Combining likelihood or p-value functions,

with or without statistical dependence

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joint work

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Statistical Sciences

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www.utstat.toronto.edu/dfraser/documents/WU2015.pdf These slides

/274.pdf Related paper

Background:

Model & Data

$\downarrow$

Likelihood

$\downarrow$

Inference

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Model & Data

$\downarrow$

Likelihood

$\downarrow$

Inference

What if $f(y;\theta)$ is "computationally inaccessible"?

Background:

Model & Data

$\downarrow$

Likelihood

$\downarrow$

Inference

What if $f(y;\theta)$ is "computationally inaccessible"?

Arose in Applications

Large literature

Composite likelihood

1. What is "Composite likelihood"?
2. Example 1
3. Example 2
4. What can asymptotics do?
5. "First-Order" Model; Combining log-likelihoods
6. Example 1 again
7. Example 2 again
8. Combining p-value functions (dependent data)
9. Example 2 again
10. Vector parameter case
11. Discussion

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data y°

moderate regularity

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<u>Often</u> <u>have</u>	Components	Model	$\log L$
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<u>Try:</u> Fix "first Bartlett" $E\{-\frac{\partial^2}{\partial\theta^2}\ell(\theta)\} = \text{var}\{\ell_\theta(\theta):\theta\}$

Examples: Just Normals

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Reciprocal info $= [-\ell_{\theta\theta}]^{-1} = \frac{1}{4}$

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<u>Actual</u>

$y_1 = x_1 + x_3$ $\qquad var(y_1 + y_2) = 6$

$y_2 = x_2 + x_3$ $\qquad$ " $cov(y_1, y_2) = 1$ "

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Actual

$y_1 = x_1 + x_3$ $\qquad var(y_1 + y_2) = 6$

$y_2 = x_2 + x_3$

$var\ \hat{\theta} = \frac{6}{16} = \frac{3}{8}$

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$y_2 \sim N(2\theta; 2)$ $\qquad$ $\ell_2(\theta) = -\theta^2 + \theta y_2$ $\qquad$ $y_2 = x_2 + x_3$

$\ell_{UCL}(\theta) = -2\theta^2 + \theta(y_1 + y_2)$ $\qquad$ $\hat{\theta} = \frac{y_1 + y_2}{4}$ $\qquad$ $\text{var}\,\hat{\theta} = \frac{6}{16} = \frac{3}{8}$

<u>not</u> equal:

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Bartlett: $\ell_{UCL}(\theta)$ ... <u>not</u> a "likelihood"

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"Force Bartlett"!

Composite lik: $\qquad \ell_{ACL}(\theta) = \dfrac{1/4}{3/8}\,\ell_{UCL}(\theta) = -\frac{4}{3}\theta^2 + \frac{2}{3}\theta(y_1+y_2)$

"$\times \frac{2}{3}$"

Like B!

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$y_2 \sim N(2\theta;2)$ $\qquad \ell_2(\theta) = -\theta^2 + \theta y_2$ $\qquad y_2 = x_2 + x_3$

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Does it work? ... in some sense?

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Does it work?

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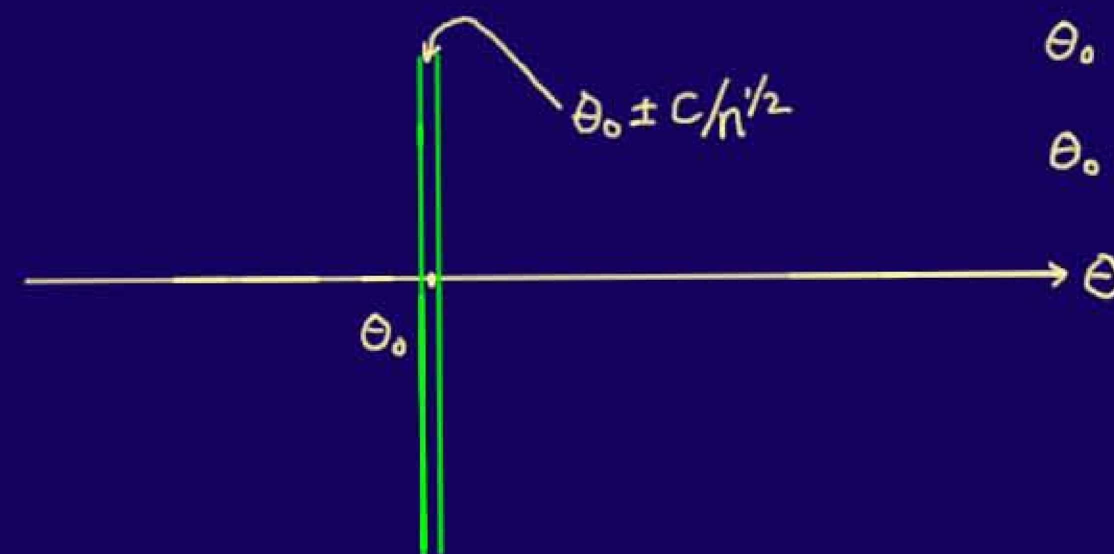

