BFF4

Harvard University

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Distributions for θ: Validity and Risks

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Colleagues

www.utstat.toronto.edu/dfraser/documents/BFF4-fraser.pdf

(~ / xxx.pdf references)

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frequentist	forever

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Xiao-Li	Appreciation: Thank you for bringing us together <u>friends</u> ... <u>eternity</u>!

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Not to forget - We have two theories!

- We have contradictions!

Future of SS (2014) F: "Why does statistics have two theories?"

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- All in Sterling 1959 60 years...

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responsibilities!

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On background... then outline, Main, Summary

Antiquity

Bayes (1763)

Fisher (1930)

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Focus here: Distributions for θ

Outline:

Background

1 Distributions for $y|\theta$

2 Distributions for $\theta|y$

3 Methods for $\theta|y$

4 Any connection $y|\theta$ and $\theta|y$?

5 Check it out: Exp't'l models ... general, simple

6 What can go wrong? Curvature

7 What can go wrong? Likelihood and Jeffreys

Summary

1 Distributions for y

Context, variable, behavior

$$\boxed{f(\cdot)} \longrightarrow y$$

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- Empirically
- In theory

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That's why we are here!

2 Distributions for θ

From somehow, somewhere.

we have a distribution $\pi(\theta; y)$ for θ ...

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Q: Is the true ψ_* in $(\hat{\psi}_p, \infty)$ or not?

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Is the proportion equal to the claimed p?

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Simulate <u>from what</u>? Try <u>Bayes case</u>

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from another, say $\pi_1(\theta)$ with $f(y; \theta)$, ... Typically No

3 Methods for Θ

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Anyway ... he was a power ....

Fisher 1930

Pivot: $F(y;\theta) = u \sim U(0,1)$

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"General discomfort" among B's ...and others

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Extended from "$\pi(\theta) = c$ for $f(y-\theta)$" to

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but he saw Problems! and suggested some resolutions!

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Subscripts y, θ - differentiation

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just math!

total differential

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Where is θ statistically re y?

survivor posterior $s(\theta) = \int_{\theta}^{\infty} \pi(\theta)\, F_y(y;\theta)\, d\theta \qquad (2)$

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Otherwise: Bayes is typically <u>wrong</u>

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$$f(s;\varphi)\,ds = \exp\{\varphi' s - \kappa(\varphi)\}\, h(s)\, ds$$

c. par $\varphi(\theta)$
c. var $s(y)$

(Example:
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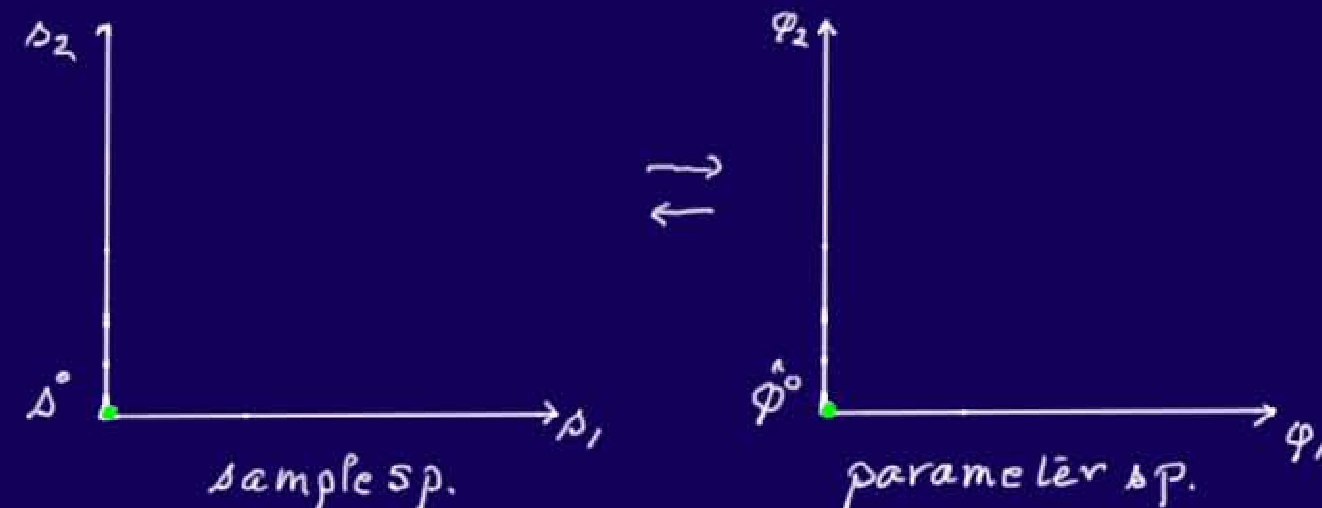

