Distributional Inference:
Reproducibility and limitations
"Can Bayes give 2nd order inference"

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www.utstat.Toronto.edu/dfraser/documents/Cadiz2014.pdf
: some references ... / xxx.pdf

What is Distributional Inference ?

Some history

What are the tools ?

Scalar interest parameter

What can Saddlepoint do for Bayes ?

Example 1

Scalar interest ψ, nuisance λ (2nd order)

Example 2 α linear
Example 3 μ rotating
Example 4 ψ curved

Concluding ...

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If "Maybe": "if θ values follow some pattern, …" genuine pattern ?

Efron: Science 2013 Jun 30

2 Some history

Two routes:

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Theme here: Laplace & Differential geometry $\overset{?}{\Longrightarrow}$ Reproducible distributional inference!

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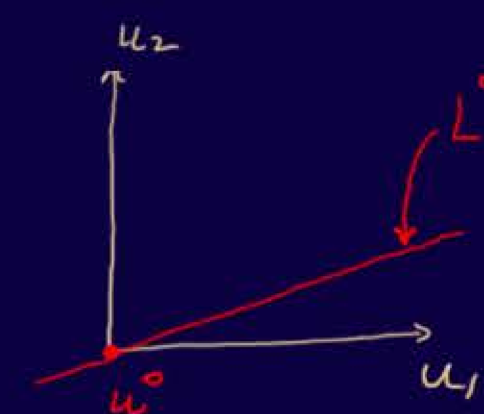

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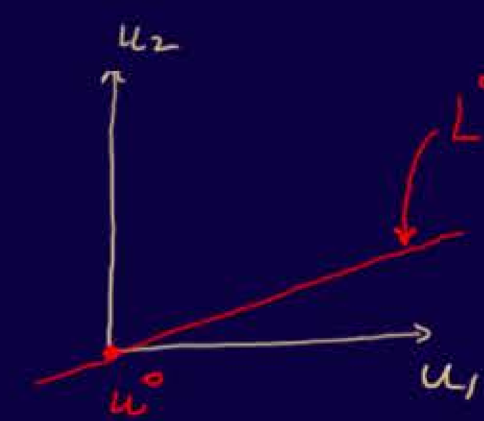

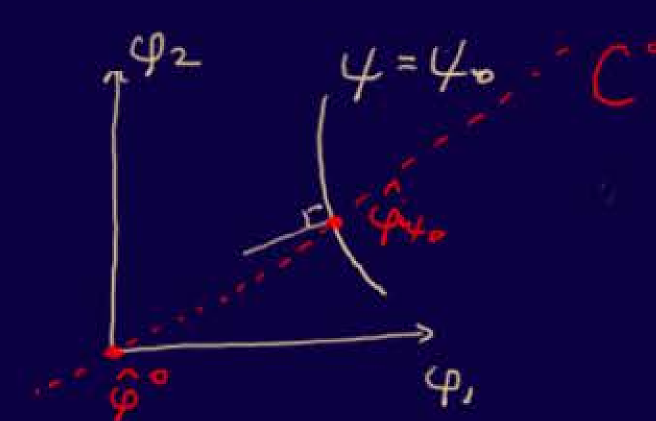

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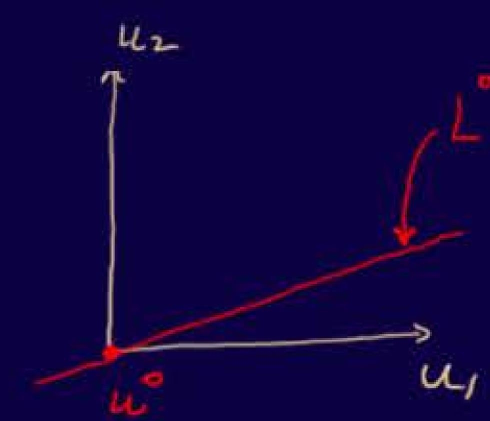

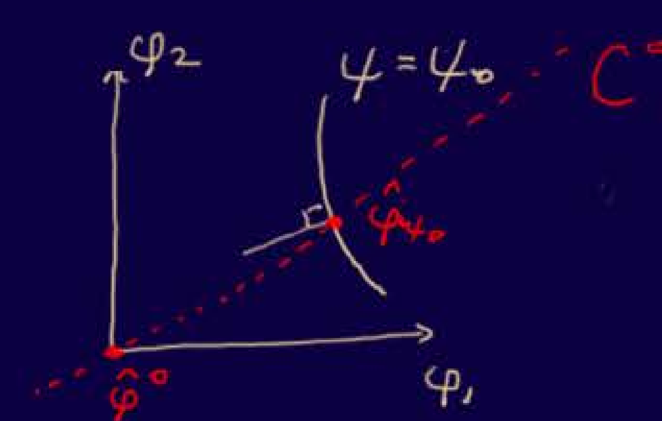

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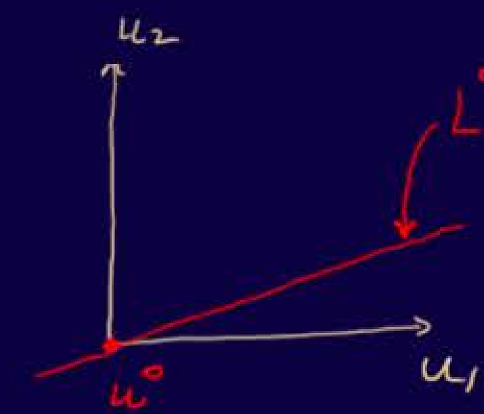

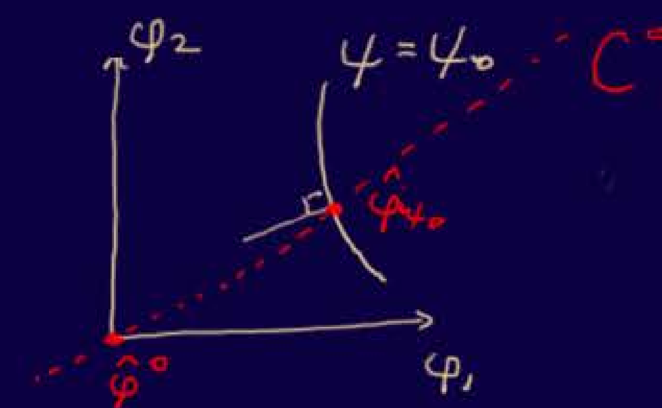

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...."where the info re ψ is located ... on the profile C° | fundamental! widely ignored !

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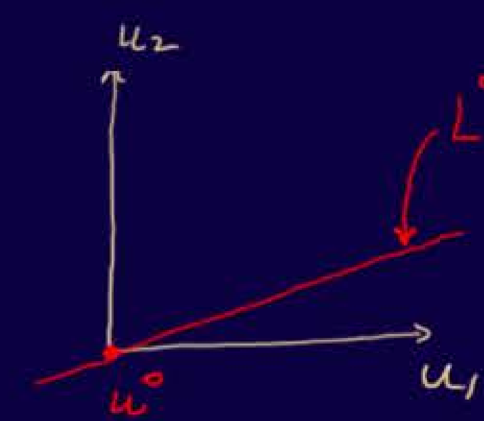

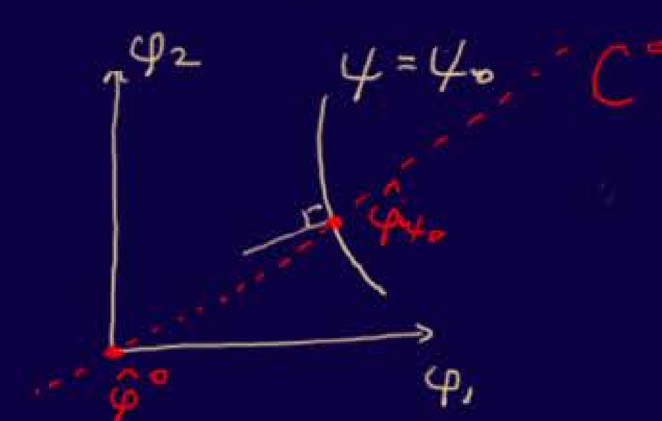

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4) Regular model ; scalar parameter ; Go profile !

Go where the info is ! (3rd)

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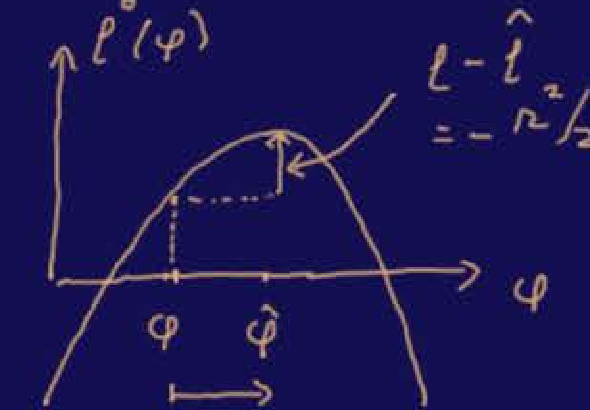

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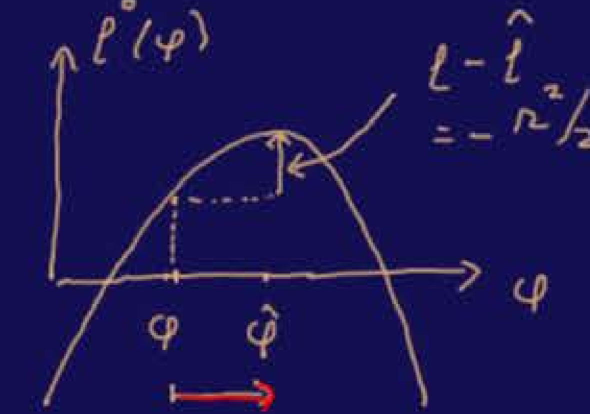

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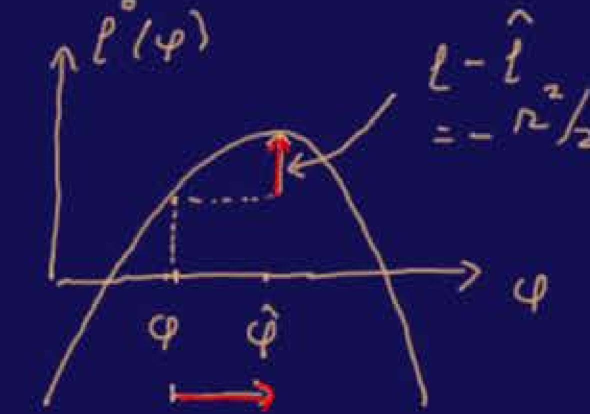

