ISI 2015
60th World Statistics Congress
Rio de Janeiro

Combining likelihood functions or p-value functions: Higher accuracy and composite likelihood

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IPS036
July 29

www.utstat.toronto.edu/dfraser/documents/WSC2015.pdf

1. Why is this of interest?
2. Model asymptotics ($n^{-1/2}$)
3. First-order model: just simple regression
4. Combining log-likelihood functions $\ell_i(\theta)$
5. Combining p-value functions $p_i(\theta)$
6. Meta-Analysis: Combining independent p-value fns
7. Vector θ
8. Summary

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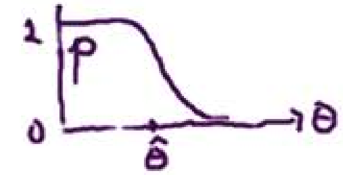

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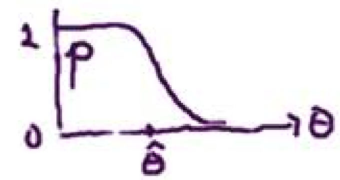
- p-value function $p(\theta)$ $\qquad p(\theta) = \Phi\left\{\dfrac{(\bar{y}-\theta)}{\sigma_0/n^{1/2}}\right\}$ 

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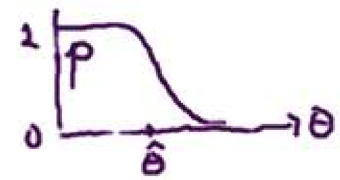
- p-<u>value</u> function $p(\theta)$ $\qquad p(\theta) = \Phi\left\{\frac{(\bar{y}-\theta)}{\sigma_0/n^{1/2}}\right\}$ 

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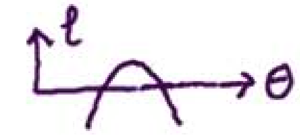

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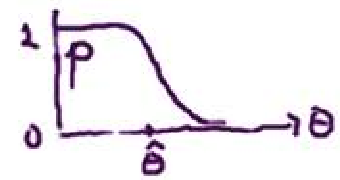

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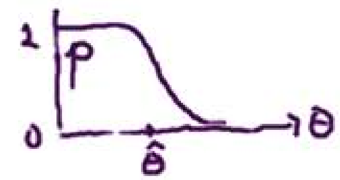

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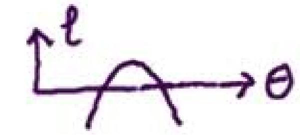

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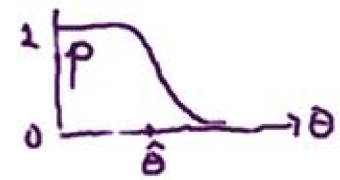

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Expand in y & θ
first order

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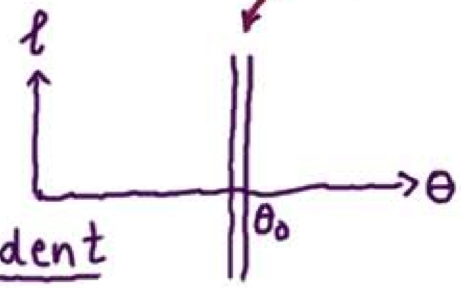

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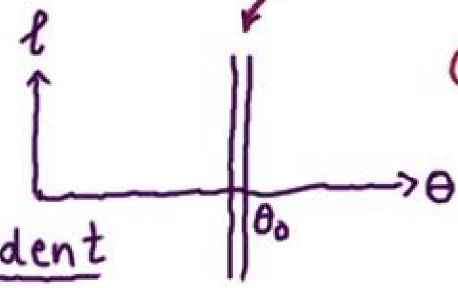

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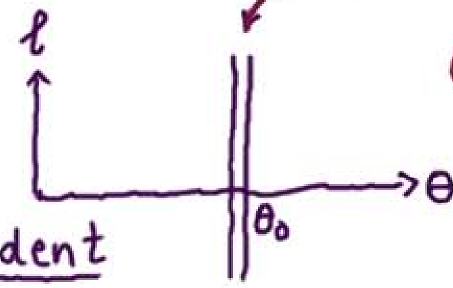

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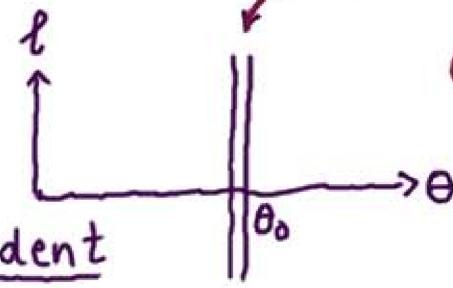