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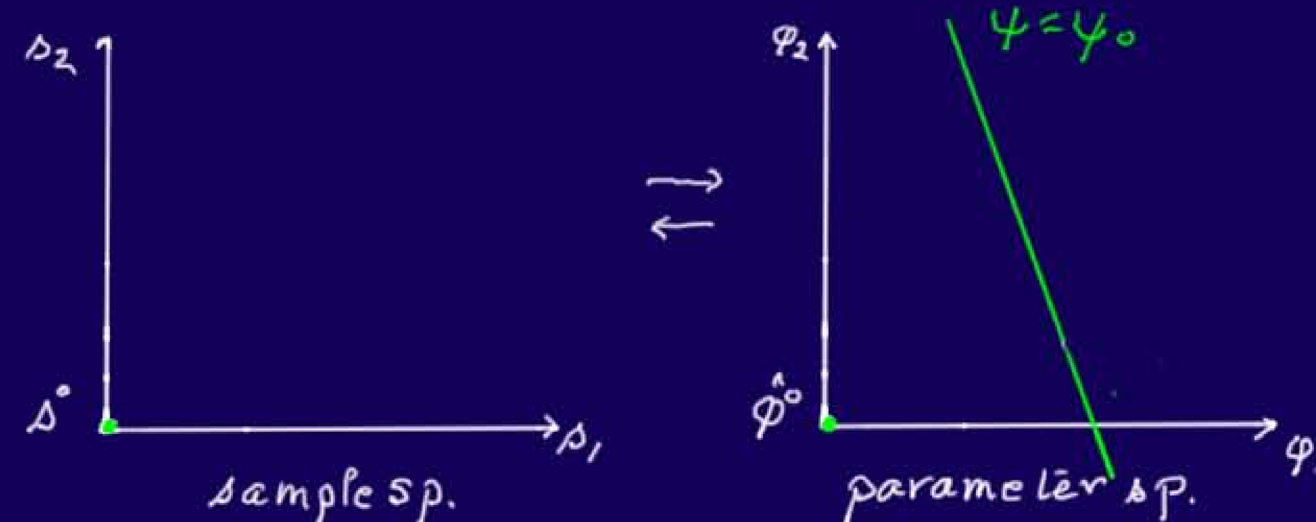
Linear interest $\psi = \psi_0$



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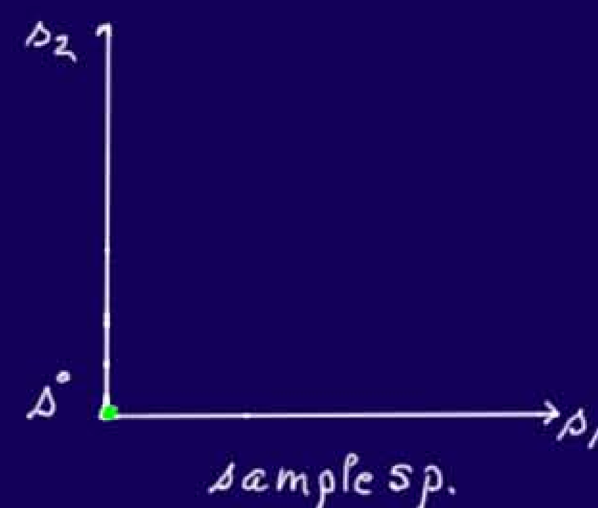

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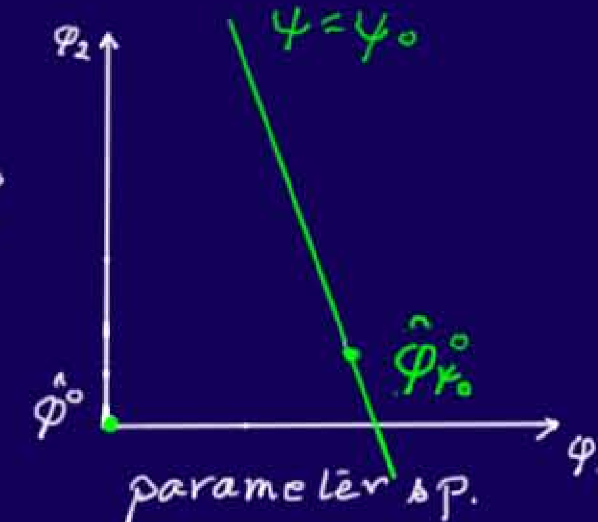

