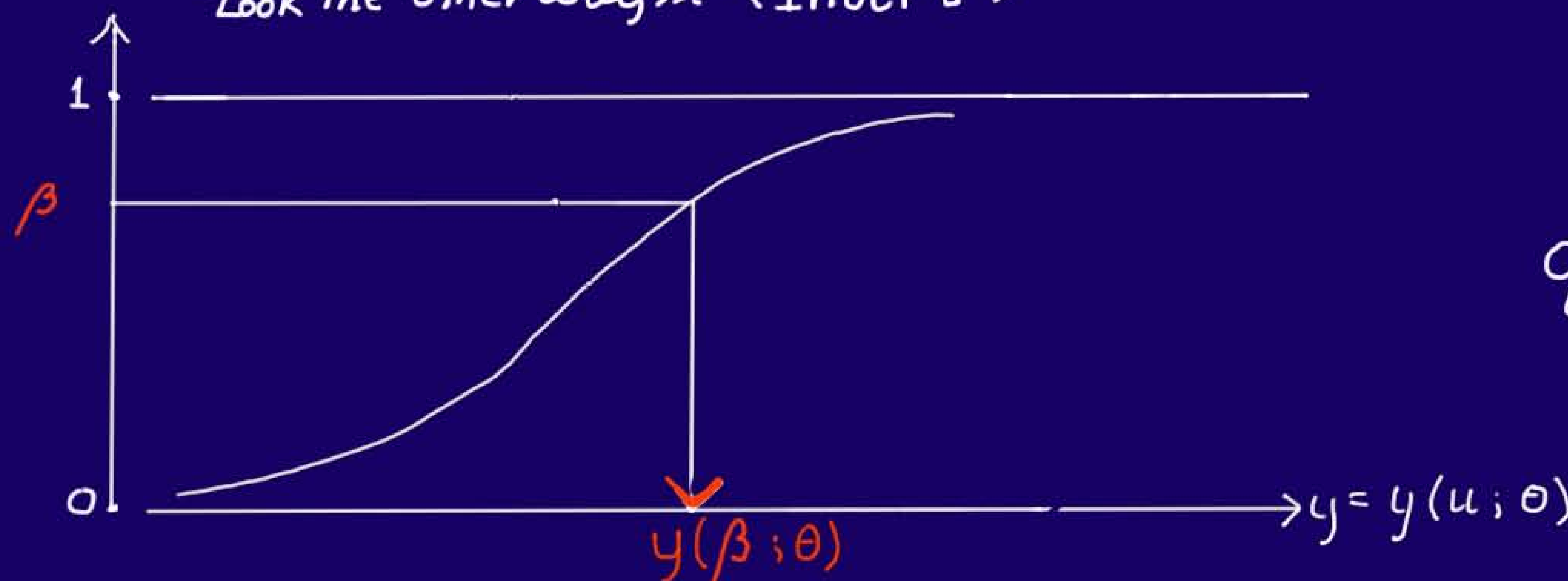


p-value dist'n fn

u

"Look the other way..." <Invert>

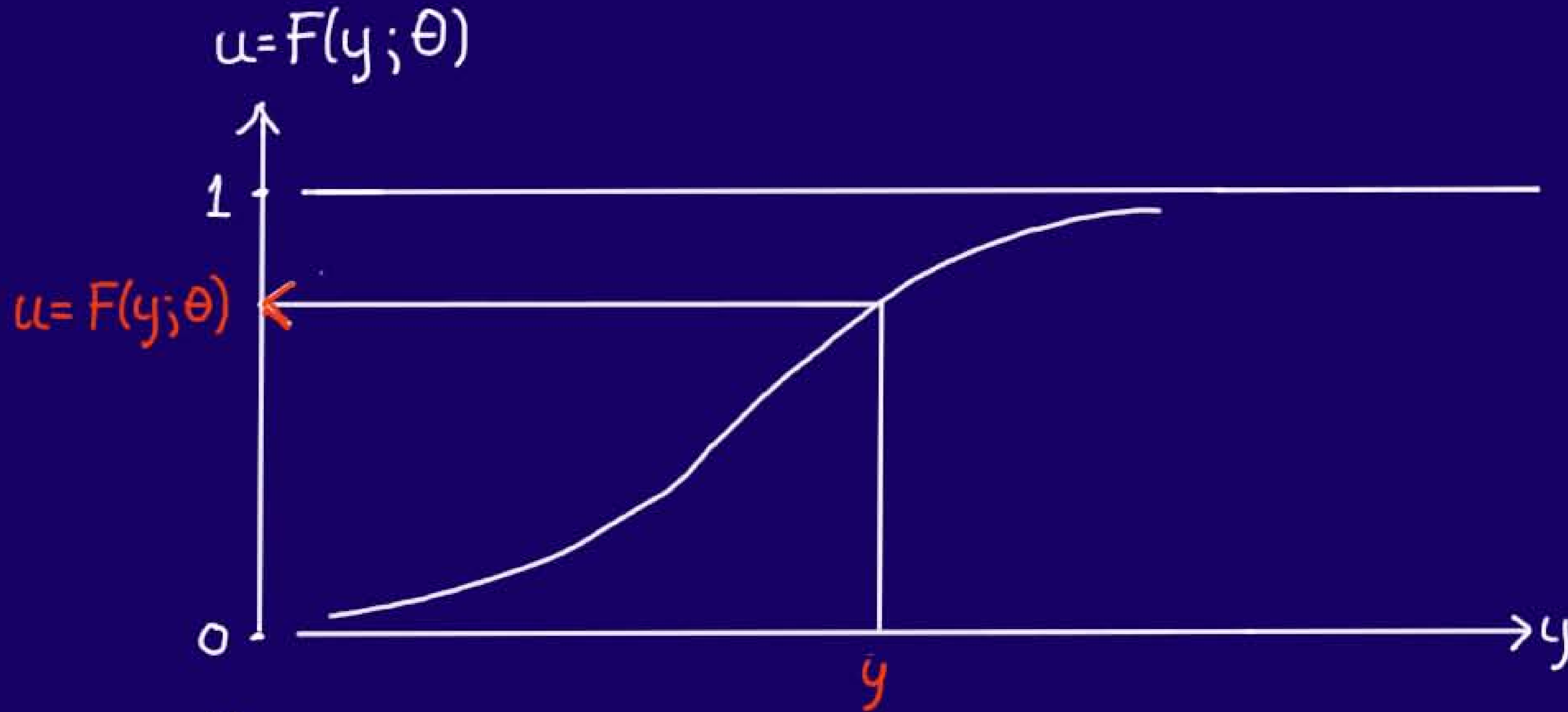


q-value quantile

3. Continuity:

Case: <u>Scalar</u>

Distn fn Continuous



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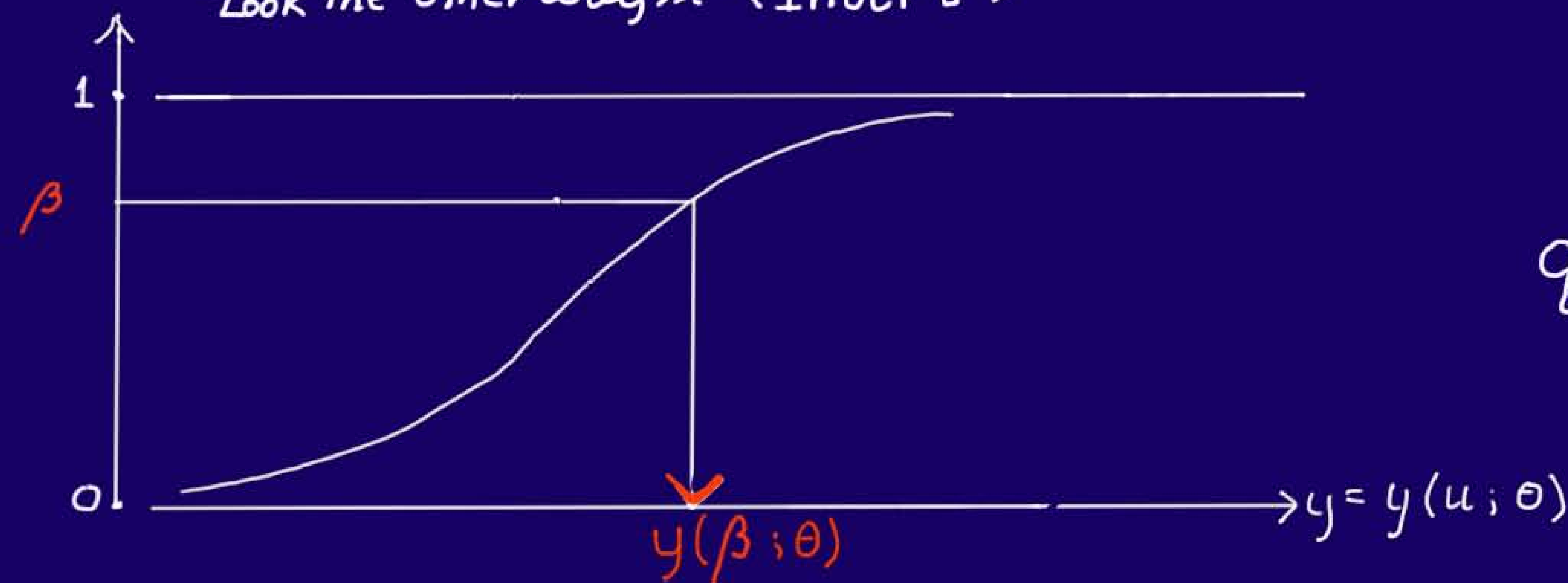