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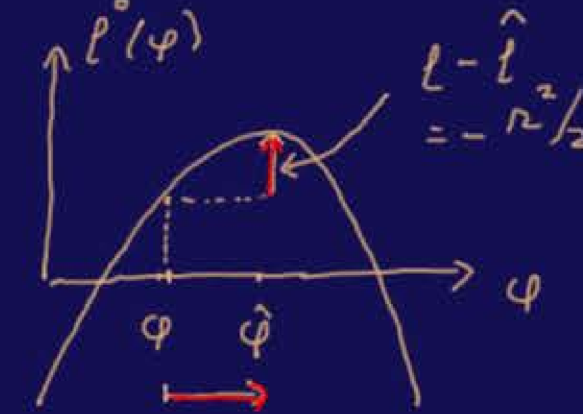

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$$r = \pm\sqrt{2(\hat{\ell} - \ell)} = \text{SLR} = \text{signed lik. root}$$

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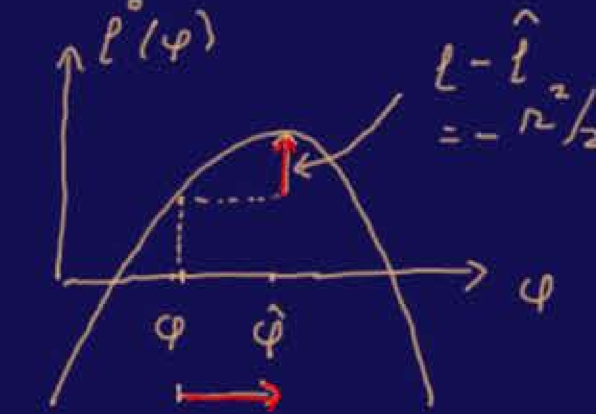

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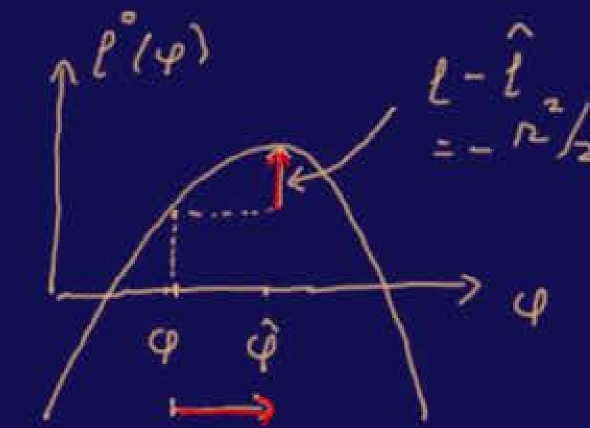

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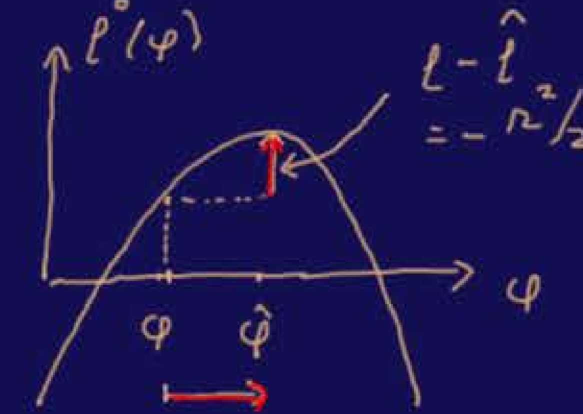
df re r $\quad F(r) = \Phi\left(r - r^{-1} \log \dfrac{r}{q}\right) = \Phi(r^*)$ $\quad \Phi(r) = \int_{-\infty}^{r} \phi(r)\, dr$

B-N (1990)

(4) For a Scalar interest: $\varphi = \varphi(\theta)$ scalar



SP $\quad \dfrac{e^{k/n}}{(2\pi)^{1/2}} e^{\ell - \hat{\ell}} \, j_{\varphi\varphi}^{-1/2}(\hat{\varphi}) \cdot du$

Horizontal departure $\quad q = (\hat{\varphi} - \varphi)\, \hat{j}_{\varphi\varphi}^{1/2}$ = mle departure

Vertical " $\quad \hat{\ell} - \ell = r^2/2$ = loglik ratio

$r = \pm\sqrt{2(\hat{\ell} - \ell)}$ = SLR = signed lik. root

Density re r $\quad f(r) \cdot dr = \underbrace{e^{k/n} \phi(r) \dfrac{r}{q} dr}$ $\qquad \phi(r) = \dfrac{1}{\sqrt{2\pi}} e^{-r^2/2}$ Calculus

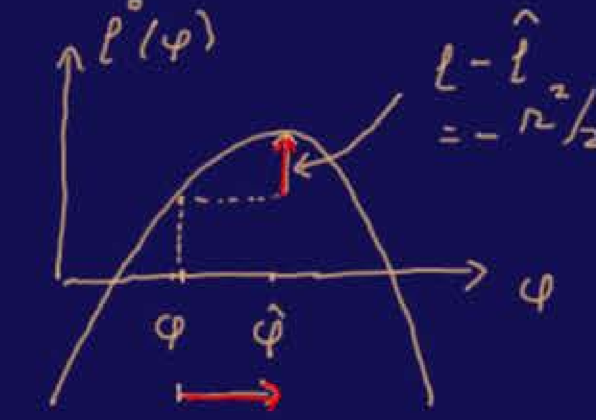
df re r $\quad F(r) = \Phi\left(r - r^{-1} \log \dfrac{r}{q}\right) = \Phi(r^*)$ $\qquad \Phi(r) = \int_{-\infty}^{r} \phi(r)\, dr$

B-N (1990)

High accuracy $\quad O(n^{-3/2})$

(4) For a <u>Scalar</u> interest: $\varphi = \varphi(\theta)$ scalar

SP $\quad \dfrac{e^{k/n}}{(2\pi)^{1/2}} e^{\ell - \hat{\ell}} \, \hat{j}_{\varphi\varphi}^{-1/2}(\hat{\varphi}) \cdot du$



<u>Horizontal</u> departure $\quad q = (\hat{\varphi} - \varphi)\, \hat{j}_{\varphi\varphi}^{1/2}$ = mle departure

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<u>Widely</u> useful $\quad$ With nuisance parameter $\quad q \longrightarrow Q$ = adjusted q

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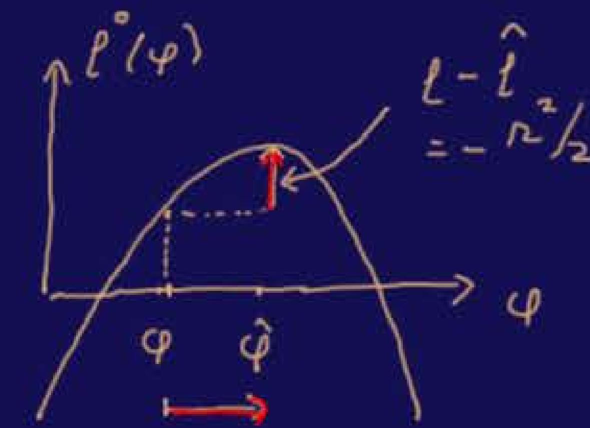

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B-N (1990)

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Widely useful $\quad$ With nuisance parameter $\quad q \longrightarrow Q$ = adjusted q

Above by: Simple change of variable
" integration

Just calculus! & Answers major questions!

5. What can SP do for Bayes? a) Scalar parameter: <u>Welch Peers</u>

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Have likelihood $L^{\circ}(\theta)$

add weight fn $\pi(\theta)$

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$\updownarrow$ switch

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Posterior (loc'n flip) $f(\beta | \hat{\beta}) = \frac{1}{\sqrt{2\pi}} \exp\left\{ \frac{(\hat{\beta}-\beta)^2}{2} - \gamma (\hat{\beta}-\beta)^3/6n^{1/2} \right\} \cdot d\beta$ $d\beta = \underline{(1 + \gamma\varphi/n^{1/2})}\, d\varphi$

root information prior

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root information prior

(Jeffreys prior)

Welch Peers 1963

Brown Cai DasGupta 2001
(binomial)

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but Jeffreys rejected his prior when $p>1$

but ...... it may be better than he expected!

5-1

Scalar parameter

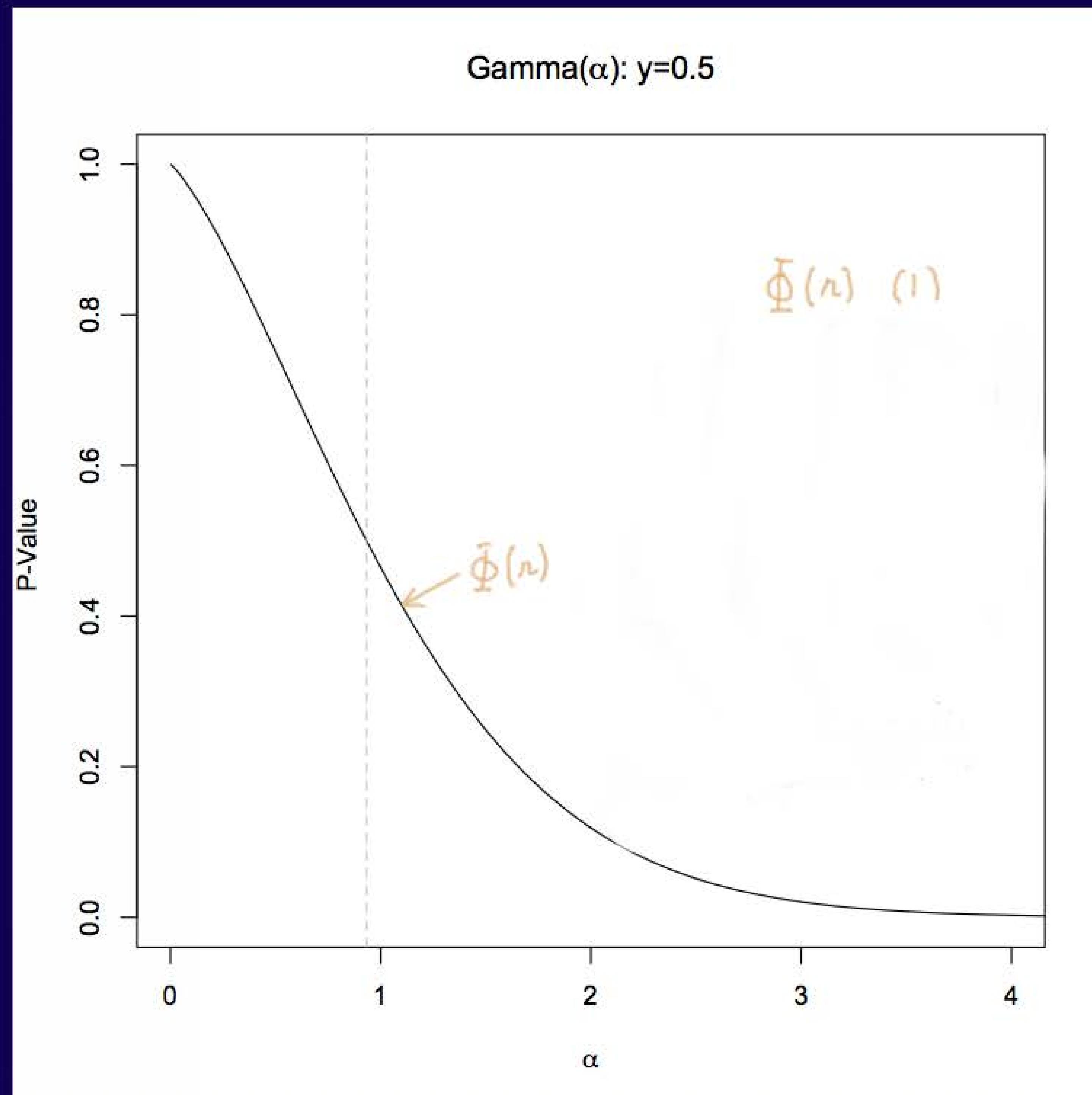
Example 1

Setup

- Model: $Y \sim Gamma(\alpha, \beta)$ where α is shape and β is rate
  pdf is $\Gamma^{-1}(\alpha)\, y^{\alpha-1} e^{-y} \cdot dy$ ... with $\beta = 1$