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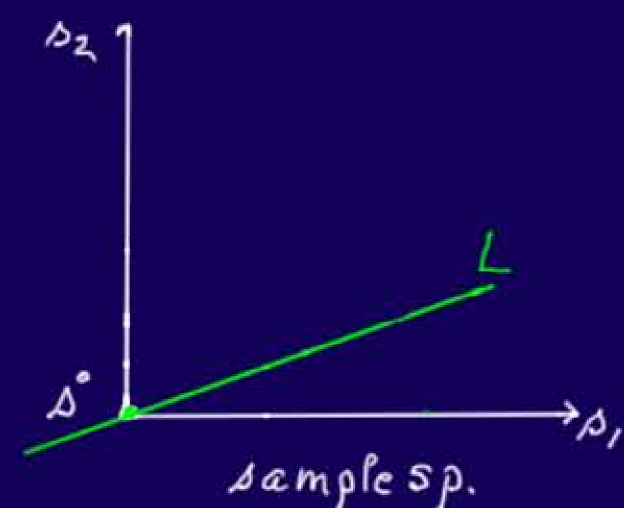

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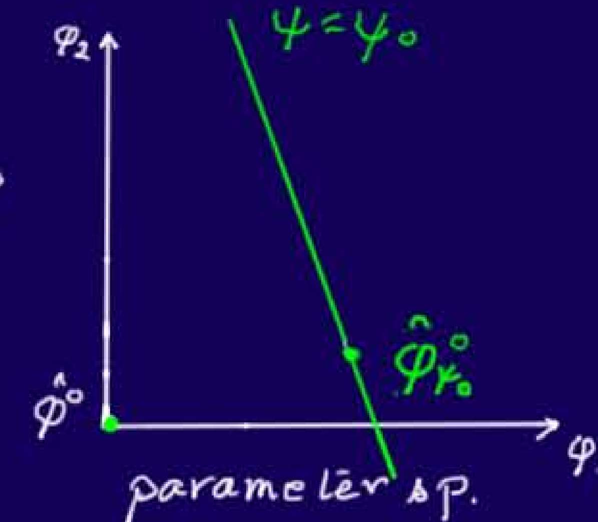

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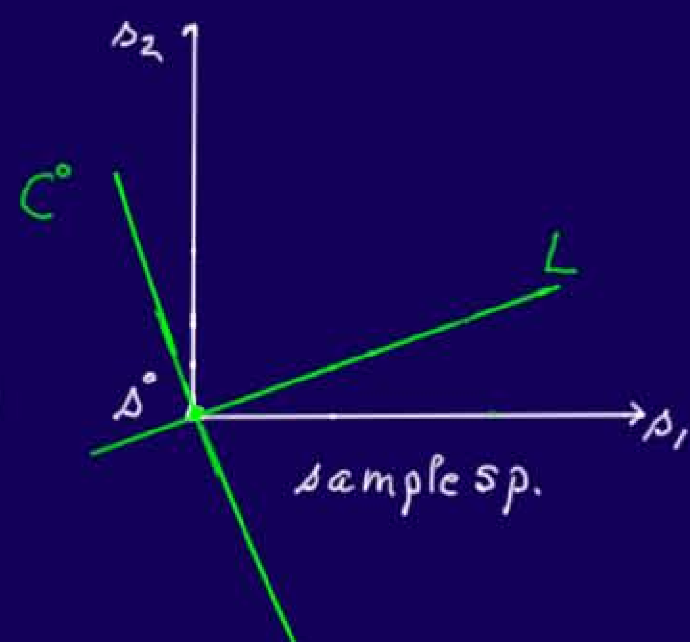