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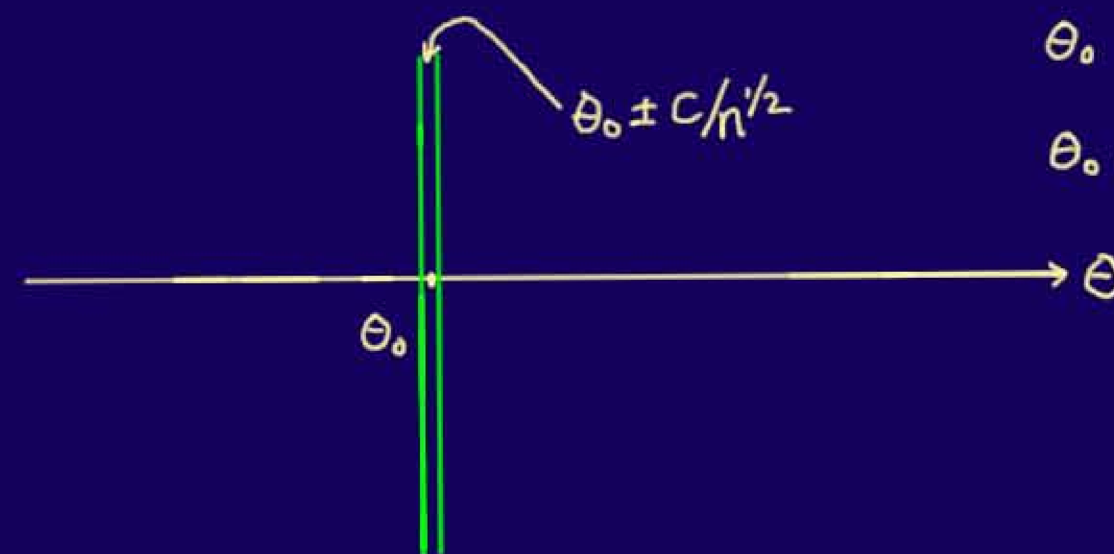

<u>Only</u> variable: $s_i = \frac{\partial}{\partial\theta} l^i(\theta_0; y_i) = l_\theta^i(\theta_0; y_i)$ ... local score variable

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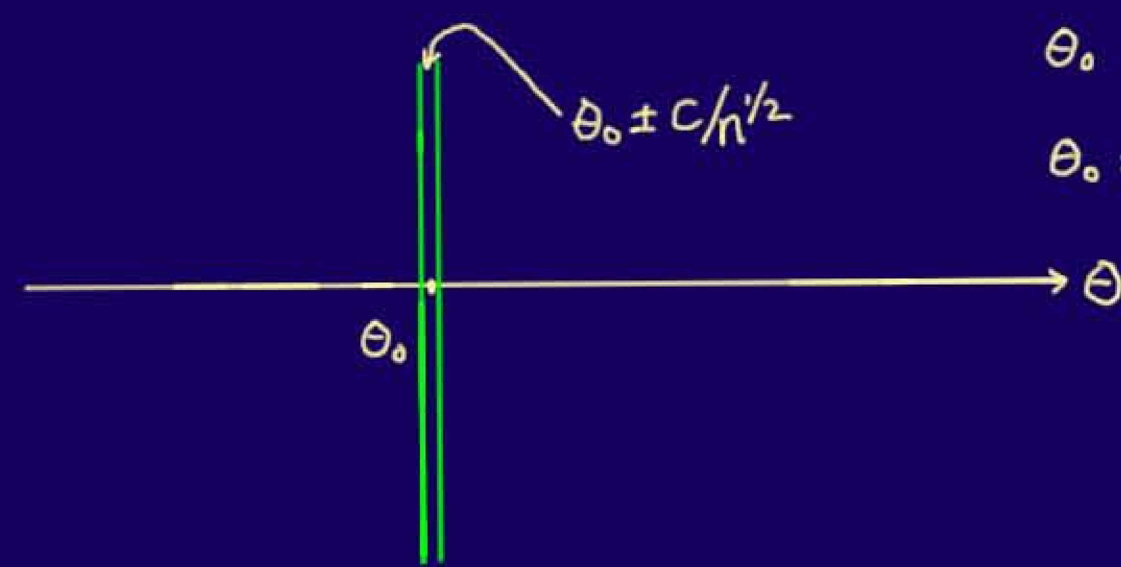