p-value dist'n fn

p-q or q-p plots

q-value quantile

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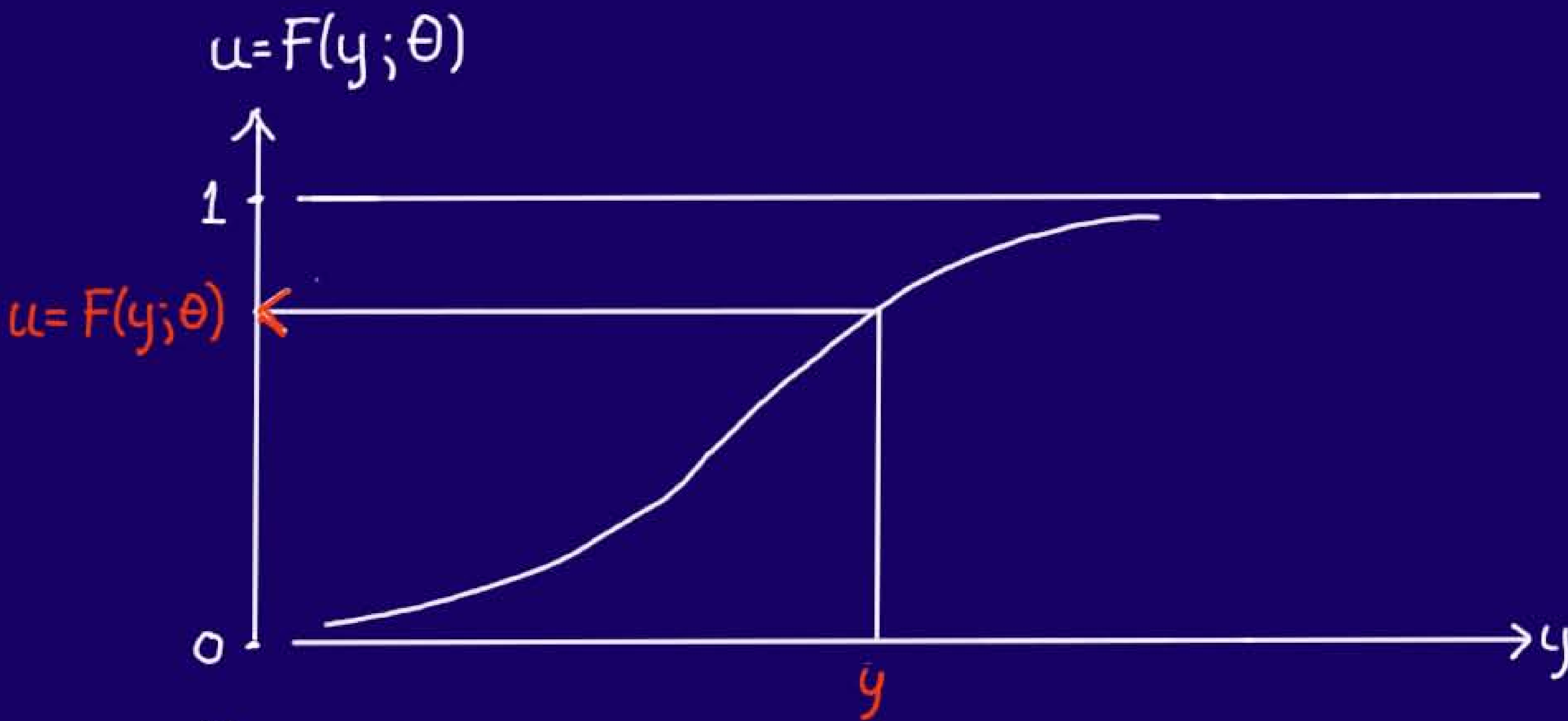

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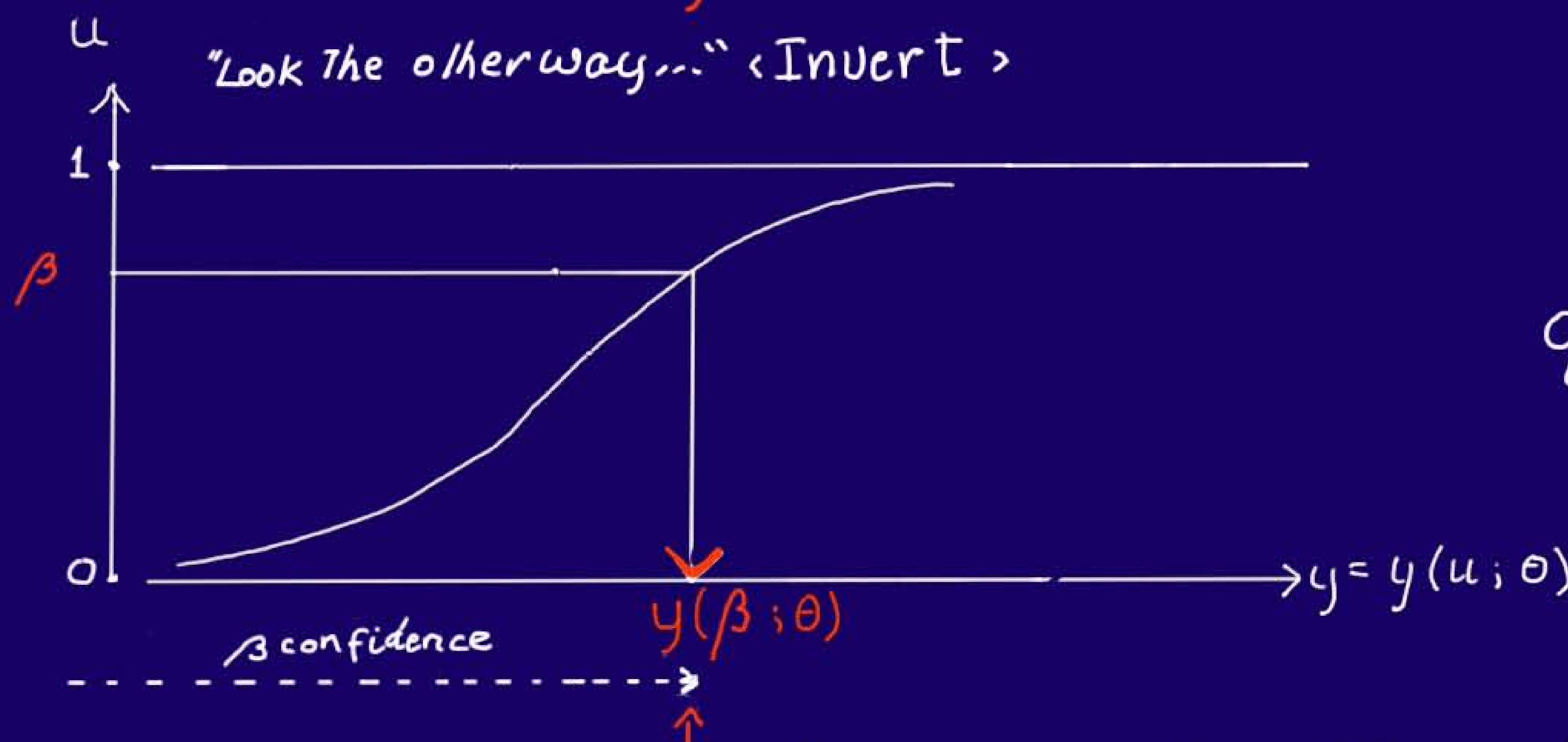