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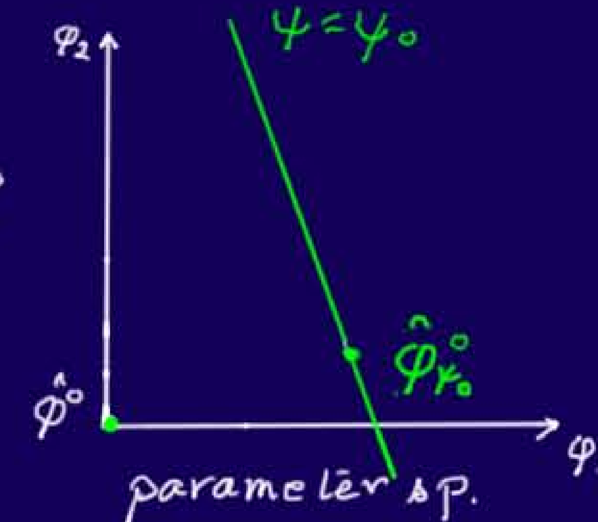

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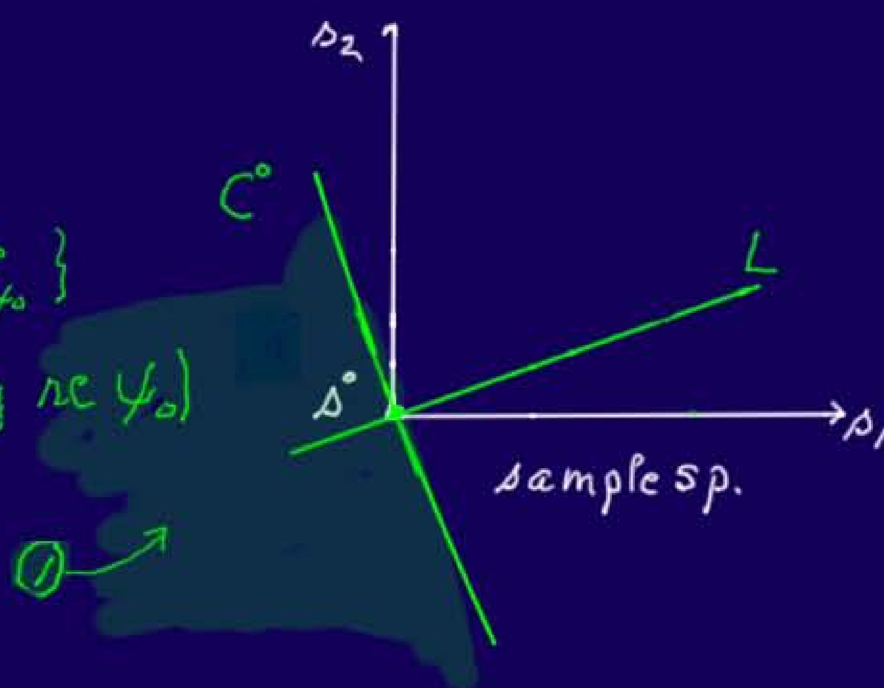

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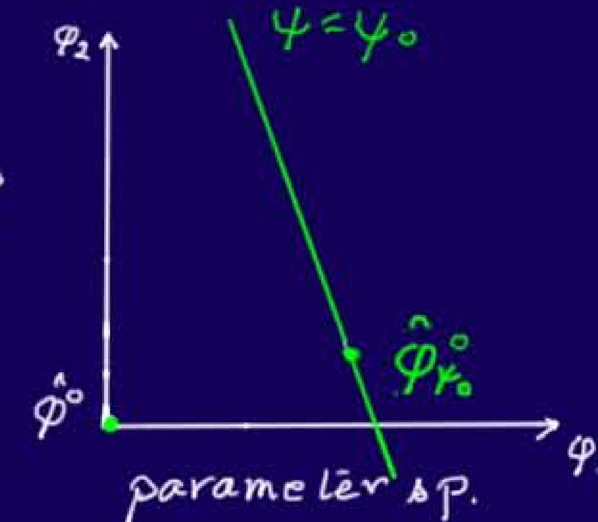