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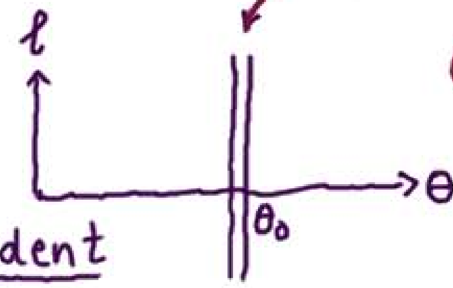

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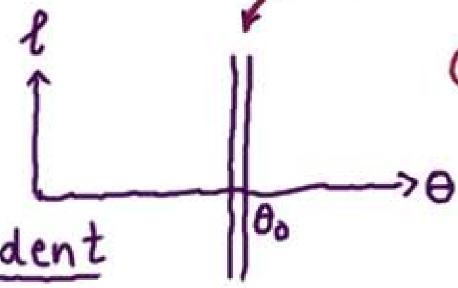

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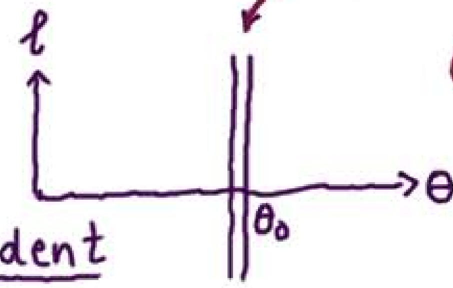

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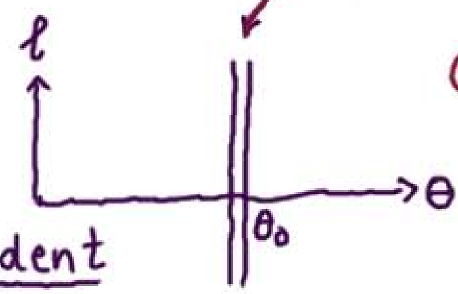

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v_{ij} from "info" in $f(y_i, y_j;\theta)$

Vector $\qquad$ Variances $\qquad$ Var. matrix

$$\underset{\sim}{s} = \begin{pmatrix} s_1 \\ \vdots \\ s_m \end{pmatrix} \qquad \underset{\sim}{v} = \begin{pmatrix} v_{11} \\ \\ v_{mm} \end{pmatrix} \qquad V = \begin{pmatrix} v_{11} & \cdots & v_{1m} \\ \vdots & & \\ v_{m1} & & v_{mm} \end{pmatrix}$$

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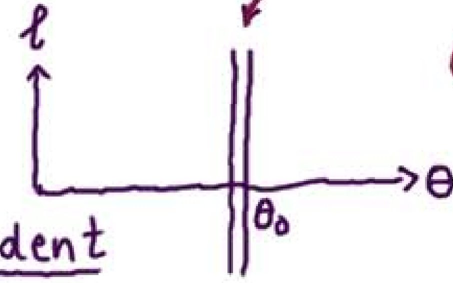

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$\underset{\sim}{s}$ distributed $\underset{\text{mean}}{\nu\theta}\,;\,\underset{\text{variance}}{V}$

Familiar?

3. Model (1st) is regression model ... in score variable

$\underset{\sim}{s} = \underset{\sim}{v}\,\Theta + \underset{\sim}{e}$ $\qquad$ $\underset{\sim}{e}$ has variance V $\quad$ (Expansion
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Model has:

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Model has

<u>Design</u> matrix $X \rightarrow$ $\qquad\qquad\qquad$ $\underset{\sim}{\nu}$

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Design matrix $X \rightarrow$ $\qquad \underset{\sim}{v}$

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Error $\sigma^2 I \rightarrow$ $\qquad$ V

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$$\hat{\theta} = (\underset{\sim}{v}' V^{-1} \underset{\sim}{v})^{-1} \underset{\sim}{v}' V^{-1} \underset{\sim}{s}$$

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Combined log-likelihood

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Apply Gauss-Markov: Get <u>Min</u>. <u>Var</u>. <u>Unb</u>. for θ

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Combined log-likelihood

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Normal examples represent "First order"; use scalar x_i

$x \sim N(n\theta; n)$ $\quad$ $\ell(\theta; x) = x\theta - n\theta^2/2$ $\quad$ i.e. linear model

4 How does it work ?

Normal examples represent "First order"

$x \sim N(v\theta; v)$ $\quad \ell(\theta; x) = x\theta - v\theta^2/2$ $\quad$ i.e. linear model

Ex 1 Independence: $s_i \sim N(v_{ii}\theta; v_{ii})$

$$v = \begin{pmatrix} v_{11} \\ v_{22} \end{pmatrix} \qquad V = \begin{pmatrix} v_{11} & 0 \\ 0 & v_{22} \end{pmatrix}$$

4 How does it work?

Normal examples represent "First order"

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<u>Usual</u> $= \ell_{UCL} = \ell_1(\theta) + \ell_2(\theta) = (y_1 + y_2)\theta - \frac{3}{2}\theta^2$

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" $\quad \ell_{ACL} = \frac{1/3}{5/9}\,\ell_{UCL} = \frac{3}{5}\theta(y_1 + y_2) - \frac{9}{10}\theta^2$ $\quad$ <u>Force</u> Bartlett ✓ Rescale

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= Not loglik from y_2 $\quad$ mle "wrong" info "wrong" $\quad$ Not OK

5 Combining p-value functions

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given: p-value fns $p_1(\theta), \ldots, p_m(\theta)$

Informations $\sim V$

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Want: Combined $\tilde{p}(\theta)$ .... What to do? $-2\ln p_i$?