q-value ← quantile

β-confidence upper quantile

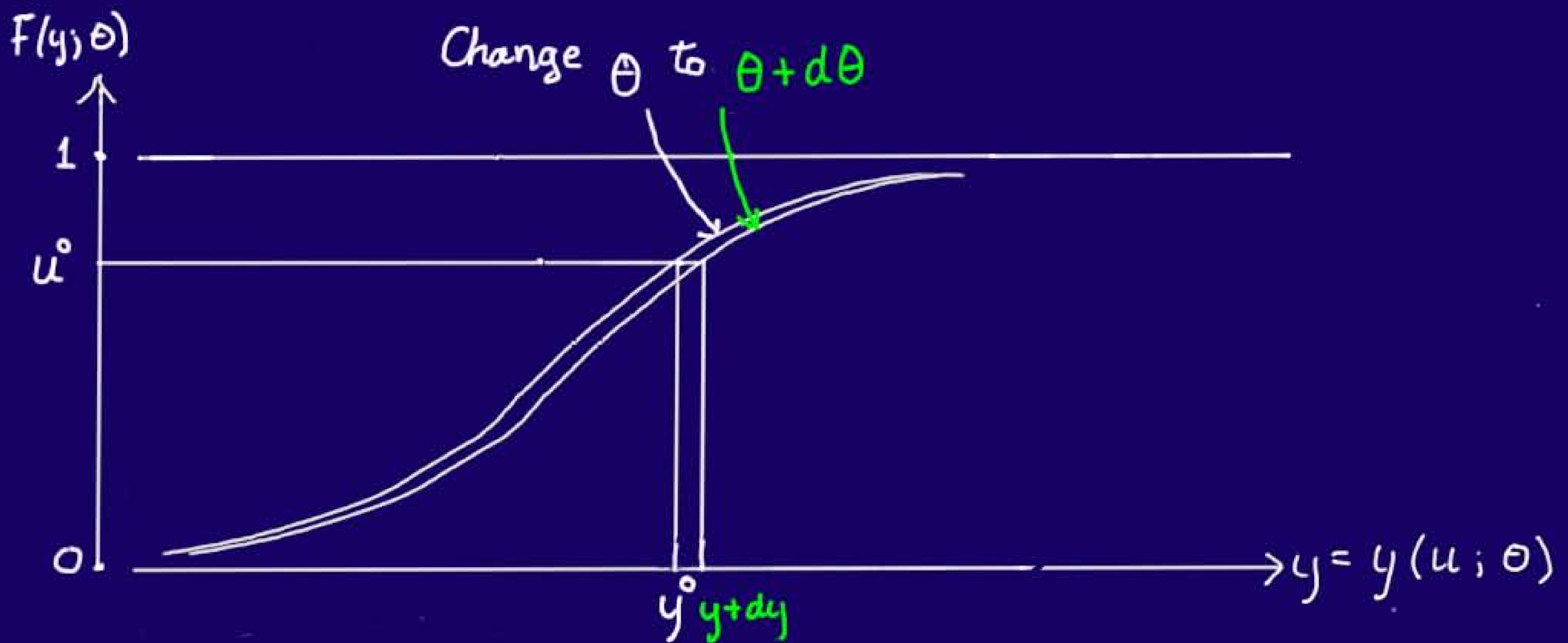
Quantiles ... of the essense

4. Continuity ... more

Case: Scalar and vector

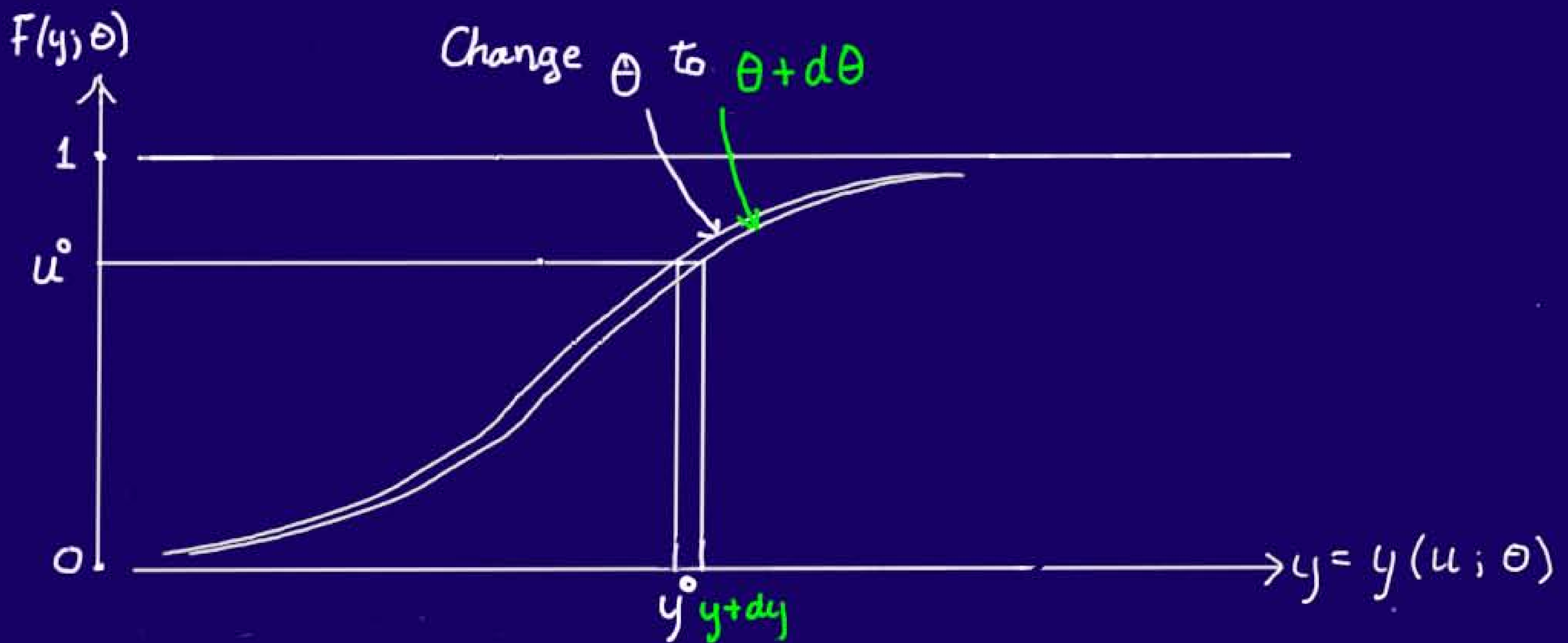
4. Continuity

Case: <u>Scalar</u> and <u>vector</u>



4. Continuity

Case: <u>Scalar</u> and <u>Vector</u>

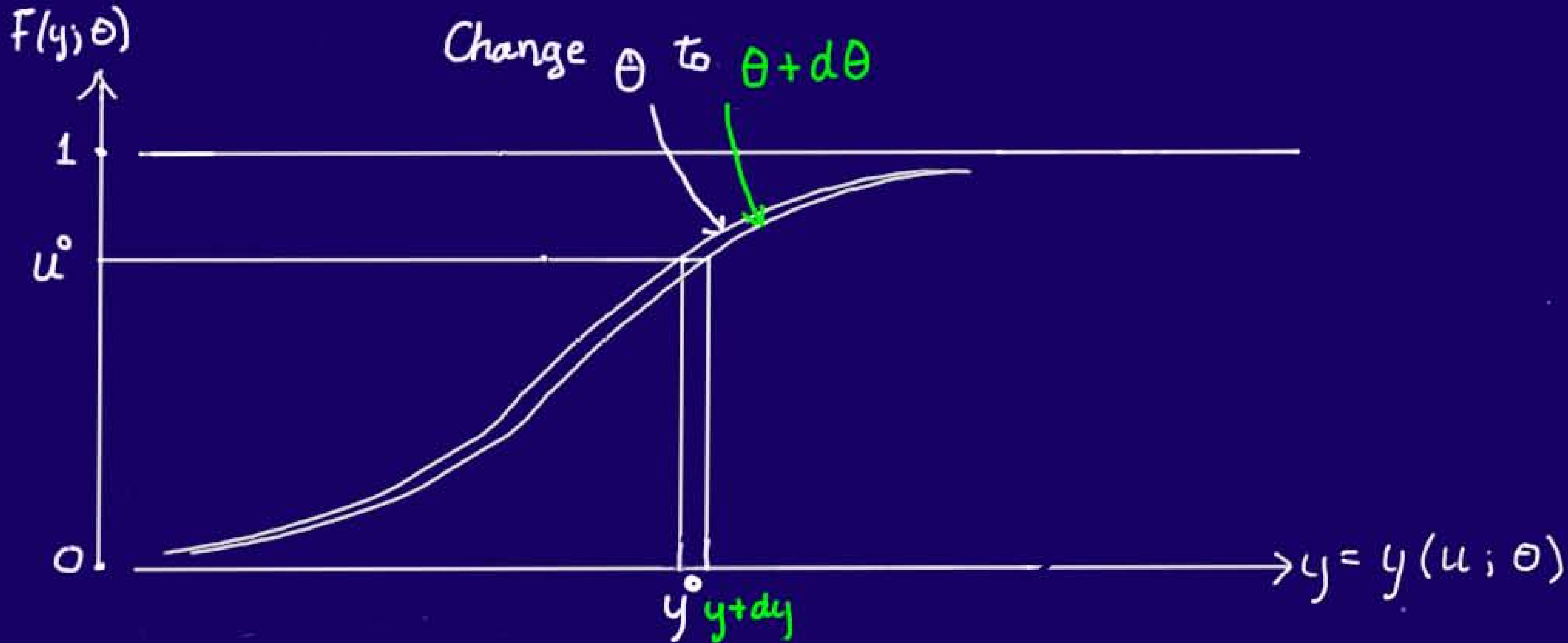


Fixed p-value u°
Change $\theta \to \theta + d\theta$

$$dy = \left.\frac{\partial y}{d\theta}\right|_{y^\circ} d\theta = v(\theta)\, d\theta$$

4. Continuity

Case: Scalar and vector



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Call it ?

$v(\theta) =$ velocity ?
sensitivity ?
"How θ moves data ?"