  Let $n = 1$ $y^\circ = .5$

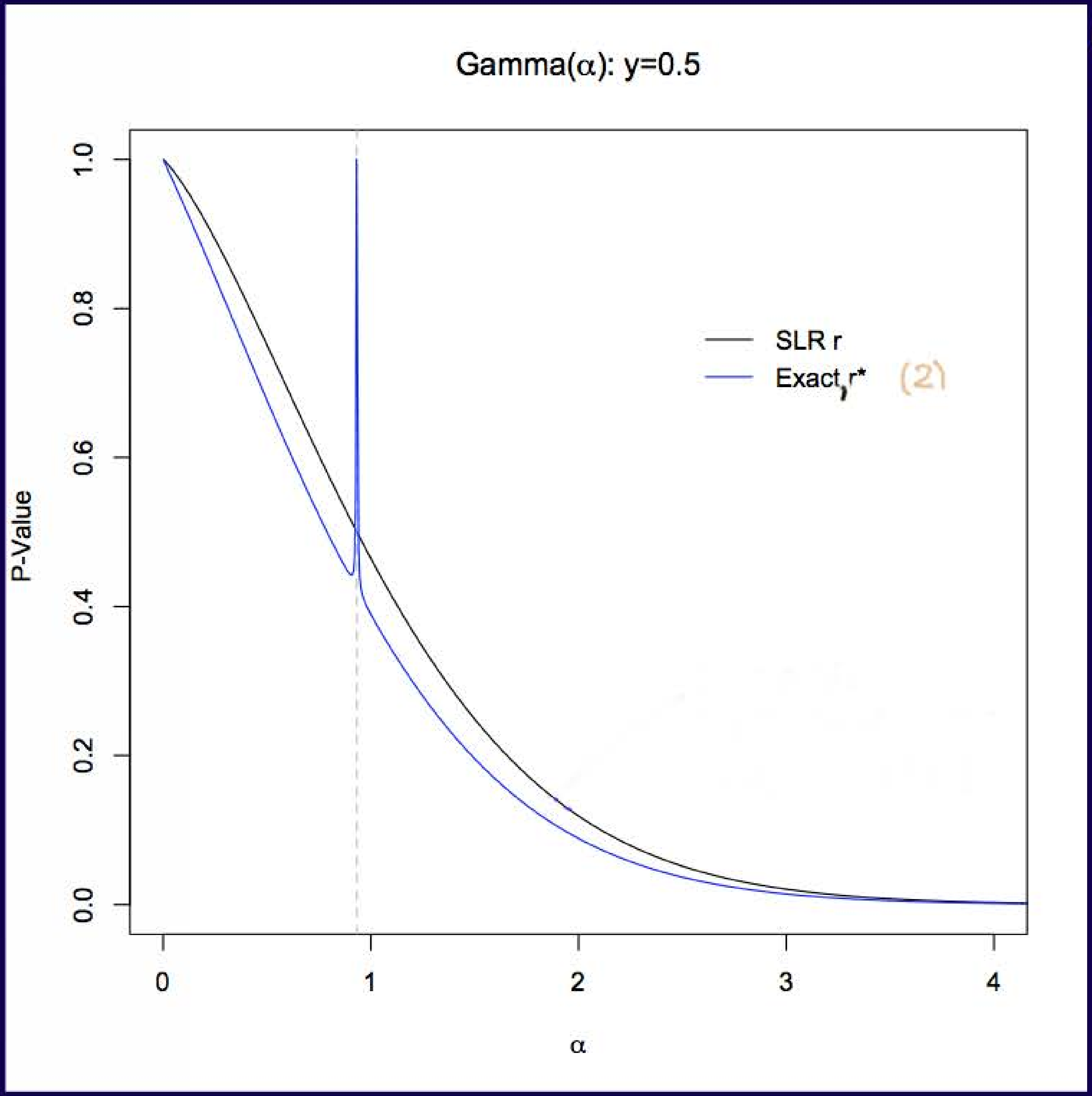
  Parameter of interest is α



(1)

What likelihood says: $\Phi(r)$

r = SLR Φ df N01



(1)

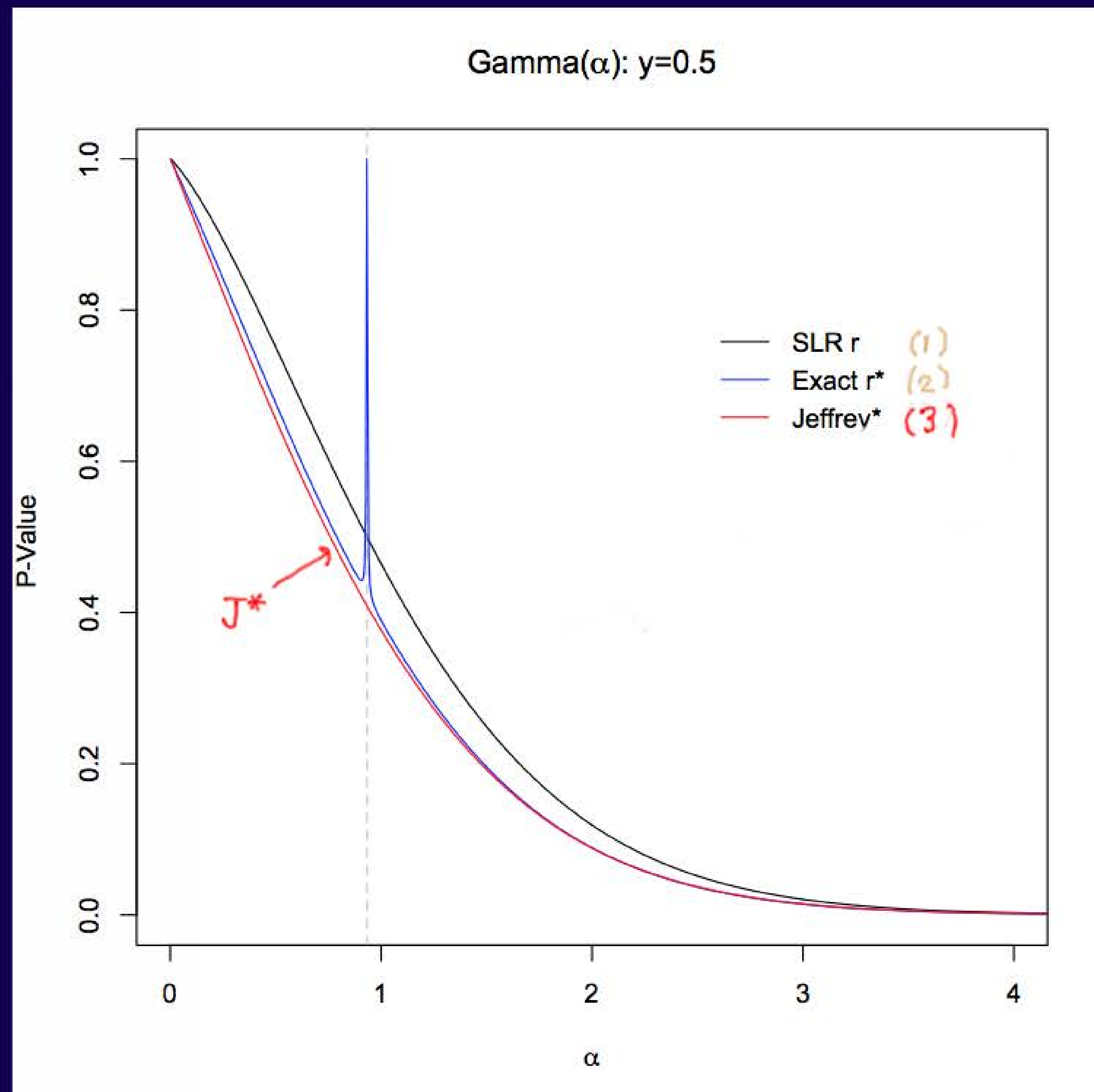
What likelihood says: $\Phi(r)$

r = SLR $\quad \Phi$ df N_{01}

(2)

Third order r^*

Exact



(1)

What likelihood says: $\Phi(r)$

r = SLR Φ df N01

(2)

Third order r^*

Exact

(3)

Jeffreys J^* "$p = 1$"

Closely hugs exact

J^* is "<u>reproducible</u>"

6 Bayes: Vector parameter

Interest $\psi = \psi(\varphi)$ Scalar

Nuisance $\lambda = \psi(\varphi)$

6 Bayes: Vector parameter

Interest $\psi = \psi(\varphi)$ Scalar

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SP separates: $$f(s;\psi)\, ds \cdot \frac{|J_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{-1/2}}{(2\pi)^{\frac{p-1}{2}}} \exp\{\ell - \tilde{\ell}\}\, dt$$

3rd

(continuity, interest, other)

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3rd

(continuity, interest, other)

Bayes: Get rid of nuisance

(1) Norming constant

$$\downarrow$$

$$\frac{|J_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{(2\pi)^{-\frac{p-1}{2}}}$$

6 Bayes: Vector parameter

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Nuisance $\lambda = \psi(\varphi)$

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3rd

(continuity, interest, other)

Bayes: Get rid of nuisance

(1) Norming constant (2) Put on profile

$$\dfrac{|J_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{(2\pi)^{\frac{p-1}{2}}} \cdot \quad 1$$

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SP separates: $\quad f(s;\psi)\,ds \;\cdot\; \dfrac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{-1/2}}{(2\pi)^{\frac{p-1}{2}}} \cdot \exp\{\ell - \hat{\ell}\}\,dt$ $\qquad$ 3rd (continuity, interest, other)

$$\frac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{(2\pi)^{\frac{p-1}{2}}} \cdot \quad 1$$