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Start: p-value: $p_i(\theta)$

Convert: z-value: $z_i(\theta) = \Phi^{-1}(p_i(\theta; s_i))$

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Combine:

Score: $\nu' V^{-1}(s - \nu\theta) =$

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Score: $\nu' V^{-1}(s - \nu\theta) = \nu' V^{-1} \nu^{1/2} \Phi^{-1}(\tilde{p})$ $\qquad \underset{\sim}{\nu}^{1/2} = \begin{pmatrix} \nu_{11}^{1/2} \\ \vdots \\ \nu_{mm}^{1/2} \end{pmatrix}$

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Convert: z-value: $\tilde{z}(\theta) = (v' V^{-1} v)^{-1/2}(S - v' V^{-1}\theta)$

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Convert: z-value: $\tilde{z}(\theta) = (v'V^{-1}v)^{-1/2}(S - v'V^{-1}\theta)$

p-value: $\tilde{p}(\theta) = \Phi\{\tilde{z}(\theta)\}$

$= \Phi\{(v'V^{-1}v)^{-1/2}\, v'V^{-1}v^{1/2}\,\Phi^{-1}(\tilde{p})\}$

Comp. p-value

6. Meta-Analysis: independent p-value functions

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given: p-value fns, independent, $p_1(\theta), \ldots, p_m(\theta)$

Informations $v = \begin{pmatrix} v_{11} \\ \\ v_{mm} \end{pmatrix}$ $V = \begin{pmatrix} v_{11} & & 0 \\ & \ddots & \\ 0 & & v_{mm} \end{pmatrix}$

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Informations $v = \begin{pmatrix} v_{11} \\ \\ v_{mm} \end{pmatrix}$ $V = \begin{pmatrix} v_{11} & \ddots & 0 \\ 0 & & v_{mm} \end{pmatrix}$

Convert to z-value $\Phi^{-1}\{p_i(\theta)\}$

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Convert to z-value $\Phi^{-1}\{p_i(\theta)\}$

score $v_{ii}^{1/2}\, \Phi^{-1}\{p_i(\theta)\}$

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Convert to z-value $\Phi^{-1}\{p_i(\theta)\}$

score $v_{ii}^{1/2}\, \Phi^{-1}\{p_i(\theta)\}$

Combine: score $\Sigma_i\, v_{ii}^{1/2}\, \Phi^{-1}\{p_i(\theta)\}$

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Convert to

z-value $\Phi^{-1}\{p_i(\theta)\}$

score $v_{ii}^{1/2}\, \Phi^{-1}\{p_i(\theta)\}$

Combine:

score $\Sigma_i\, v_{ii}^{1/2}\, \Phi^{-1}\{p_i(\theta)\}$

z-value $(v_{11} + \cdots + v_{mm})^{-1/2}\, \Sigma_i\, v_{ii}^{1/2}\, \Phi^{-1}\{p_i(\theta)\}$

6. Meta-Analysis: independent p-value functions

given: p-value fns, independent, $p_1(\theta), \ldots, p_m(\theta)$

Informations $v = \begin{pmatrix} v_{11} \\ \\ v_{mm} \end{pmatrix}$ $\quad V = \begin{pmatrix} v_{11} & & 0 \\ & \ddots & \\ 0 & & v_{mm} \end{pmatrix}$

Convert to

	z-value	$\Phi^{-1}\{p_i(\theta)\}$
	score	$v_{ii}^{1/2}\, \Phi^{-1}\{p_i(\theta)\}$

Combine:

	score	$\Sigma_i\, v_{ii}^{1/2}\, \Phi^{-1}\{p_i(\theta)\}$
	z-value	$(v_{11} + \cdots + v_{mm})^{-1/2}\, \Sigma_i\, v_{ii}^{1/2}\, \Phi^{-1}\{p_i(\theta)\}$
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Convert p-values to score; Add; Convert back to p-value!