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2nd derivative don't really need! ... info available later!

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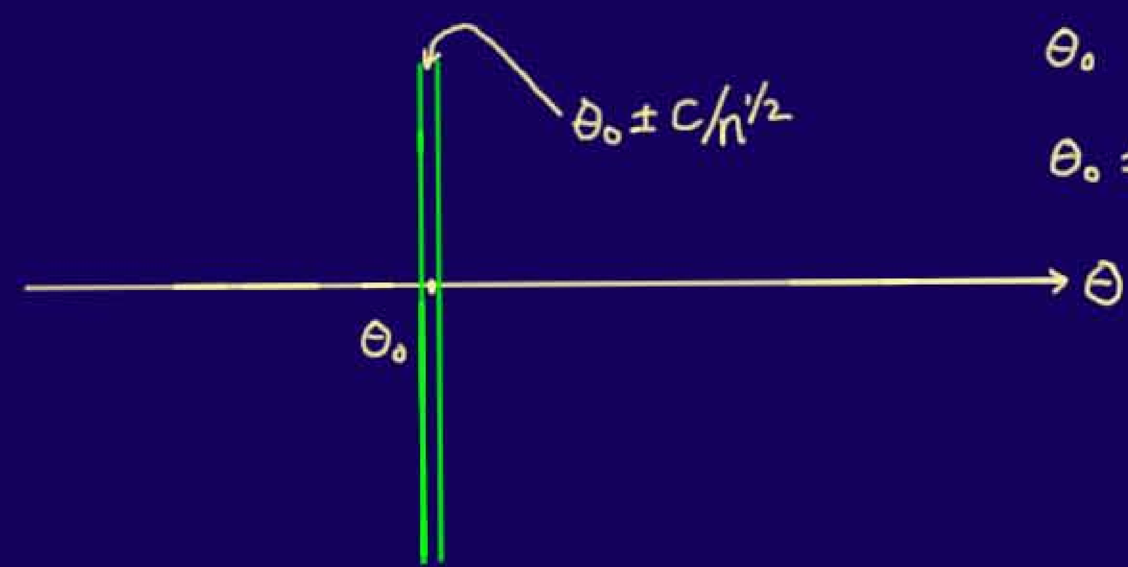

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$E\{s_i; \theta_0\} = 0$

$\mathrm{var}\{s_i; \theta_0\} = N_{ii} = \mathrm{info} = -E\{\ell^i_{\theta\theta}(\theta_0); \theta_0\}$; $\mathrm{cov}\{s_i, s_j; \theta_0\} = N_{ij}$ "available from $f_{ij}(y_i, y_j; \theta)$"