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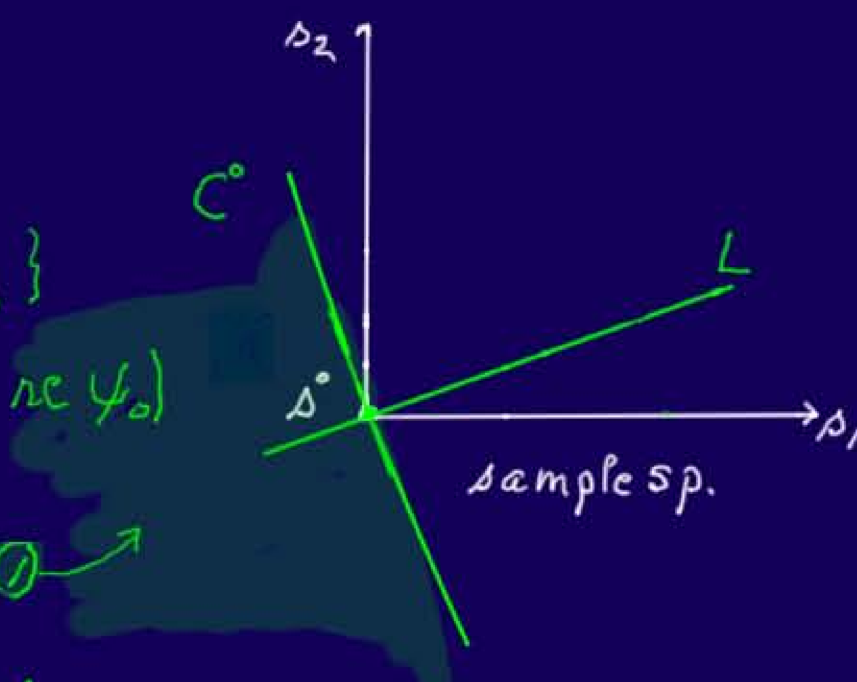

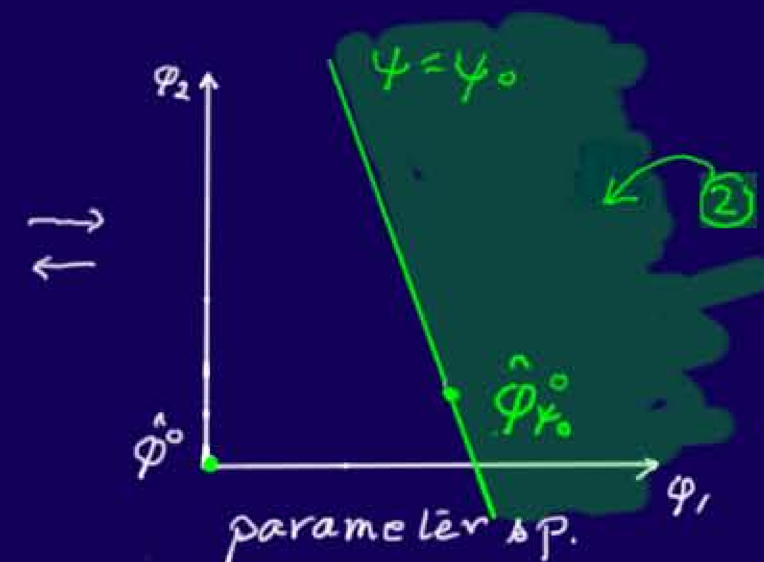

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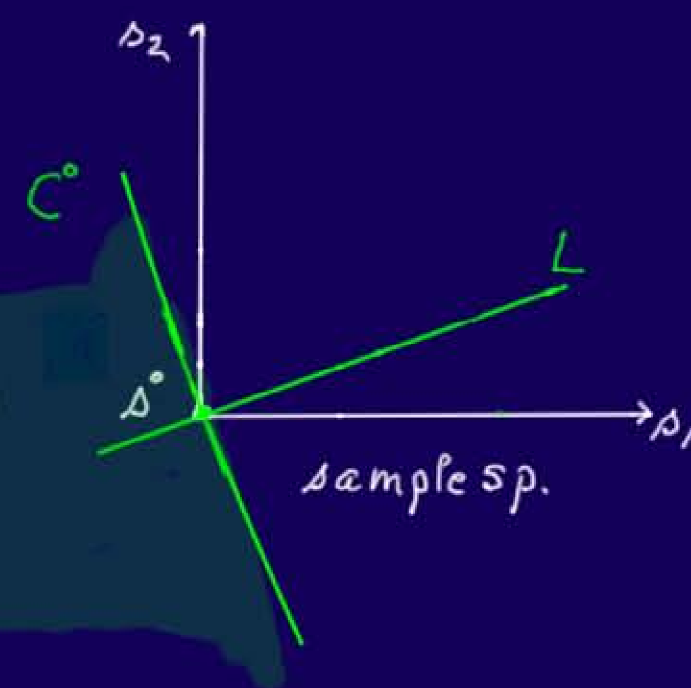