Bayes: Get rid of nuisance

(1) Norming constant (2) Put on profile

Welch-Peers for ψ $\qquad |j^{p}_{(\psi\psi)}(\psi)|^{1/2}$

6 Bayes: Vector parameter

Interest $\psi = \psi(\varphi)$ Scalar

Nuisance $\lambda = \psi(\varphi)$

SP separates: $f(s;\psi)\, ds \cdot \dfrac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{-1/2}}{(2\pi)^{\frac{p-1}{2}}} \cdot \exp\{\ell - \hat{\ell}\}\, dt$ 3rd (continuity, interest, other)

$$\dfrac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{(2\pi)^{\frac{p-1}{2}}} \cdot 1$$

Bayes: Get rid of nuisance

(1) Norming constant (2) Put on profile

Welch-Peers for ψ $|j^{p}_{(\psi\psi)}(\psi)|^{1/2}$

Combine (full prior)

- On profile
- One dimensional

$$|j_{\varphi\varphi}(\hat{\varphi}_\psi)|^{1/2} \frac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{|j_{[\lambda\lambda]}(\hat{\varphi}_\psi)|^{1/2}}$$

6 Bayes: Vector parameter

Interest $\psi = \psi(\varphi)$ Scalar

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3rd

(continuity, interest, other)

Bayes: Get rid of nuisance

(1) Norming constant (2) Put on profile

$$\frac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{(2\pi)^{\frac{p-1}{2}}} \cdot \quad 1$$

Welch-Peers for ψ $\quad |j^{p}_{[\psi\psi]}(\psi)|^{1/2}$

Combine (full prior)

- On profile
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$$|j_{\varphi\varphi}(\hat{\varphi}_\psi)|^{1/2} \cdot \frac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{|j_{[\lambda\lambda]}(\hat{\varphi}_\psi)|^{1/2}}$$

Regular Jeffreys $\qquad$ Nonlinearity adjustment

6 Bayes: Vector parameter

Interest $\psi = \psi(\varphi)$ Scalar

Nuisance $\lambda = \psi(\varphi)$

SP separates: $f(s;\psi)\, ds \cdot \dfrac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{-1/2}}{(2\pi)^{\frac{p-1}{2}}} \cdot \exp\{\ell - \tilde{\ell}\}\, dt$ 3rd (continuity, interest, other)

$$\frac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{(2\pi)^{\frac{p-1}{2}}} \cdot 1$$

Bayes: Get rid of nuisance

(1) Norming constant (2) Put on profile

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Combine (full prior)

- On profile
- One dimensional

$$|j_{\varphi\varphi}(\hat{\varphi}_\psi)|^{1/2} \frac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{|j_{[\lambda\lambda]}(\hat{\varphi}_\psi)|^{1/2}}$$

Root nuisance info re ψ

Root nuisance info re linear χ

Regular Jeffreys

Nonlinearity adjustment

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$$\downarrow \qquad \frac{|J_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{(2\pi)^{\frac{p-1}{2}}} \cdot 1$$

Bayes: Get rid of nuisance

(1) Norming constant (2) Put on profile

Welch-Peers for ψ $\quad |j^{p}_{[\psi\psi]}(\psi)|^{1/2}$

Combine (full prior)

- On profile
- One dimensional

$$\underbrace{|J_{\varphi\varphi}(\hat{\varphi}_\psi)|^{1/2}}_{\text{Regular Jeffreys}} \cdot \underbrace{\frac{|j_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}}{|j_{[\lambda\lambda]}(\hat{\varphi}_\psi)|^{1/2}}}_{\text{Nonlinearity adjustment}}$$

Root nuisance info re ψ

Root nuisance info re linear χ

Second order

Jeffreys works but use only on one-dim profile

i.e. a delta function ----- on

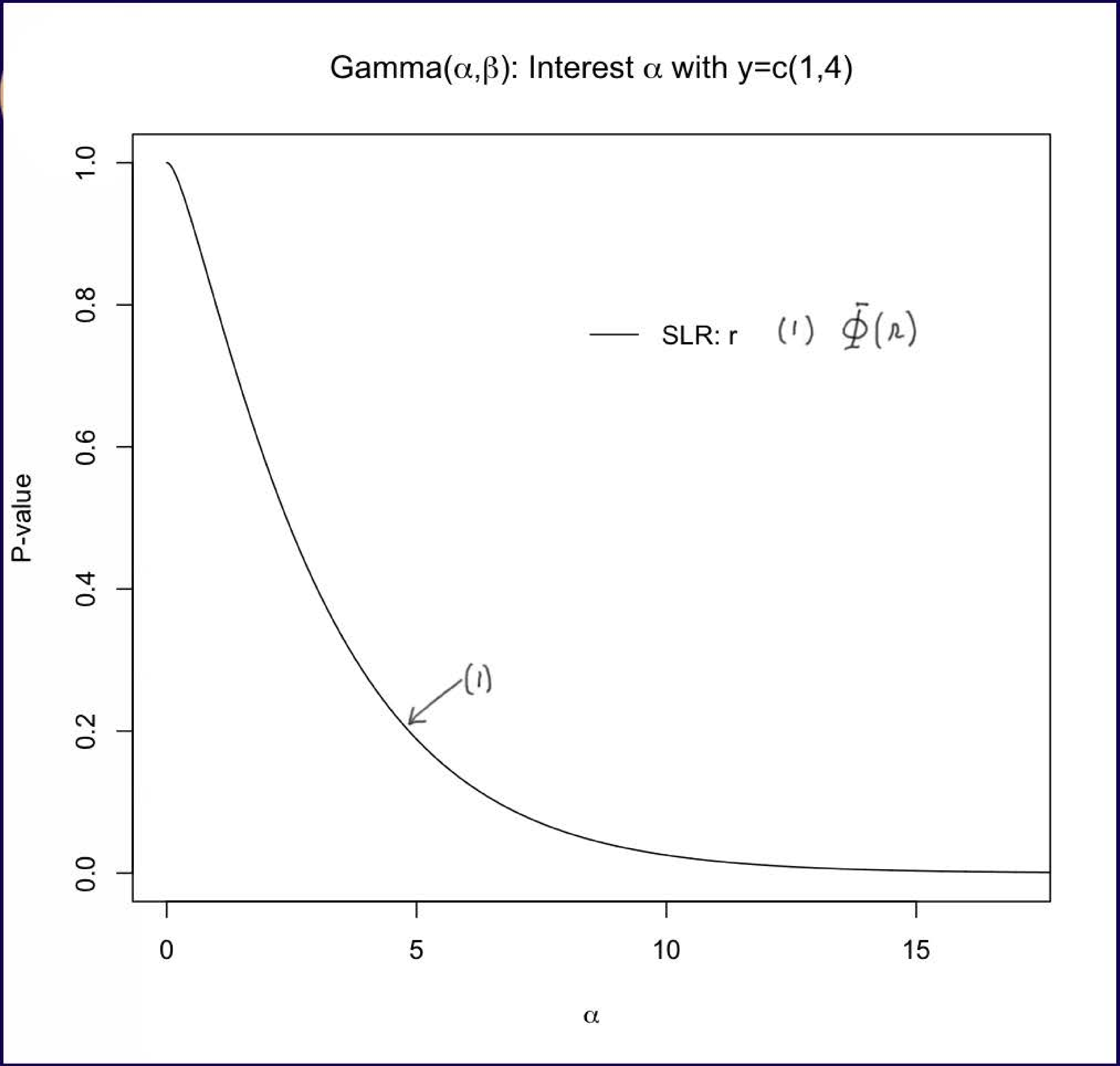
6-2 Vector parameter (α, β); Scalar linear interest α

Example 2

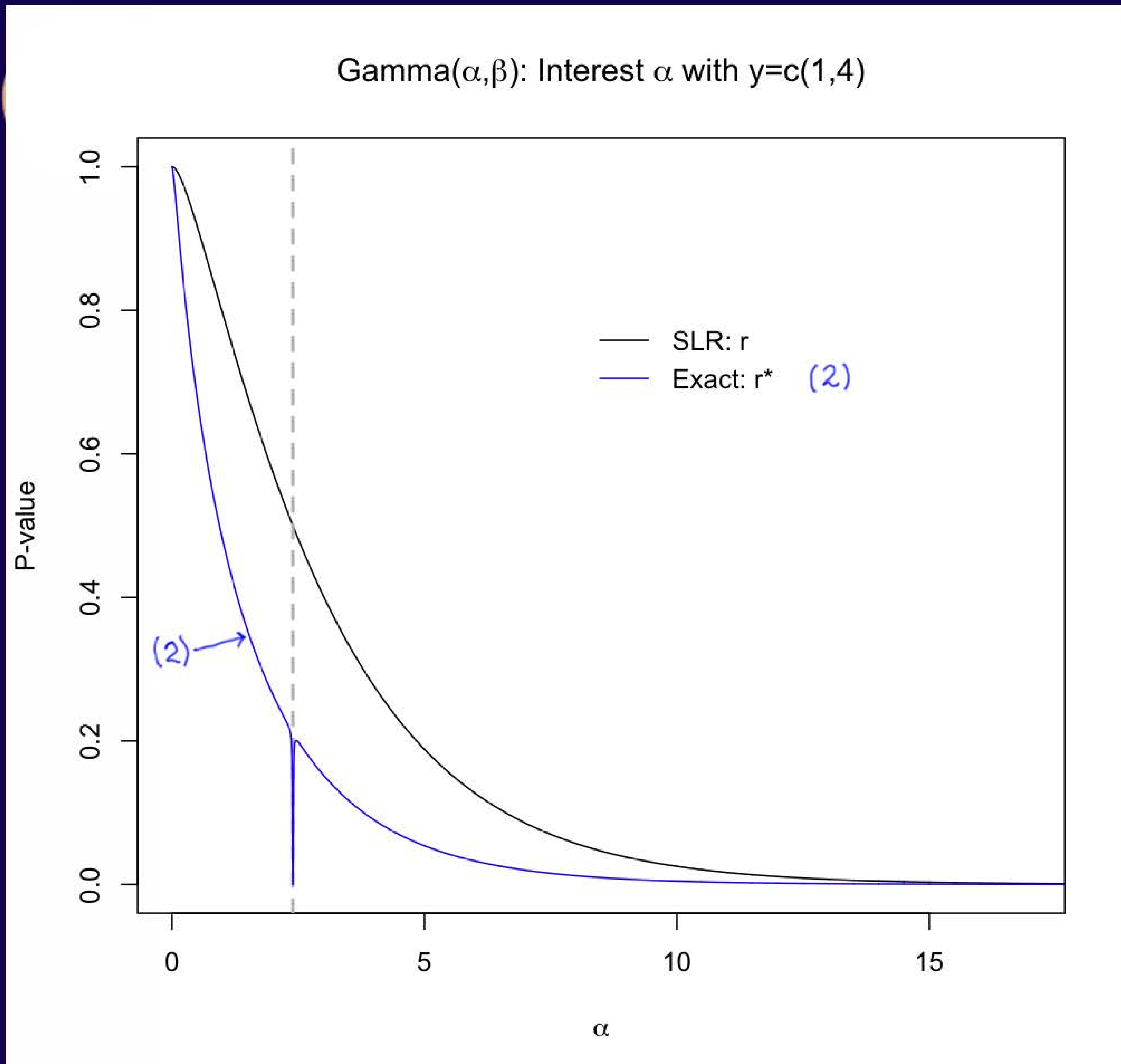
Setup

- Model: $Y \sim Gamma(\alpha, \beta)$ where α is shape and β is rate
  pdf is $\frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y}$
  Let $n = 2$ and data is $(y_1, y_2) = (1, 4)$