4. Continuity

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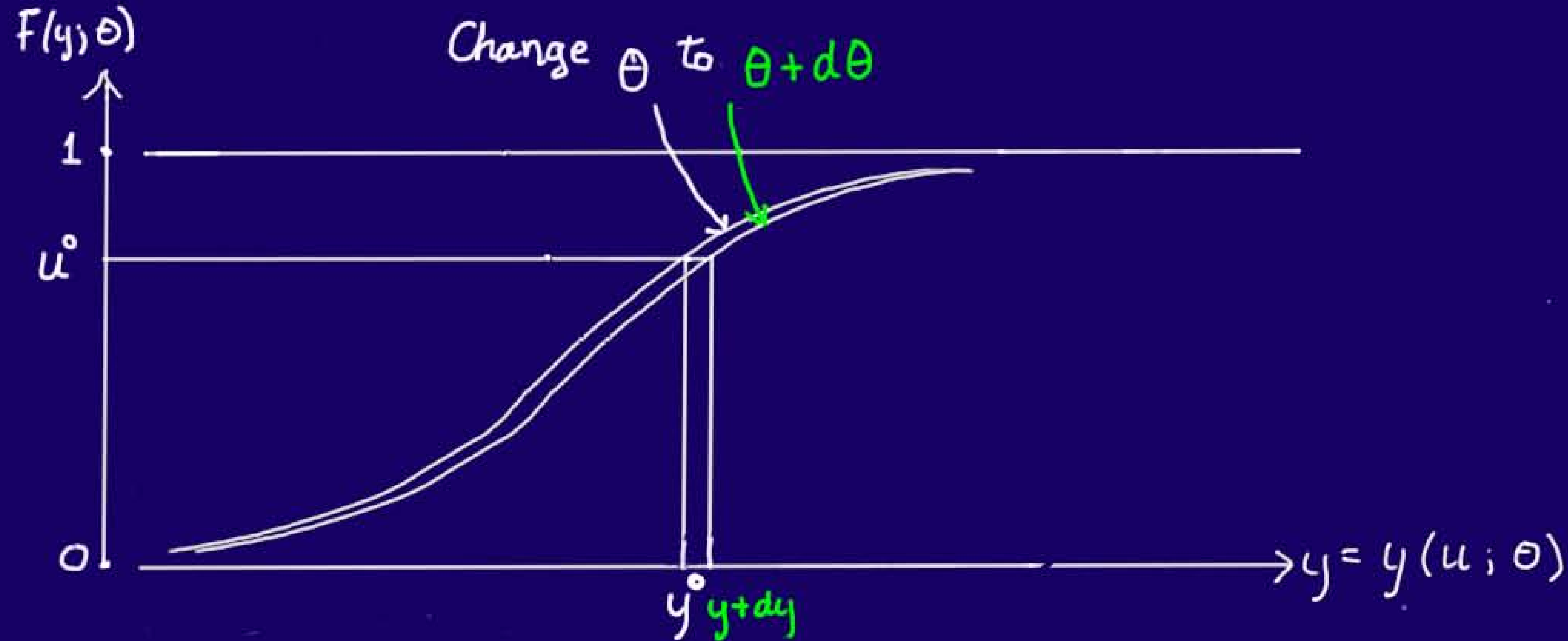

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"Unit <u>probability</u> <u>mass</u>" How does it move? A consequence of continuity!

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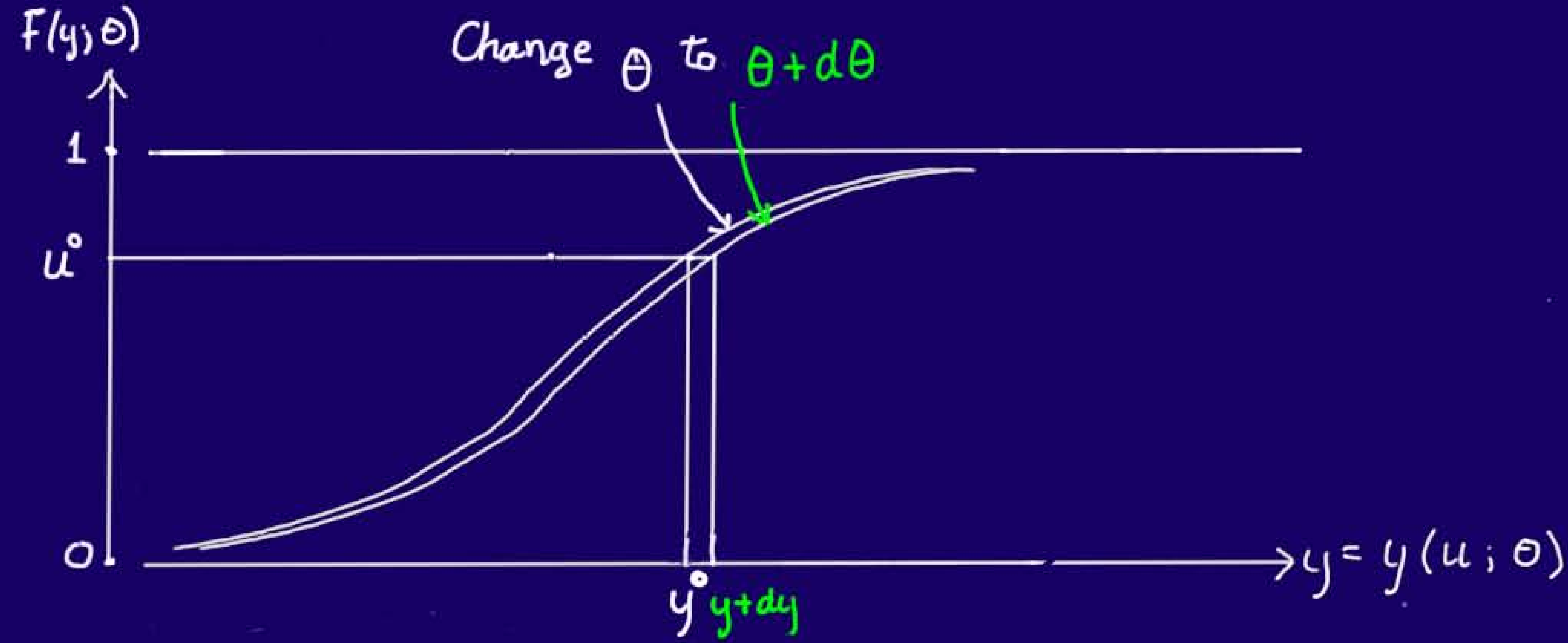

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<u>Now R^n</u>

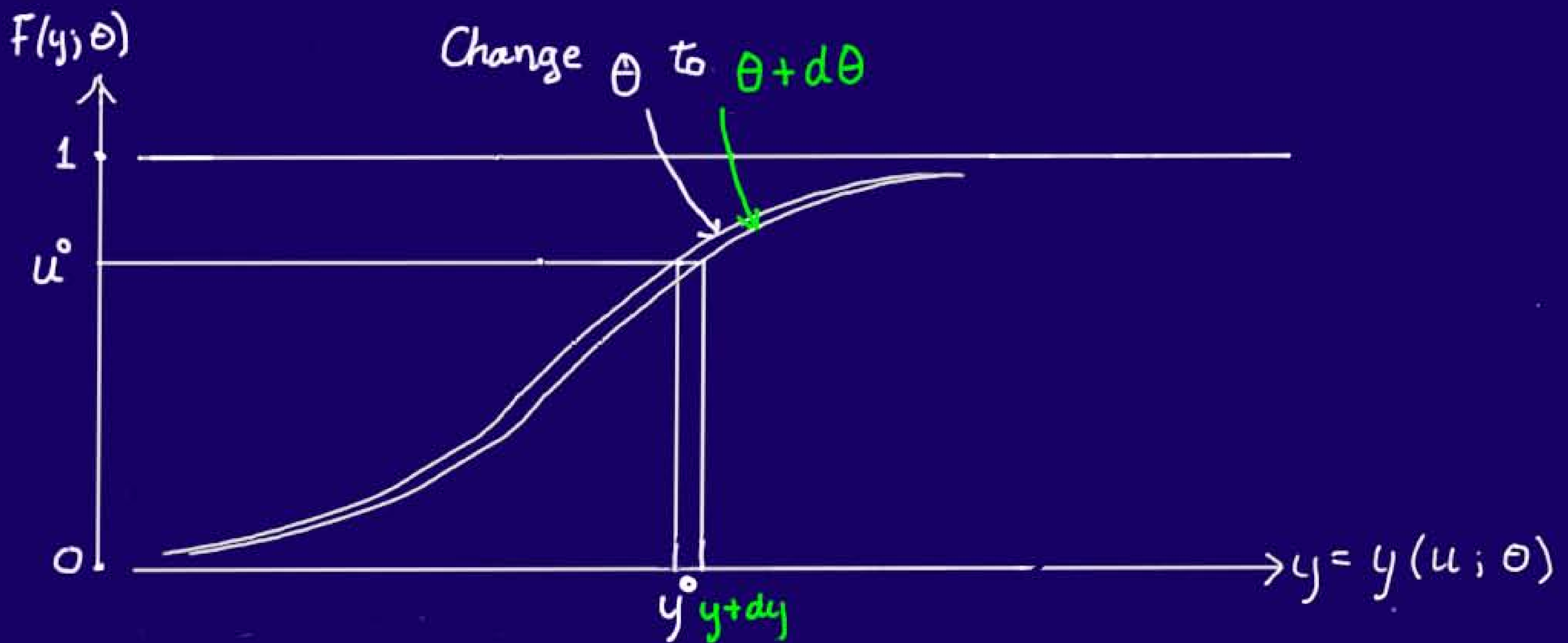
$f(y;\theta)$ dim $y = n$
dim $\theta = p$
Indep coord's say

Data y°

y°

4. Continuity

Case: Scalar and vector



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Change $\theta \to \theta + d\theta$

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R^n $f(y;\theta)$ $\dim y = n$
$\dim \theta = p$
Indep coord's say

Change θ: $\theta \to \theta + d\theta$
$y^\circ + dy$

$$\frac{dy}{d\theta} = \underset{n \times p}{V(\theta)} = (\underset{\sim}{v}_1(\theta) \cdots \underset{\sim}{v}_p(\theta))$$

$$dy = V(\theta)\, d\theta$$

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Case: <u>Scalar</u> and <u>vector</u>