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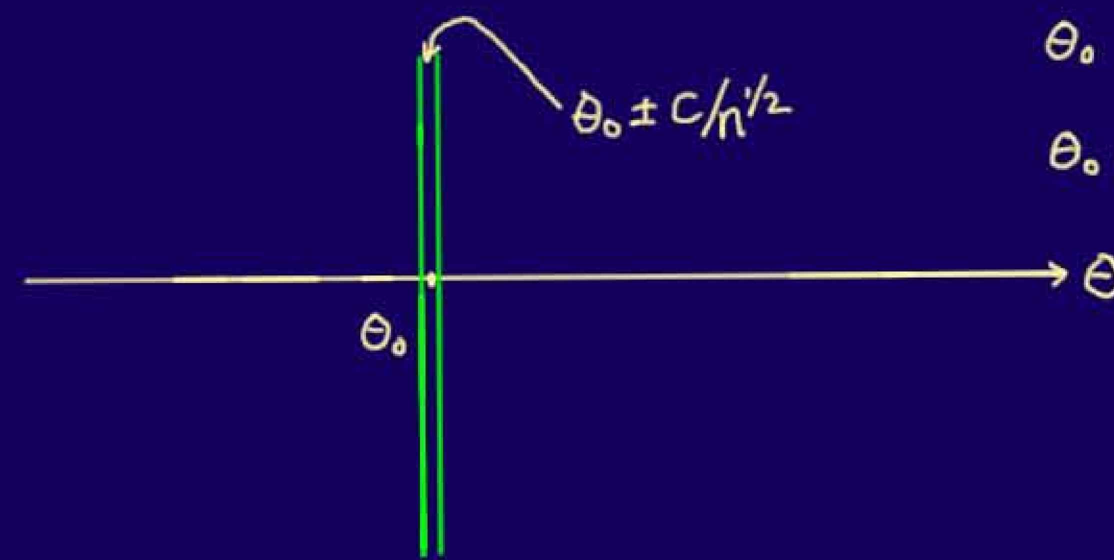

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"available from $f_{ij}(y_i, y_j; \theta)$"

$$\underset{\sim}{s} = \begin{pmatrix} s_1 \\ \vdots \\ s_m \end{pmatrix} \qquad E(\underset{\sim}{s}; \theta) = (\theta - \theta_0)\underset{\sim}{\nu} = (\theta - \theta_0)\begin{pmatrix} \nu_{11} \\ \vdots \\ \nu_{mm} \end{pmatrix}$$

$$\text{Var}(\underset{\sim}{s}; \theta_0) = V = \begin{pmatrix} \nu_{11} & & \nu_{1m} \\ & & \\ \nu_{m1} & & \nu_{mm} \end{pmatrix}$$

First Order Model

<u>More</u> on local score variable $s_i = \ell_\theta^i(\theta_0; y_i)$

First Order Model

More on local score variable $s_i = l_\theta^i(\theta_0; y_i)$

$$\frac{d}{d\theta} E\{s_i ; \theta\} = (\theta - \theta_0)\, \nu_{ii} = \theta\, \nu_{ii} + O(n^{-\frac{1}{2}})$$

Easier notation:

use $\underline{\theta_0 = 0}$

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use $\theta_0 = 0$

Model: $E(s;\theta) = \theta\, i$

$\mathrm{var}(s;\theta) = V$

ordinary linear model

But scalar θ

$O(n^{-\frac{1}{2}})$

First Order Model

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Easier notation: use $\theta_0 = 0$

$$\frac{d}{d\theta} E\{s_i ; \theta\} = (\theta - \theta_0)\, v_{ii} = \theta\, v_{ii} + O(n^{-\frac{1}{2}})$$

Model: $E(s;\theta) = \theta v$ Ordinary linear model

$\mathrm{var}(s;\theta) = V$ But scalar θ

$O(n^{-\frac{1}{2}})$

	Data	E	var	$\hat{\theta}$	var $\hat{\theta}$
LS	y	$X\theta$	$\sigma^2 I$	$(X'X)^{-1}X'y$	$\sigma^2(X'X)^{-1}$

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LS	y	$X\theta$	$\sigma^2 I$	$(X'X)^{-1}X'y$	$\sigma^2(X'X)^{-1}$
GM	y	$X\theta$	Σ	Σ^{-1} Σ^{-1}	Σ^{-1}

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Model: $E(s;\theta) = \theta\,\nu$ Ordinary linear model

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	Data	E	var	$\hat\theta$	var $\hat\theta$	
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				Σ^{-1} Σ^{-1}	Σ^{-1}	
GM	y	$X\theta$	Σ			
Now	s	$\nu\theta$	V	$(\nu' V^{-1}\nu)\,\nu' V^{-1} s$	$(\nu' V^{-1}\nu)^{-1}$	Just substitute

5 First Order Model

More on local score variable $s_i = \ell_\theta^i(\theta_0; y_i)$

Easier notation: use $\theta_0 = 0$

$$\frac{d}{d\theta} E\{s_i ; \theta\} = (\theta - \theta_0)\, \nu_{ii} = \theta\, \nu_{ii} + O(n^{-\frac{1}{2}})$$