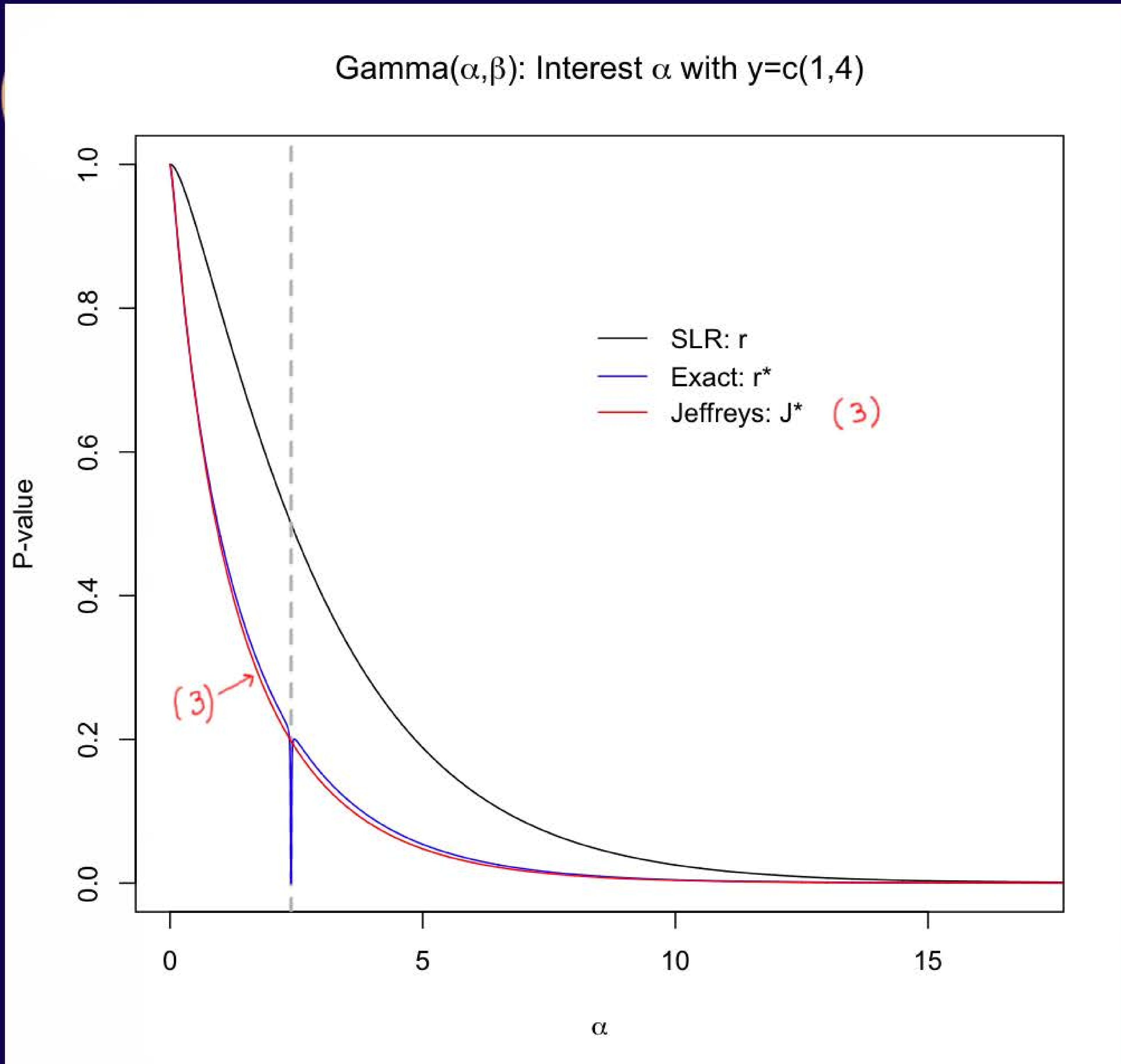
  Parameter of interest is α, β free nuisance



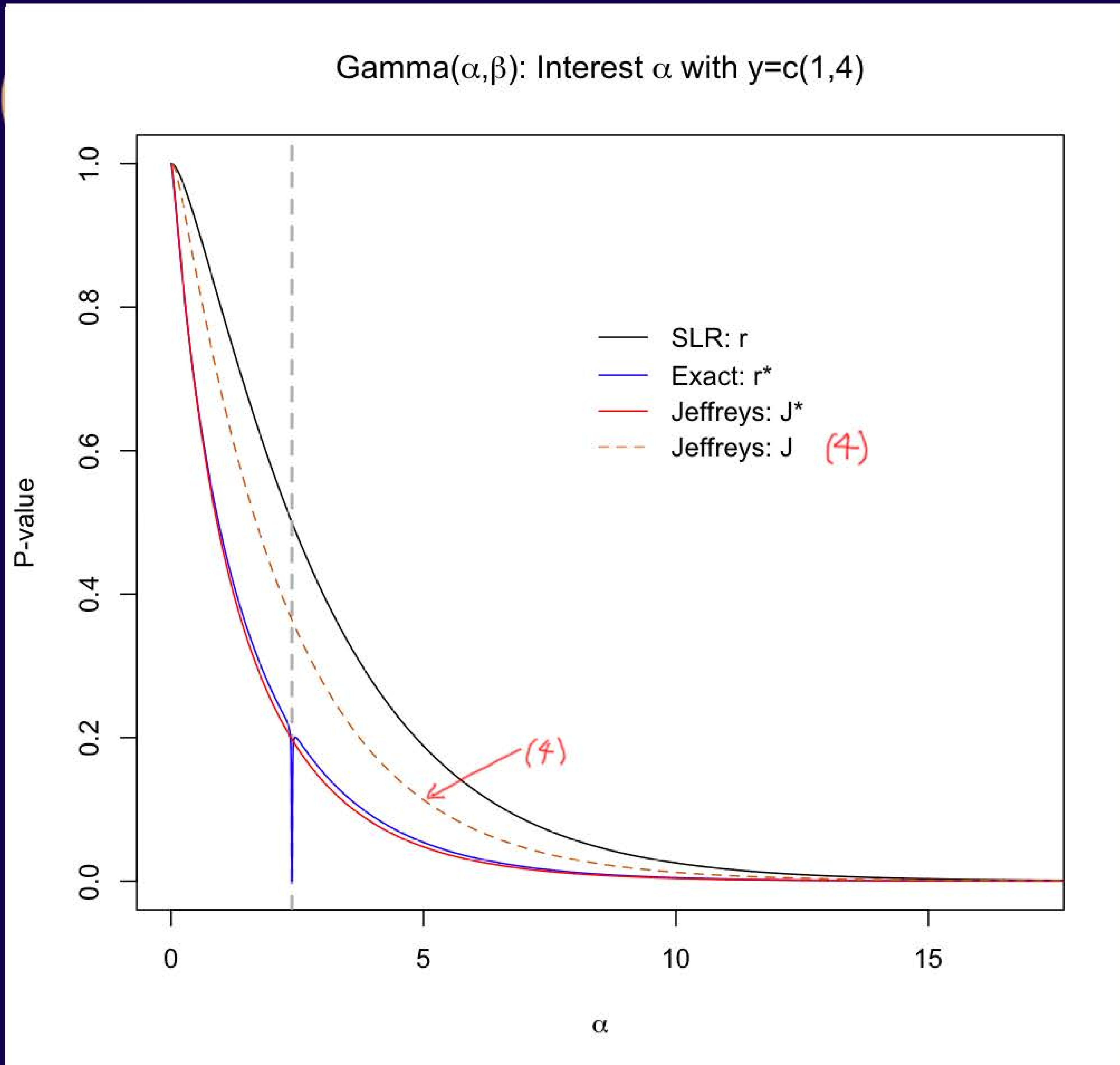
Gamma(α,β): Interest α with y=c(1,4)
P-value
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
α
SLR: r (1) $\bar{\Phi}(r)$
(1)


Gamma(α,β): Interest α with y=c(1,4)
SLR: r
Exact: r*
(2)
(2)
P-value
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
α