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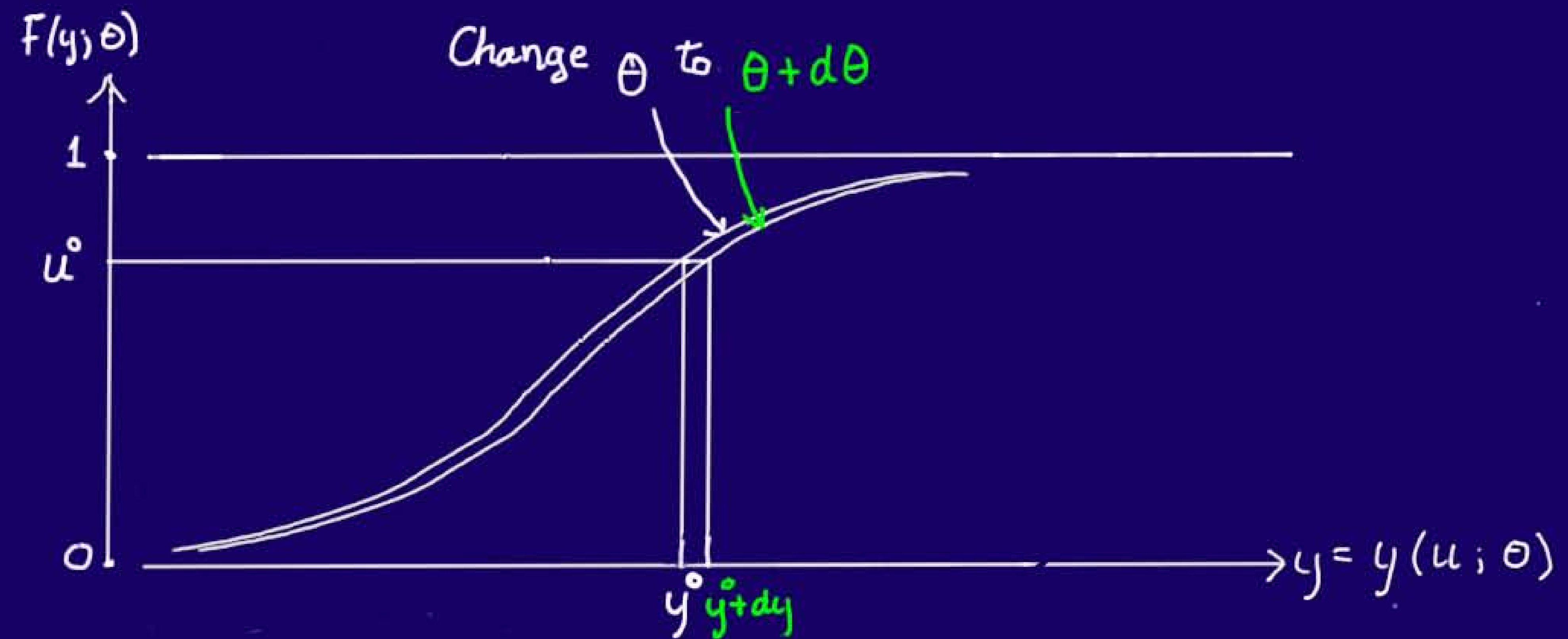

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$\nearrow y^\circ + dy$

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"Ultimate consequence of continuity! $\quad V(\theta) \quad V = V(\hat{\theta}^\circ)$

What's it do for you?

$y^0 \to y^0 + dy$

$$\underset{n\times 1}{dy} = \underset{n\times p}{V(\theta)}\,\underset{p\times 1}{d\theta}$$

What's it do for you ?

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Gives Highly-Accurate SP-type tests of $\psi(\theta)=\psi$ $O(n^{-3/2})$
(cgf-type)

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C Linear parameters re Bayes & Curved

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Fraser F Fraser (2010) J. Statist Res Efron vol. 44 to appear

An example:

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$y^0 \rightarrow y^0 + dy$

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Ex: Regression $y = X\beta + \sigma Z$ say Non-N error $f_i(z_i)$

An example:

$y^0 \to y^0 + dy$

$$\underset{n\times 1}{dy} = \underset{n\times p}{V(\theta)}\ \underset{p\times 1}{d\theta}$$

Differentiate quantile fn

Ex: Regression $y = X\beta + \sigma z$ say Non-N error $f_i(z_i)$

at y^0

$$\underset{n\times 1}{dy} = \Big(\underset{n\times k}{X}\quad \underset{n\times 1}{z^0(\beta,\sigma)}\Big)\underset{(k+1)\times 1}{\begin{pmatrix} d\beta \\ d\sigma \end{pmatrix}}$$

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at y^0 $\underset{n\times 1}{dy} = \left(\underset{n\times r}{X} \quad \underset{n\times 1}{z^0(\beta,\sigma)} \right) \underset{(r+1)\times 1}{\begin{pmatrix} d\beta \\ d\sigma \end{pmatrix}}$

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Can put in terms of $d\hat\beta$, $d\hat\sigma$ at Observed ...

at y^0 $d\hat\beta$ =

$d\hat\sigma$ =

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$$d\hat\sigma = \qquad \hat\sigma\, d\sigma/\sigma$$

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Absolute value $|d\hat\theta| = \left|\ \downarrow\ \right| d\theta = \underset{\text{prior}}{\pi(\theta)}\,d\theta = \dfrac{\hat\sigma}{\sigma}\,d\beta\,d\sigma$

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Absolute value $|d\hat\theta| = \left| \downarrow \right| d\theta = \underset{\text{prior}}{\pi(\theta)}\, d\theta = \frac{\hat\sigma}{\sigma}\, d\beta\, d\sigma$

"Standard" Bayesian right-invariant prior

Also: in general $\Rightarrow$ 2nd order prior

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A Conditioning on data information

Model: $f(y;\theta)$ y in R^n $\dim\theta = p$

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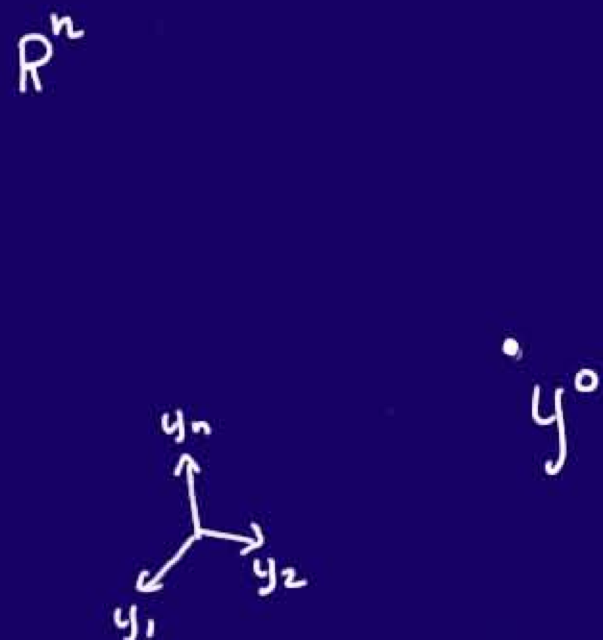

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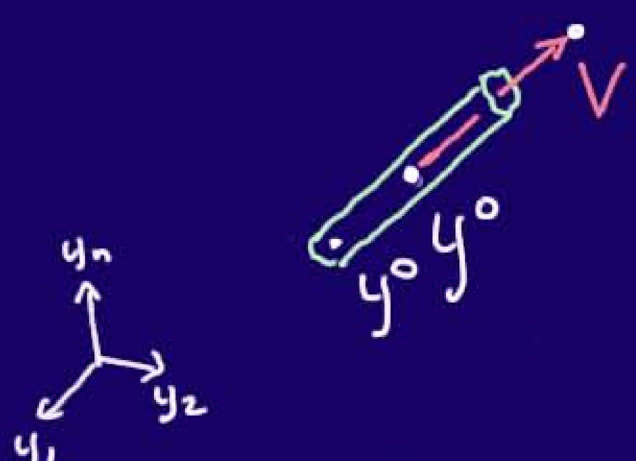

Continuity available:

Take θ from $\hat\theta^0$ to $\hat\theta^0 + d\theta$

$$dy = V\, d\theta \qquad V = V(\hat\theta^0) = \left.\frac{dy}{d\theta}\right|_{y^0\,\hat\theta^0}$$

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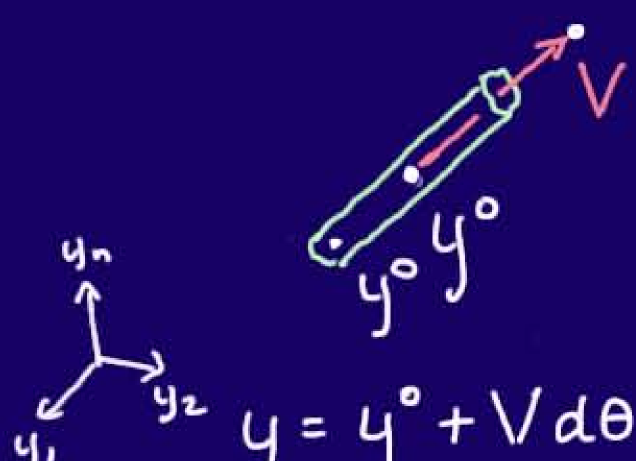

$y = y^0 + V\,d\theta$

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Take θ from $\hat\theta^0$ to $\hat\theta^0 + d\theta$

$$dy = V\,d\theta \qquad V = V(\hat\theta^0) = \left.\frac{dy}{d\theta}\right|_{y^0\,\hat\theta^0}$$