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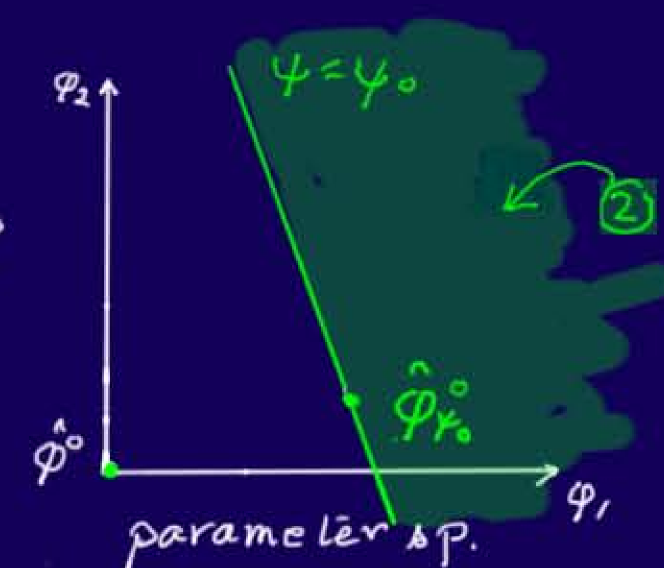

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Welch Peers 1963
Recent "L" extension

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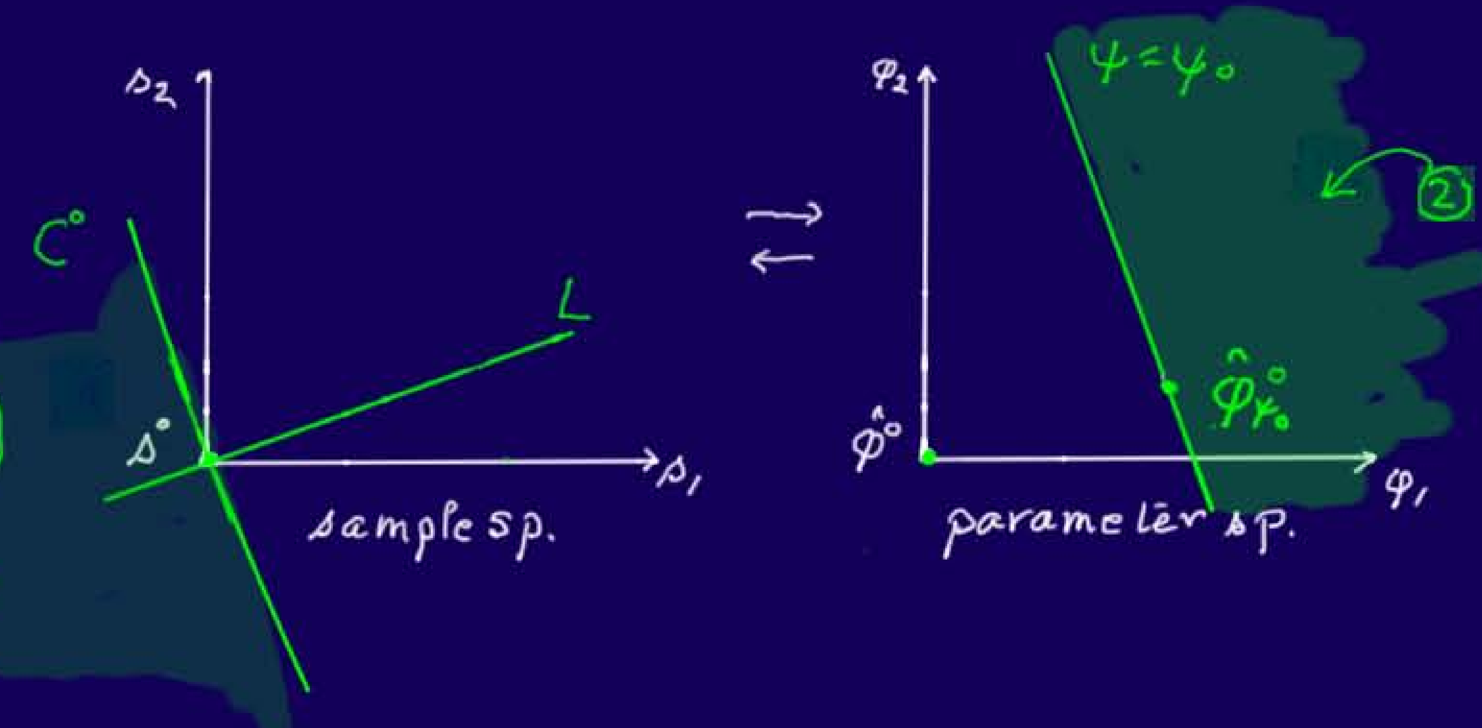