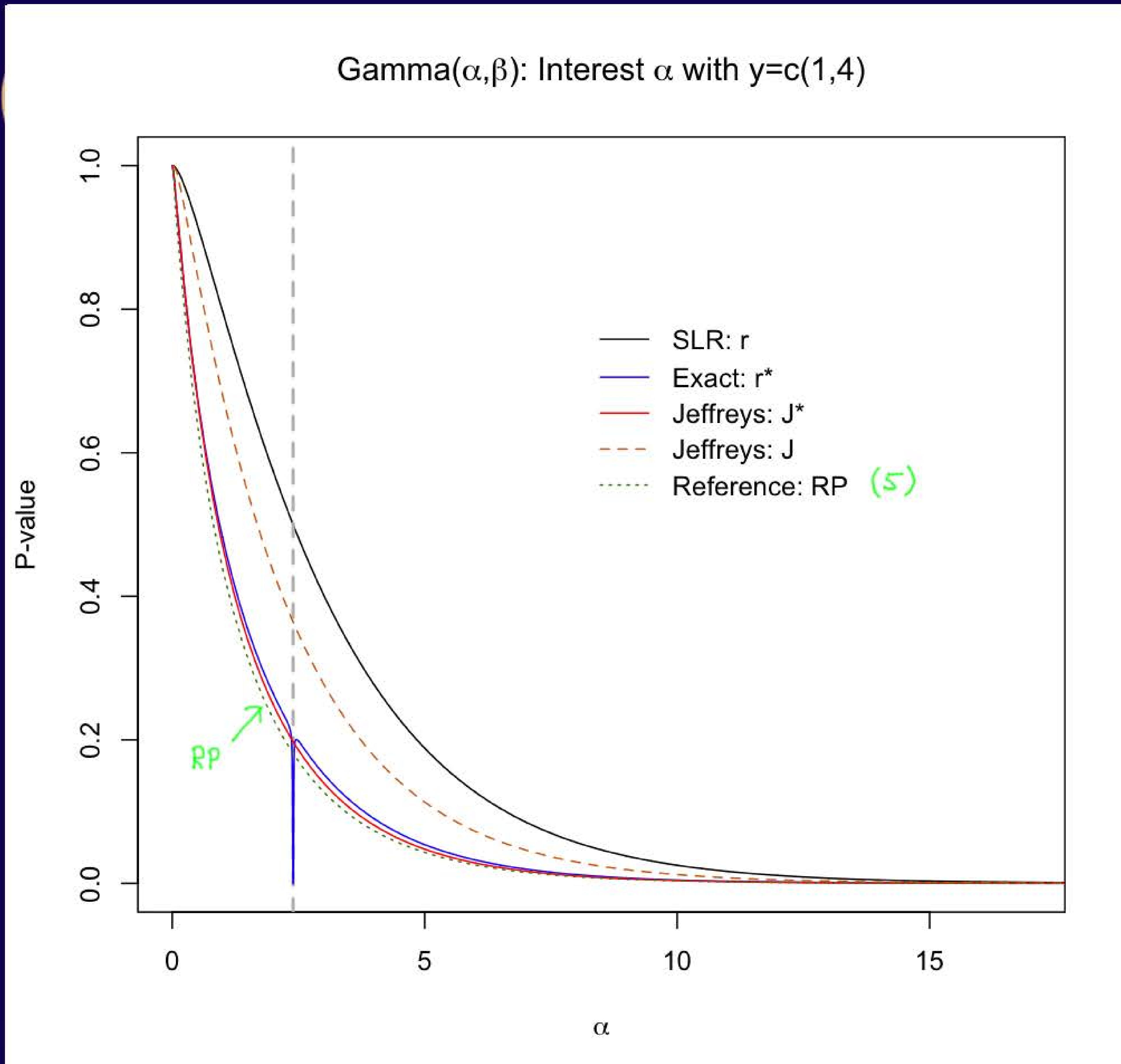
(2) $\Phi(r^*)$



(3) Profile Jeffreys J^*



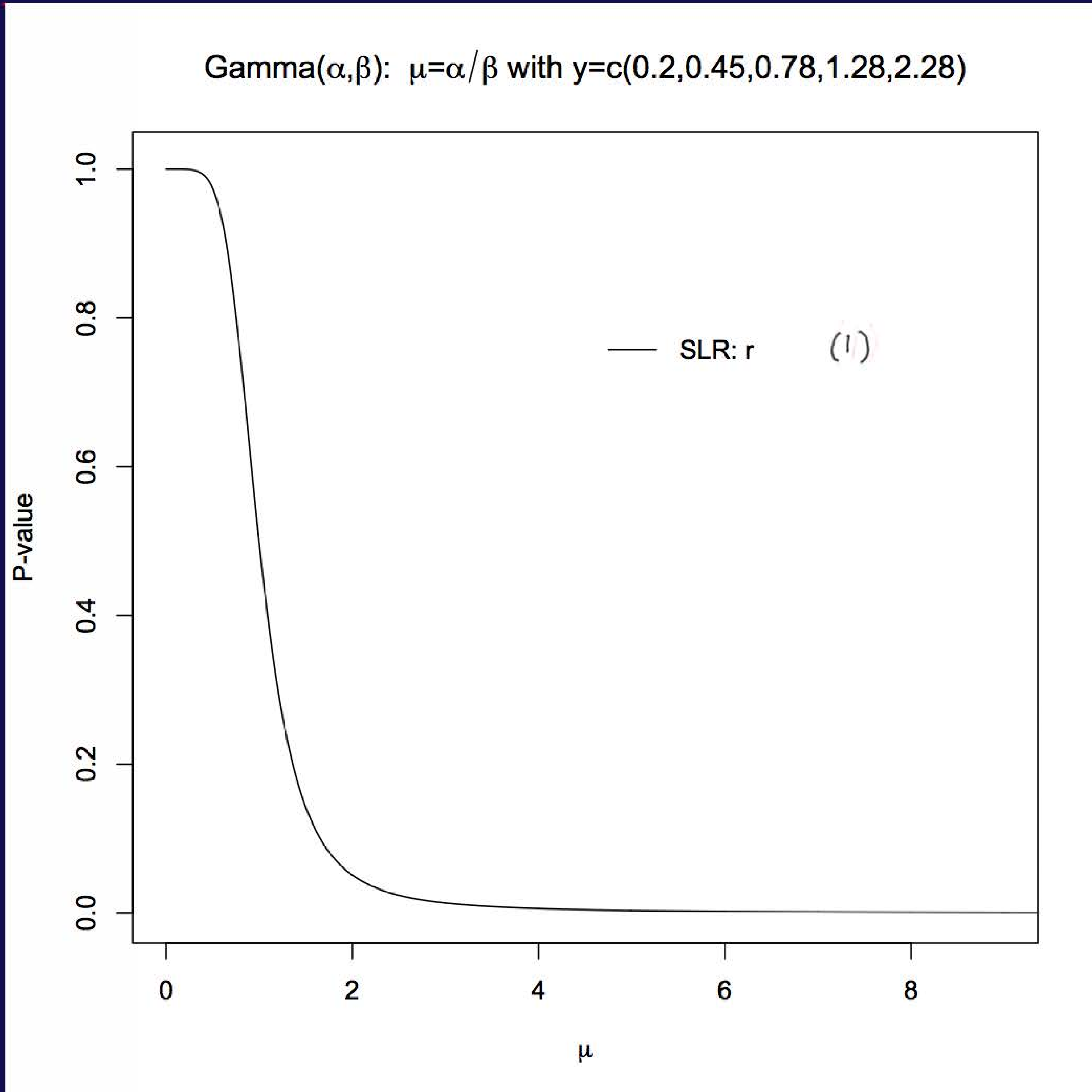
(4) Full Jeffreys



(5) Targeted Reference

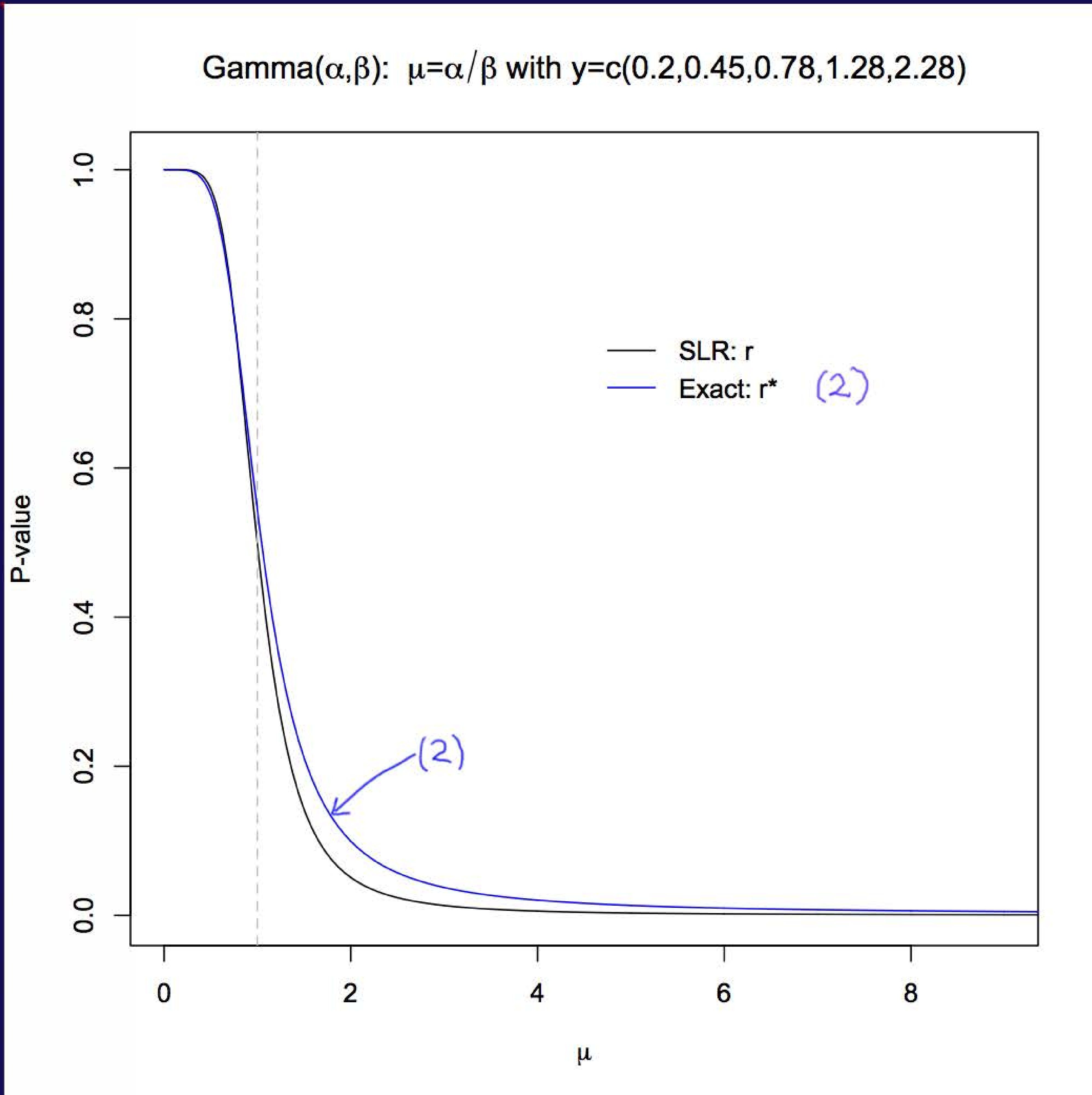
6-3 Gamma(α, β) : Interest μ ... <u>rotating</u>

Gamma(α,β): $\mu=\alpha/\beta$ with y=c(0.2,0.45,0.78,1.28,2.28)



Gamma(α,β): μ=α/β with y=c(0.2,0.45,0.78,1.28,2.28)
1.0
0.8
0.6
0.4
0.2
0.0
P-value
SLR: r
(1)
0
2
4
6
8
μ