Distribution moves in tube

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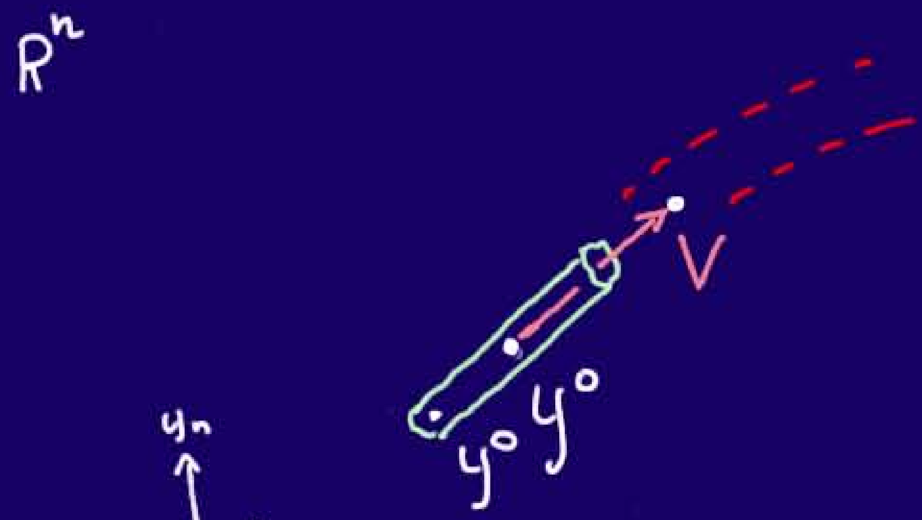
A Conditioning on data information

Model: $f(y;\theta)$ $\quad$ y in R^n $\quad$ $\dim \theta = p$

Data: y°



Continuity available:

Take θ from $\hat\theta^\circ$ to $\hat\theta^\circ + d\theta$

$dy = V\,d\theta \qquad V = V(\hat\theta^\circ) = \frac{dy}{d\theta}\Big|_{y^\circ \hat\theta^\circ}$

Distribution moves <u>in</u> tube

$\left\{ \begin{array}{l} W = V_\theta(\hat\theta^\circ) \;\; p\times p \text{ of } p \\ \text{Frobenius} \end{array} \right.$

$y_1 \quad y_2 \quad y = y^\circ + V d\theta \;+\; d\theta' W d\theta\, n^{-1/2} = y(d\theta, \hat u^\circ)$ $\qquad$ W = curvature $= V_\theta(\hat\theta^\circ)$

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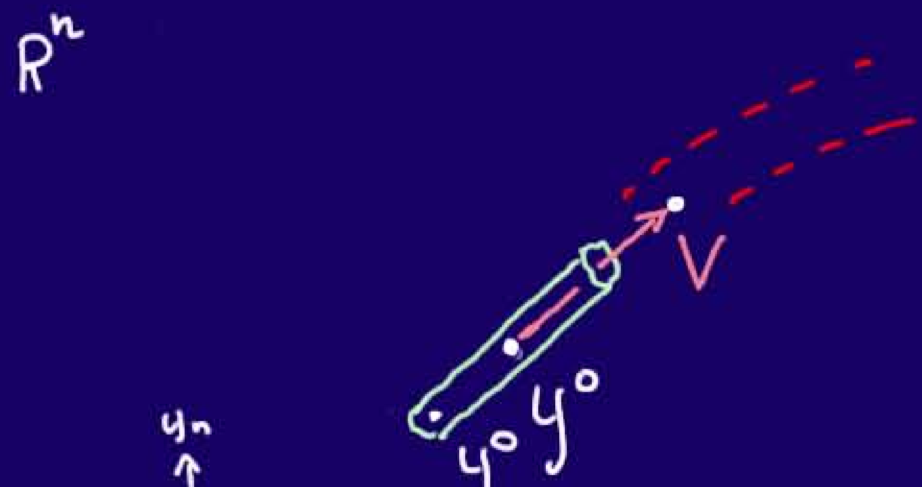

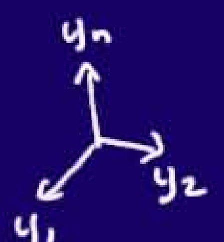

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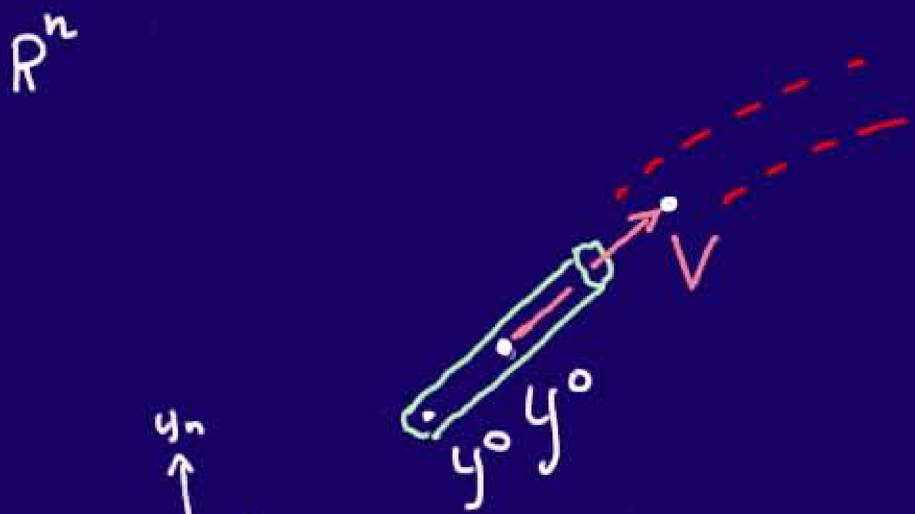

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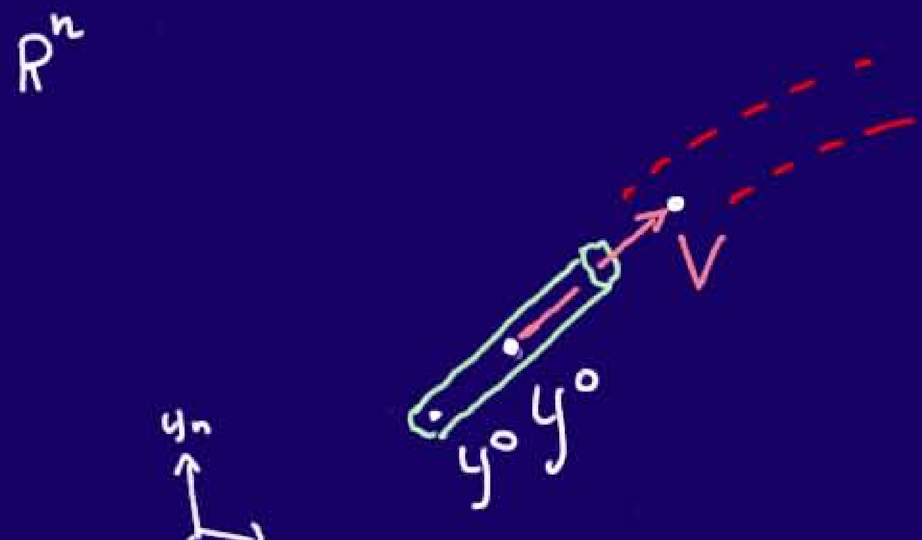

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$s^0 = 0$

Just an "Exp model"... Easy... $O(n^{-3/2})$

Reid F (2010) Biometrika 159-170
Fraser F Staicu (2010) Bernoulli 1208-1223

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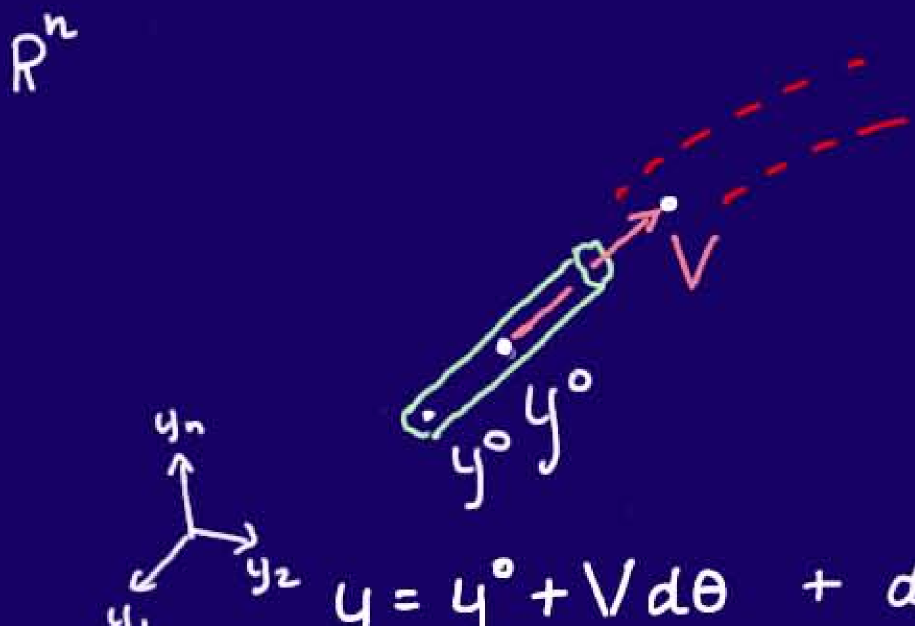

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All p-values $p(\psi)$ for scalar $\psi(\theta)$ available