Model: $E(s;\theta) = \theta \nu$, $\mathrm{var}(s;\theta) = V$

Ordinary linear model
But scalar θ

$O(n^{-\frac{1}{2}})$

	Data	E	var	$\hat\theta$	var $\hat\theta$
LS	y	$X\theta$	$\sigma^2 I$	$(X'X)^{-1} X'y$	$\sigma^2 (X'X)^{-1}$
GM	y	$X\theta$	Σ	Σ^{-1} Σ^{-1}	Σ^{-1}
Now	s	$\nu\theta$	V	$(\nu' V^{-1} \nu)\, \nu' V^{-1} s$	$(\nu' V^{-1} \nu)^{-1}$

Just substitute

Combined loglikelihood $\tilde{\ell}(\theta) = \nu' V^{-1} \underset{\sim}{\ell}(\theta)$

$s_i \to \nu_{ii}\theta = \ell^i(\theta)$

Example 1 again

$y_1 \sim N(2\theta; 2)$ $\quad \ell_1(\theta) = -\theta^2 + \theta y_1$ $\quad y_1 = x_1 + x_3$ $\quad \mathrm{var}(y_1 + y_2) = 6$

$y_2 \sim N(2\theta; 2)$ $\quad \ell_2(\theta) = -\theta^2 + \theta y_2$ $\quad y_2 = x_2 + x_3$

$\ell_{UCL}(\theta) = -2\theta^2 + \theta(y_1 + y_2)$ $\quad \hat{\theta} = \frac{y_1 + y_2}{4}$ $\quad \mathrm{var}\,\hat{\theta} = \frac{6}{16} = \frac{3}{8}$

Reciprocal info $= [-\ell_{\theta\theta}]^{-1} = \frac{1}{4}$

$\mathrm{var}\,\hat{\theta} = \frac{3}{8}$

Bartlett: $\ell_{UCL}(\theta)$ ... <u>not</u> a likelihood

Force Bartlett!

Composite lik: $\quad \ell_{ACL}(\theta) = \frac{1/4}{3/8}\,\ell_{UCL}(\theta) = -\frac{4}{3}\theta^2 + \frac{2}{3}\theta(y_1 + y_2)$

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New $\quad \nu = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad V = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad \nu' V^{-1} = (2/3, 2/3)$

$y_1 = x_1 + x_3$

$y_2 = x_2 + x_3$

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$$\ell_{UCL}(\theta) = -2\theta^2 + \theta(y_1 + y_2) \longrightarrow \hat{\theta} = \frac{y_1 + y_2}{4} \quad \mathrm{var}\,\hat{\theta} = \frac{6}{16} = \frac{3}{8}$$

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$\tilde\ell(\theta) = -\frac{4}{3}\theta^2 + \frac{2}{3}\theta(y_1 + y_2)$ $\longleftarrow$ agree!

But a lot of symmetry!

7 Example 2 again

$y_1 \sim N(\theta; 1)$ $\quad \ell_1(\theta) = -\frac{1}{2}\theta^2 + \theta y_1$ $\quad y_1 = x_1$ $\quad \mathrm{var}(y_1 + y_2) = 5$

$y_2 \sim N(2\theta; 2)$ $\quad \ell_2(\theta) = -\theta^2 + \theta y_2$ $\quad y_2 = x_1 + x_3$

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$\mathrm{var}\,\hat{\theta} = \frac{5}{9}$ Bartlett: $\ell_{UCL}(\theta)$ ... not a likelihood

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Just uses y_2

7 <u>Example 2</u> again

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Composite lik:

$$\ell_{ACL}(\theta) = \frac{1/3}{5/9}\,\ell_{UCL}(\theta) = -\frac{9}{10}\theta^2 + \frac{3}{5}\theta(y_1+y_2)$$

wrong

(apparent info = $9/5$)

<u>New</u> $\quad \mu = \begin{pmatrix}1\\2\end{pmatrix} \quad V = \begin{pmatrix}1 & 1\\1 & 2\end{pmatrix} \quad \mu' V^{-1} = (0, 1)$

$$y_1 = x_1 + x_3$$
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$$V^{-1} = \begin{pmatrix}2 & -1\\-1 & 1\end{pmatrix}$$

$$\tilde\ell(\theta) = \ell_2(\theta) = -\theta^2 + \theta y_2 \longleftarrow \text{available}$$

(Info = 2)

Just uses y_2

8 Combining p-value functions (dependent data)

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Compound	$S - v\theta = \underline{v' V^{-1}}(s - v\theta)$