(1) $p(\mu) = \Phi(r)$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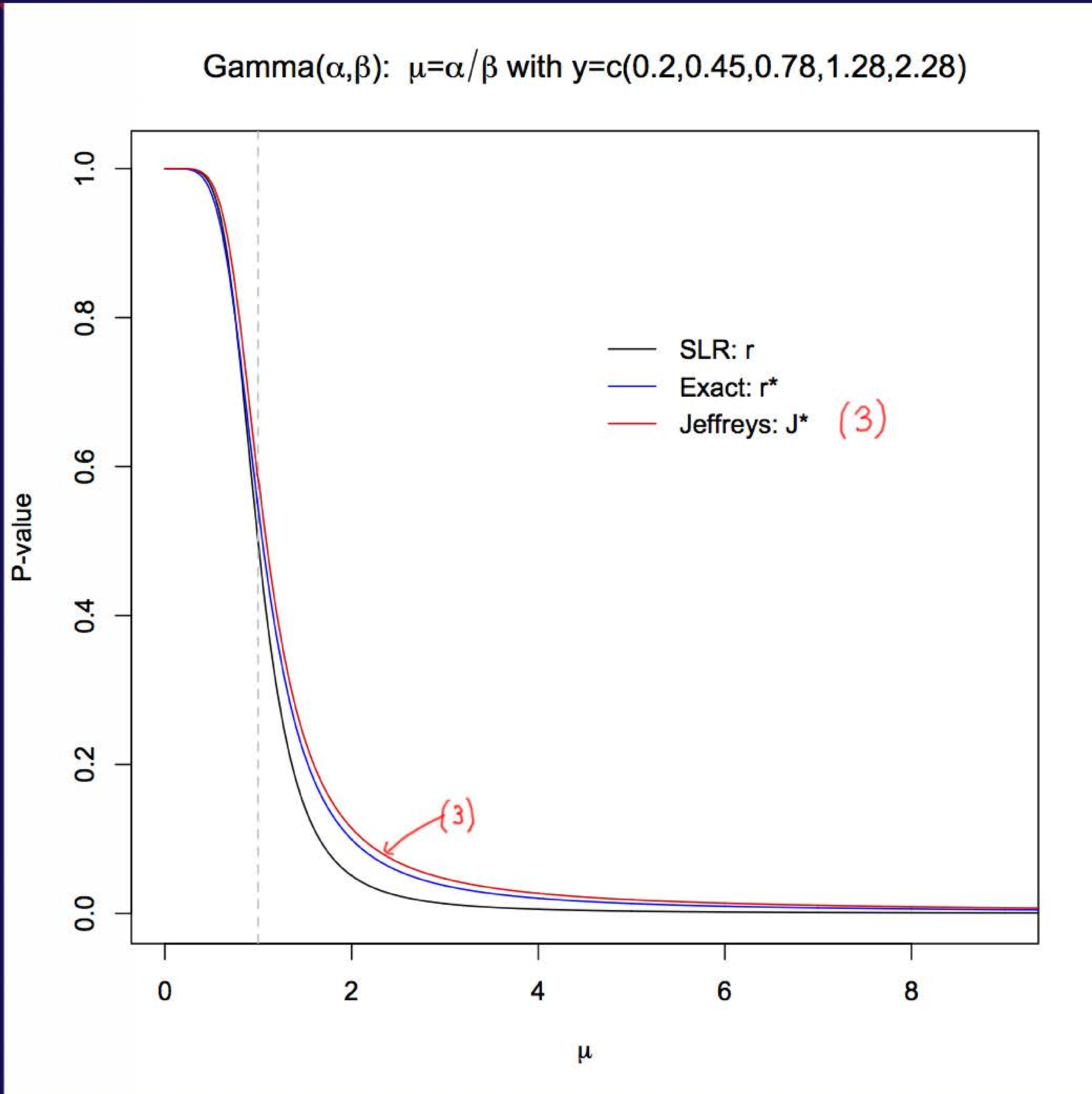
P-value
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
μ
SLR: r
Exact: r*
(2)
(2)

$p(\mu) = \Phi(r^*)$



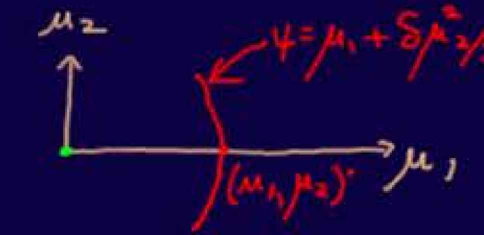
$p(\mu) = \text{Jeff}^*$

6-4

$N\begin{pmatrix}\mu_1 \\ \mu_2\end{pmatrix}; I)$ $\quad \psi = \mu_1 + \delta \mu_2^2/2$

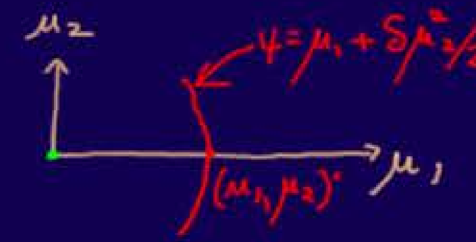
$\psi^0 = \begin{pmatrix}0 \\ 0\end{pmatrix}$

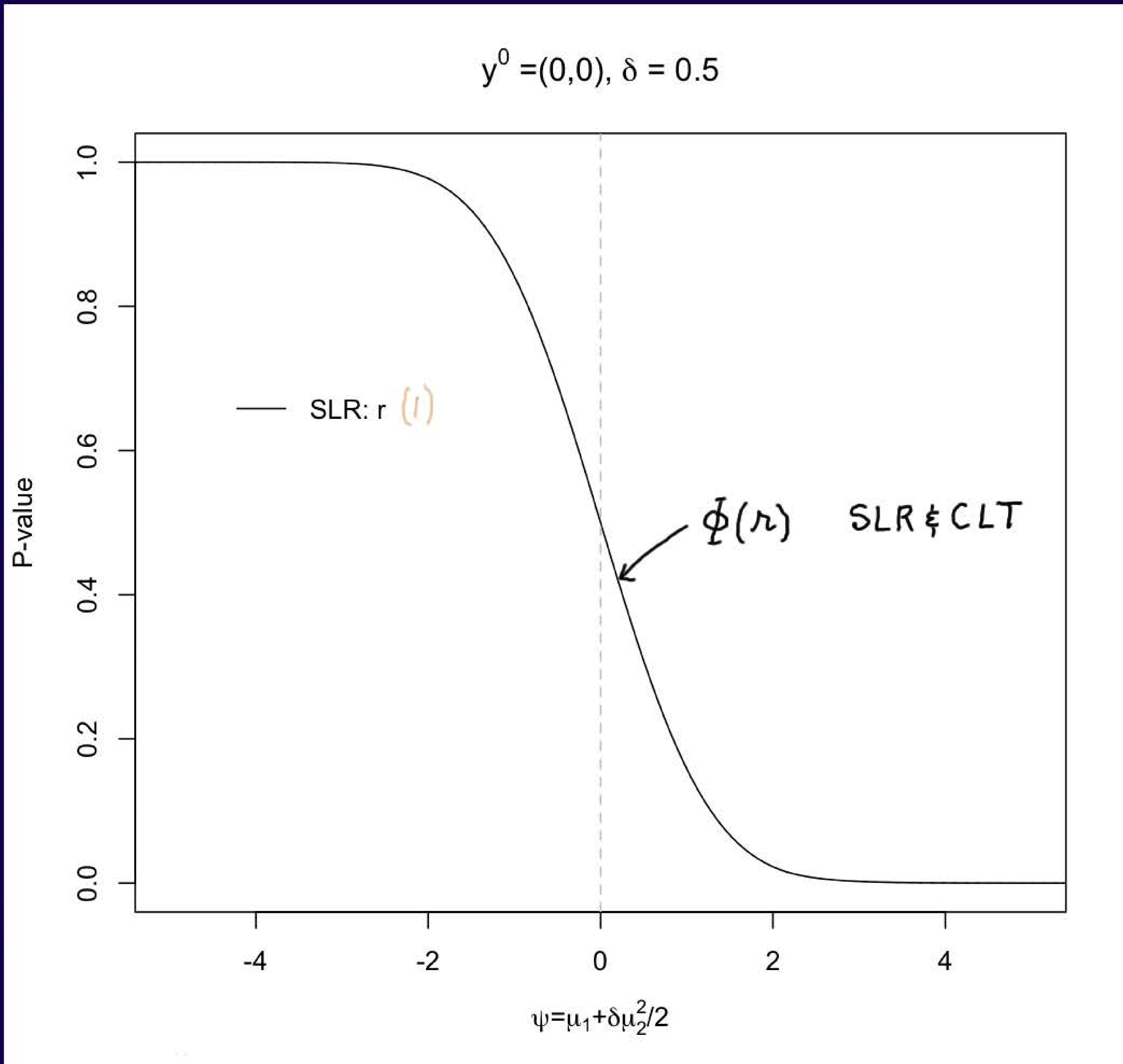
Curved Interest (A story in itself!)



6-4 $\quad N\left(\begin{matrix}\mu_1\\ \mu_2\end{matrix}; I\right) \quad \psi = \mu_1 + \delta\mu_2^2/2$

$y^0 = \left(\begin{matrix}0\\ 0\end{matrix}\right)$ Curved Interest (A story in itself!)





(1) SLR

$\Phi(r)$

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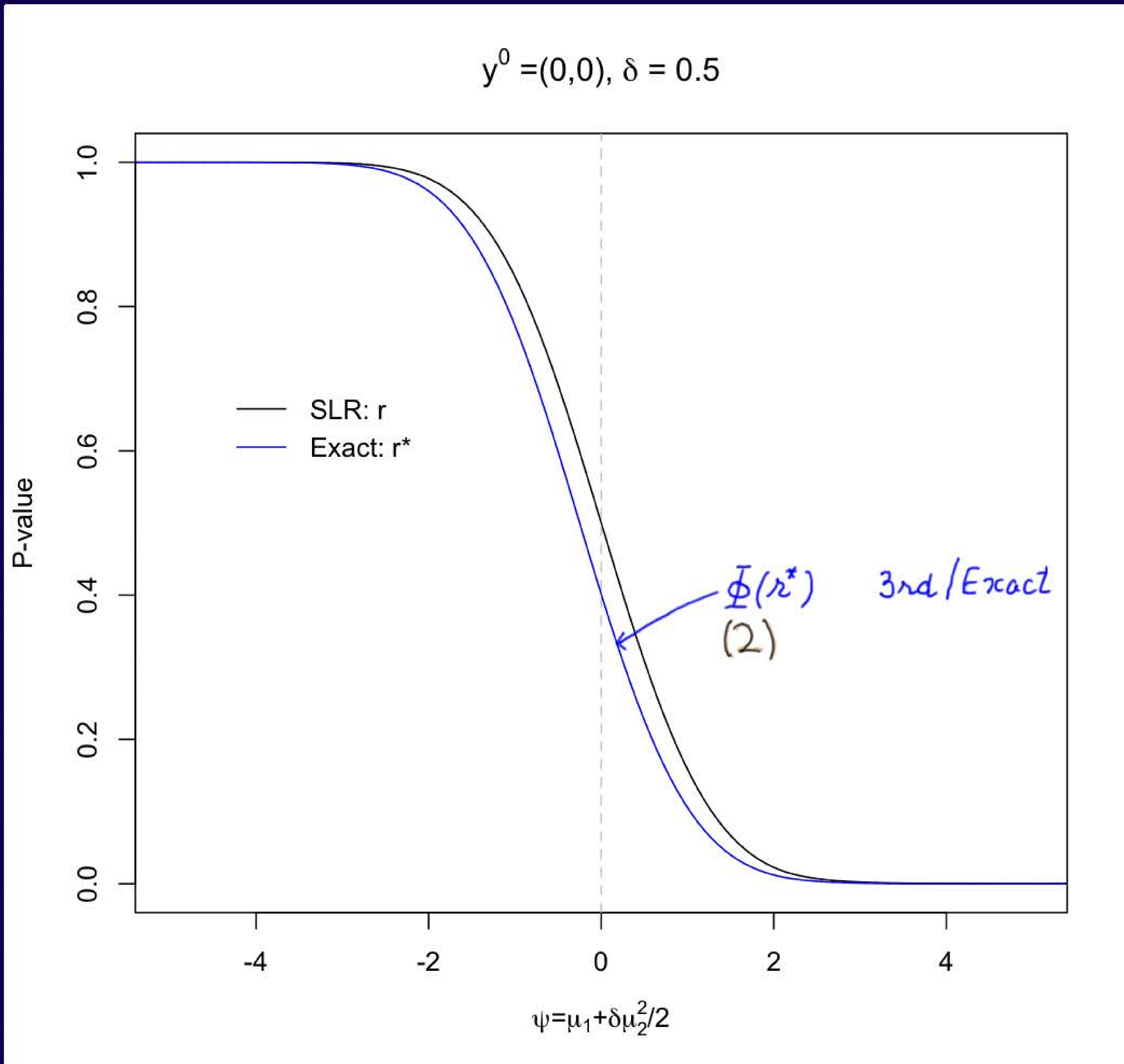