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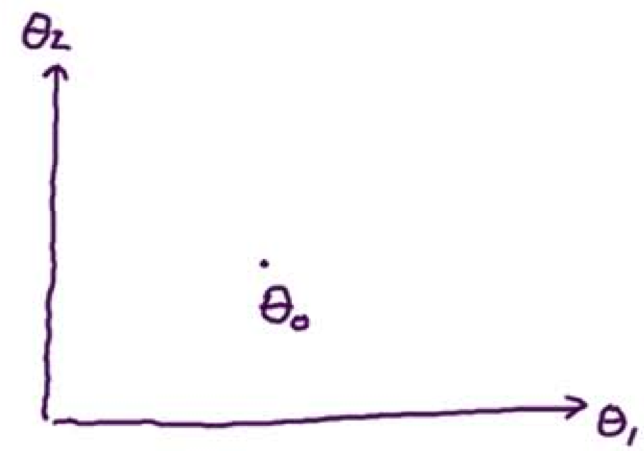

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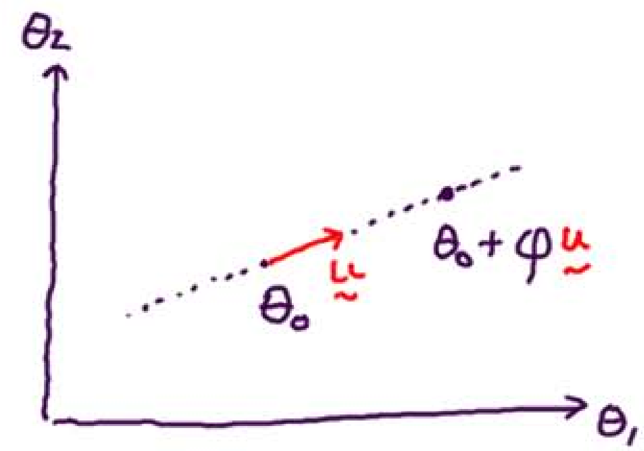

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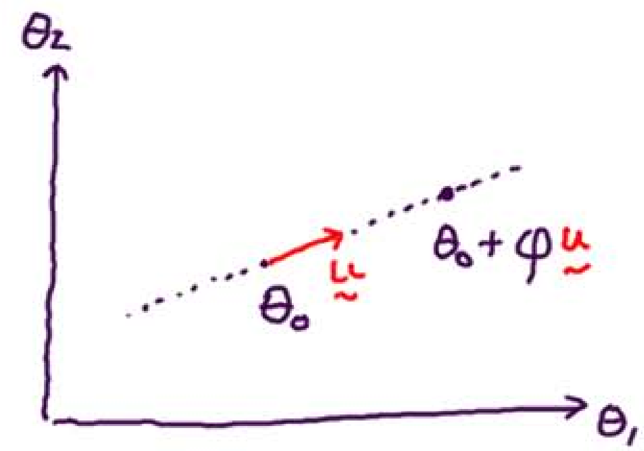

Evaluate $\Theta = \Theta_0 + \varphi \underset{\sim}{u}$ Scalar φ ... Section 3

<u>get</u> $\tilde{\ell}(\Theta_0 + \varphi \underset{\sim}{u}) = \tilde{\ell}(\Theta)$

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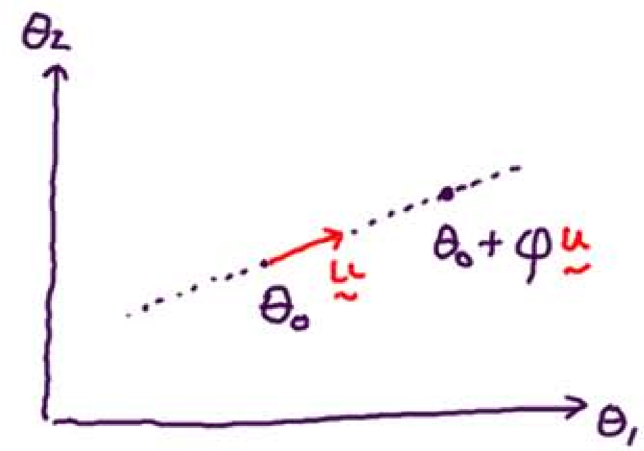

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Vector log-likelihood for θ : Easy!

First order : conditional, marginal

8 Summary

a) <u>log-Liks</u>: Have $\ell_1(\theta), \dots, \ell_m(\theta)$ Info's: $v = \begin{pmatrix} v_{11} \\ \vdots \\ v_{mm} \end{pmatrix}$ $V = (v_{ij})$

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c) Meta-Analysis: $p_1(\theta), \dots, p_m(\theta)$ Info's: v Independence

New: $\tilde{p}(\theta) = \frac{\sum_i v_{ii}^{1/2} \Phi^{-1}\{p_i(\theta)\}}{\{\sum_i v_{ii}\}^{1/2}}$

Thank you

Thank you

www.utstat.toronto.edu/dfraser/documents/WSC2015.pdf

/266.pdf

/274.pdf