Reid F (2010) Biometrika 159-170
Fraser F Staicu (2010) Bernoulli 1208-1223

B Default priors : "Nec. & Suff" conditions

Case: Location $f(\underset{\sim}{y}-\underset{\sim}{\beta}(\theta))$

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Case: Location $f(\underset{\sim}{y} - \underset{\sim}{\beta}(\theta))$ Laplace / Obvious / routine ... "flat in β"

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Default prior: $\pi(\theta) d\theta = |V(\theta)|\, d\theta$ Or switch $y \longrightarrow \hat{\theta}$

Examples: Non Normal $y = X\beta + \sigma z$ $\pi = |X\, \hat{z}^\circ(\beta,\sigma)| = \sigma^{-1}$ Widely prefered

Non Linear $y = x(\beta) + \sigma z$ $\pi = |V(\theta)|$

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same

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$|V(\theta)|\, d\theta$ gives 2nd order Accurate for linear parameters ψ

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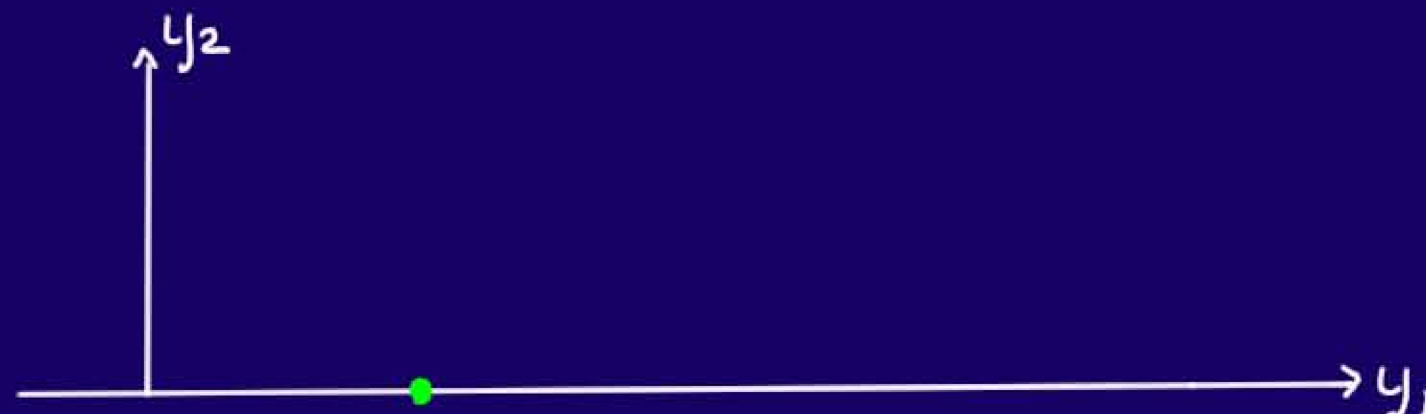

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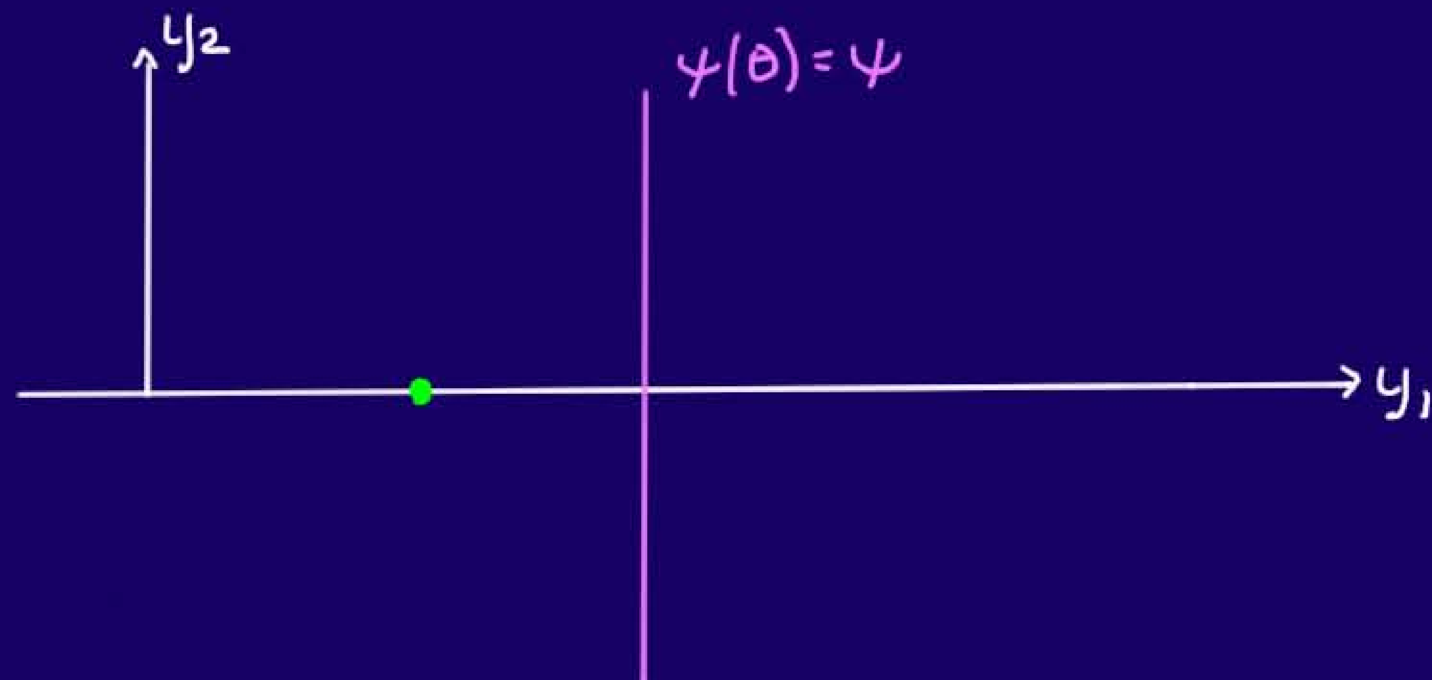
① Linear: Assess $\psi(\theta_1, \theta_2) = \theta_1$ say



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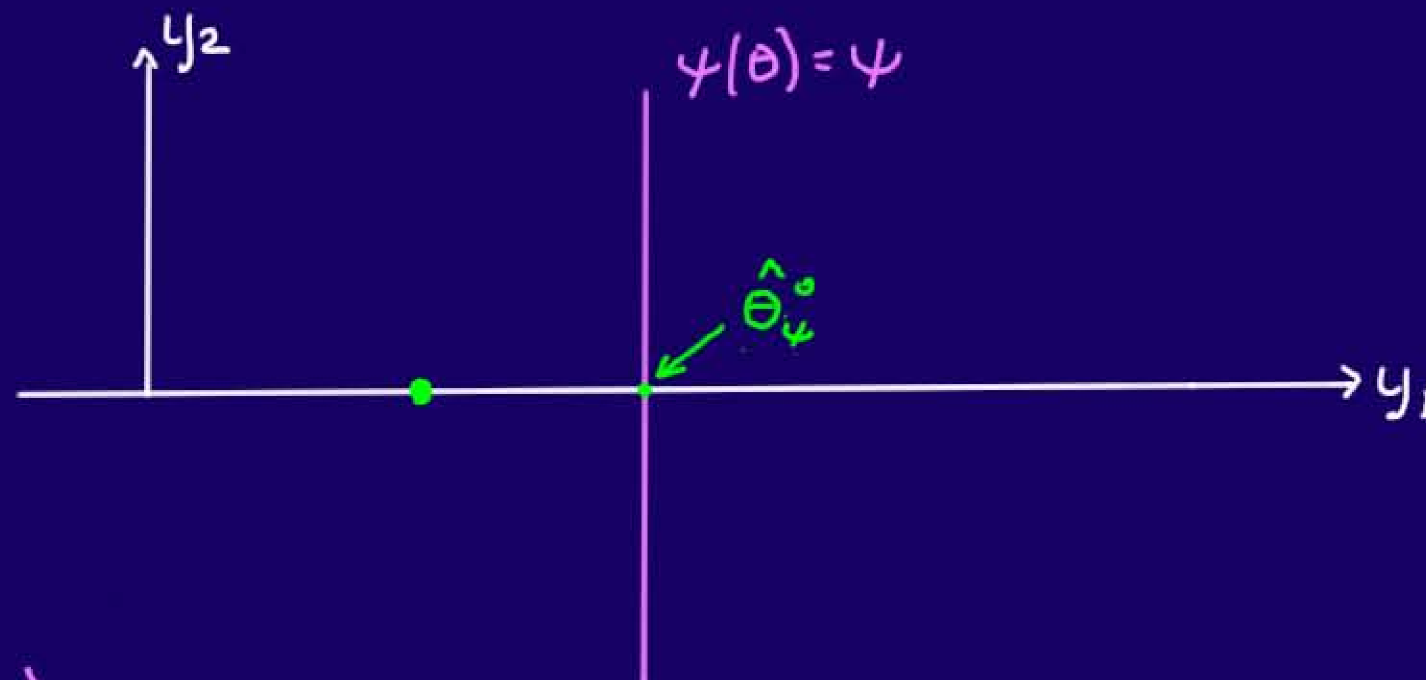