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Compound $S - v\theta = v' V^{-1}(s - v\theta)$

$= v' V^{-1} \{v_{ii}^{1/2} z_i(\theta; s_i)\}$

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Compound $\quad S - v\theta = v' V^{-1}(s - v\theta)$

$$= v' V^{-1}\{v_{ii}^{1/2} z_i(\theta; s_i)\}$$

$$= v' V^{-1} v^{1/2} \cdot \Phi^{-1/2} p_i(\theta; s_i)$$

$$v^{1/2} = \begin{pmatrix} v_{11}^{1/2} & \\ & v_{mm}^{1/2} \end{pmatrix}$$

9 Example 2 again

$y_1 \sim N(\theta; 1)$ $\qquad \ell_1(\theta) = -\frac{1}{2}\theta^2 + \theta y_1$ $\qquad y_1 = x_1$

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$$\tilde{\ell}(\theta) = -\theta^2 + \theta y_2$$ ... the New log Likelihood

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Score quantity $\qquad z = 2^{-1/2}(y_2 - 2\theta)$

Composite p-value $\qquad \tilde{p}(\theta; S) = \Phi(z) = \Phi(2^{-1/2}(y_2 - 2\theta))$ $\qquad$ again just based on y_2

10 Vector parameter case

Above: Scalar θ Component models $f_i(y_i;\theta)$ with "overlap"

scores $s_i = \ell_\theta^i(\theta; y_i)$ & w V

$\tilde{\ell}(\theta) = wV^{-1}\underset{\sim}{\ell}(\theta)$

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scores $s_i = \ell^i_\theta(\theta; y_i)$ & $\sim V$

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Now: Vector $\theta = \theta_0 + \delta\alpha$ α = unit vector δ = coord. on $\theta_0 + \delta\alpha$

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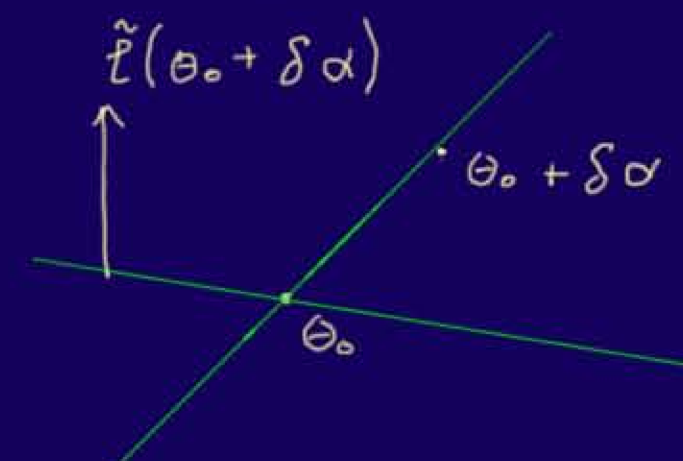
Seek: First-order... Normal... marginal

Apply to δ for given α; then all α



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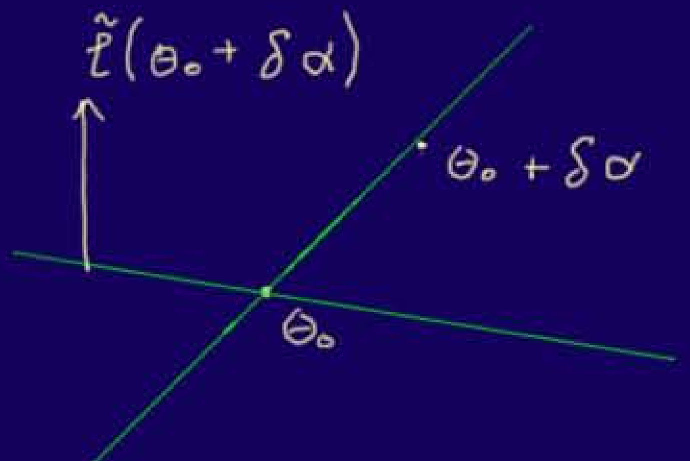

Get: Full first-order accurate $\tilde{\ell}(\theta)$

11 Discussion

- Focus on first-order... get full first order accuracy

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Thank you

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Thank you

www.utstat.toronto.edu/dfraser/documents/WU2015.pdf
... /274.pdf

Padova 2012 UCL 2012 Padova 2013