(1) SLR
$\Phi(r)$

(2) Exact
$\Phi(r^*)$

$N\left(\begin{smallmatrix}\mu_1\\ \mu_2\end{smallmatrix}; I\right)$ $\psi = \mu_1 + \delta\mu_2^2/2$

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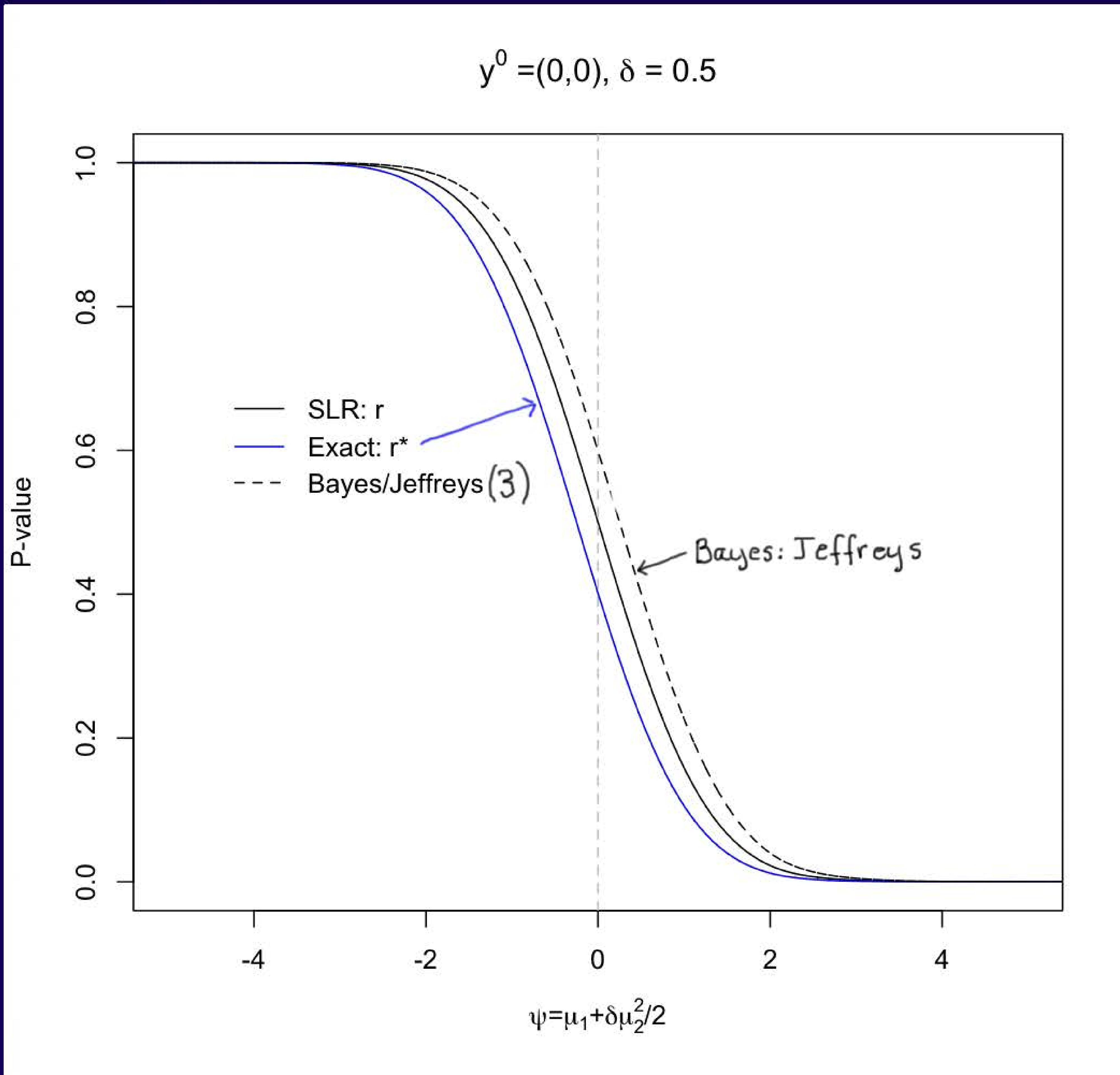

(1) SLR

$\Phi(r)$

(2) Exact

$\Phi(r^*)$

(3) Bayes

Plain Jeffreys

(Goes farther away from the Exact)

Worse than just L

$N\left(\begin{smallmatrix}\mu_1\\ \mu_2\end{smallmatrix}; I\right)$ $\psi = \mu_1 + \delta\mu_2^2/2$

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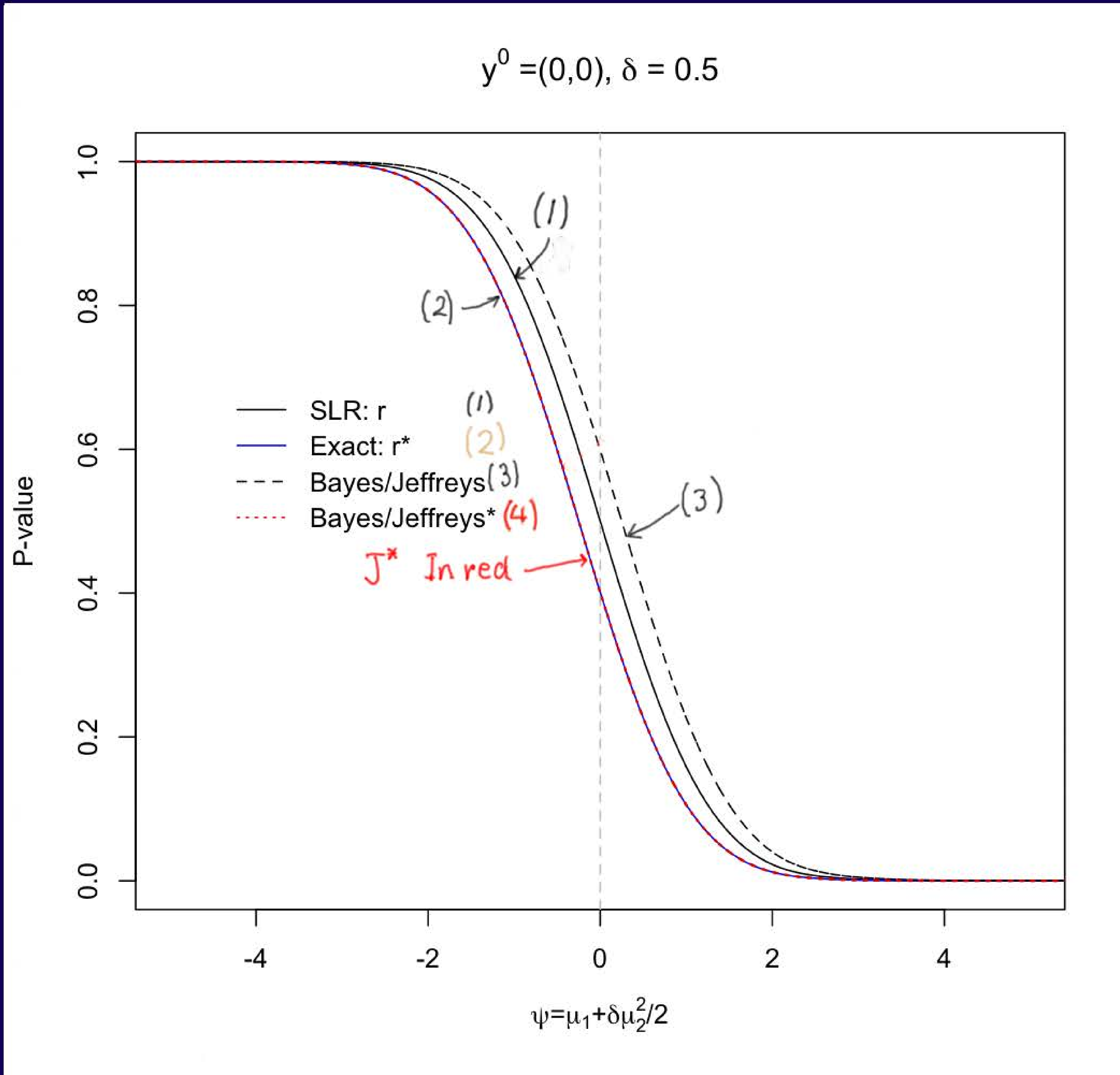

(1) SLR

$\Phi(r)$

(2) Exact

$\bar{\Phi}(r^*)$

(3) Bayes

Plain Jeffreys

(goes farther away from the Exact)

(4) J^*

Jeffreys but just on profile curve for interest α

J^* almost duplicates the exact

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3. For <u>scalar interest</u> with nuisance, put Jeffreys (1946) <u>on profile</u> (2nd order)
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   - for rotating parameter (Need a Jacobian)
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6. Fisher got it right!

How Bayes can deliver 2nd order Accuracy!

A long history

great collaborators: Nancy

A Wong	York
M Bédard	U de Montréal
W Lin	Toronto
A M Fraser	UBC
M J Fraser	Toronto

(Preliminary report)

Support from NSERC
senior scholar funding: York Univ