$\Phi(-1)$

p-value $= p(\psi) = .16$

s-value $= s(\psi) = .16$

freq p-value and Bayes posterior survivor equal

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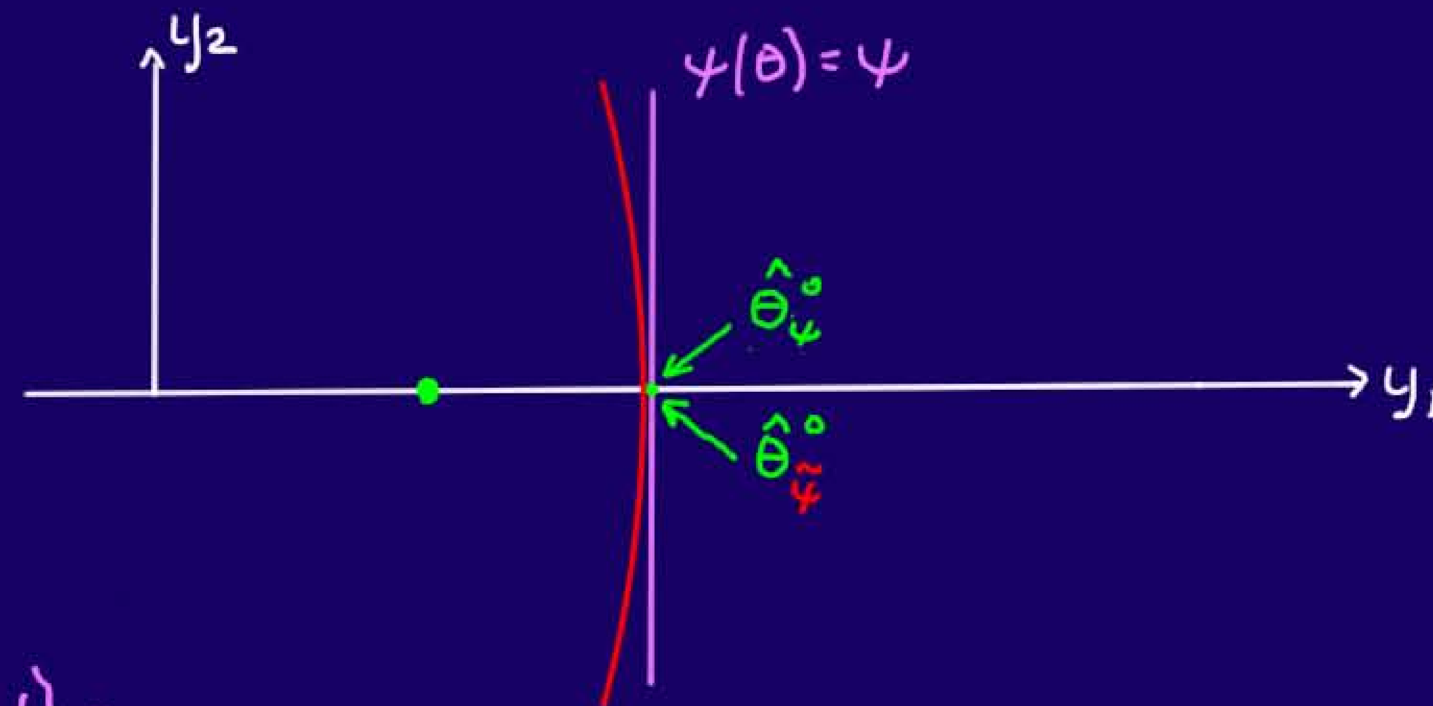
② Curved

Assess $\tilde{\psi}(\theta_1, \theta_2) = \{(\theta_1+5)^2 + \theta_2^2\}^{1/2} = \psi$



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s-value $= .21$ ↑

pvalue & s-value are not equal

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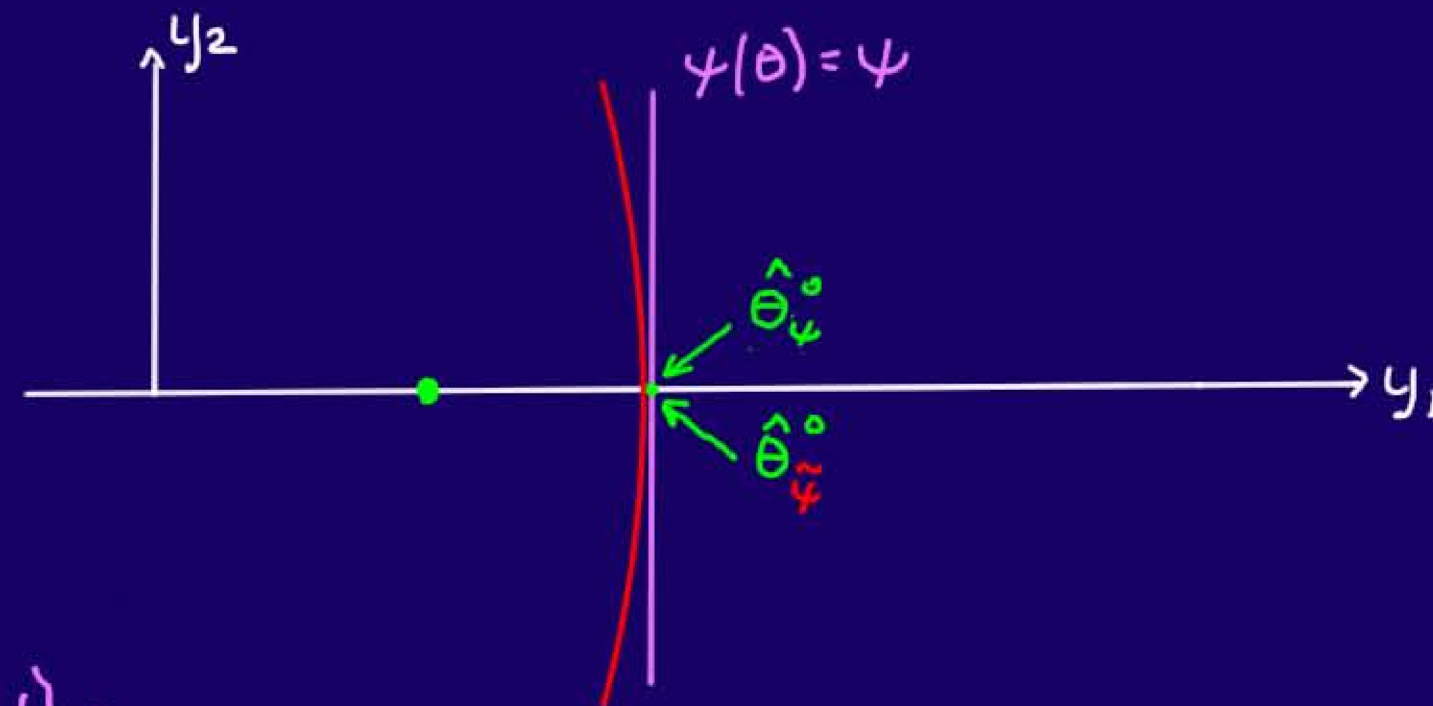

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p-value & s-value move in opposite directions

Accuracy?

10. Curvature of parameters $O(n^{-1})$ and its Bias on Bayes

Asymptotics says:

1) Model $O(n^{-1})$ is $\exp\{\ell(\theta) + \varphi'(\theta)s\}|\hat{j}_{\varphi\varphi}(s)|^{-\frac{1}{2}} \cdot ds$

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10. Curvature of parameters $O(n^{-1})$ and its Bias on Bayes

Asymptotics says:

1) Model $O(n^{-1})$ is

$$\exp\{\underbrace{\ell(\theta)}_{\text{Obs } \ell} + \underbrace{\varphi'(\theta)}_{\varphi \text{ "repar"}} s\} \, |\underbrace{\hat{j}_{\varphi\varphi}(s)}_{\text{Obs info}}|^{-\frac{1}{2}} \cdot ds \quad \leftarrow \text{SP}$$

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Obs ℓ — φ "repar" — Obs info

1) Model $O(n^{-1})$ is $\quad \exp\{\ell(\theta) + \varphi'(\theta)s\}\,|\hat{j}_{\varphi\varphi}(s)|^{-\frac{1}{2}} \cdot ds \quad$ SP

2) Can find linear par. β near $\hat{\varphi}^0$

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Fraser F Fraser (2010) J. Stat. Res. Efron Vol.

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Continuity gives $d\hat{\varphi} = M(\varphi)\,d\varphi$ $\mathbb{M}^0 = M_\varphi(\hat{\varphi}^0)$

Integrate $O(n^{-1})$ $\beta(\varphi) = \underset{p\times 1}{M^0} d\varphi + \frac{1}{2n^{1/2}}\, d\varphi' \underset{p\times p}{\mathbb{M}^0} d\varphi$ arrays of vectors

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Get linear contours ... So that Vector Bayes works!

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// Summary & Directions

Have: Sample space conditioning

Have 3rd order p-value inference

Have default priors

11 Summary & Directions

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Dawid Stone Zidek (1973)

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Dawid Stone Zidek (1973)

// Summary & Directions

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But They don't handle parameter curvature DSZ

Now have: Curvature measure $\gamma(\hat{\theta}^0)$

<u>Develop</u>: Bayes adjustment for curvature

$s(\psi) +$ "<u>Curv. corr.</u>" .... $O(n^{-1})$

Removes $O(n^{-1/2})$ DSZ bias

F. Sun (2010)

Dawid Stone Zidek (1973)

Thank you

Thank you

and Thank you 

Thank you

and Thank you

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