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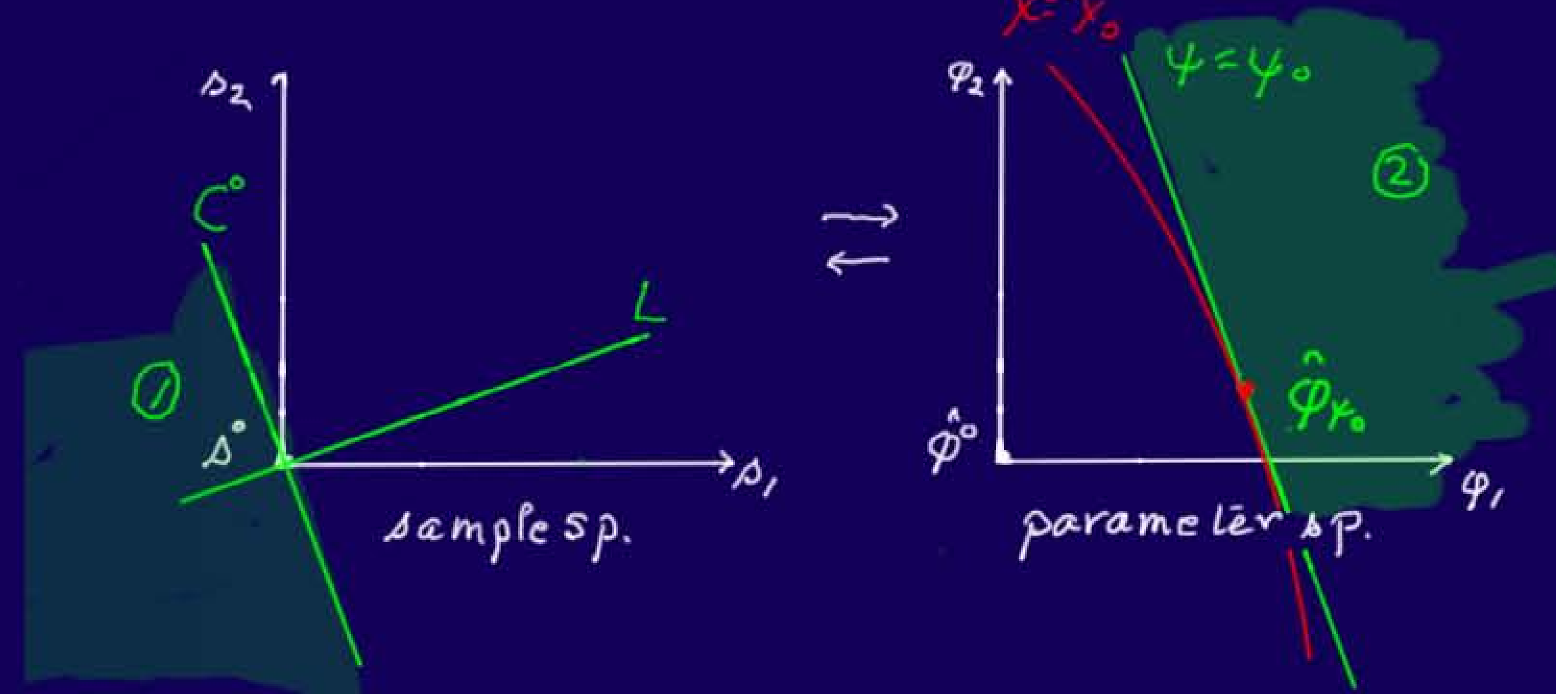
B and f agree .... linear ψ!

(but for curved ψ? ... different story?)

6 "and" Curvature : What can go wrong?

What about curved $\chi(\theta)=\chi_0$



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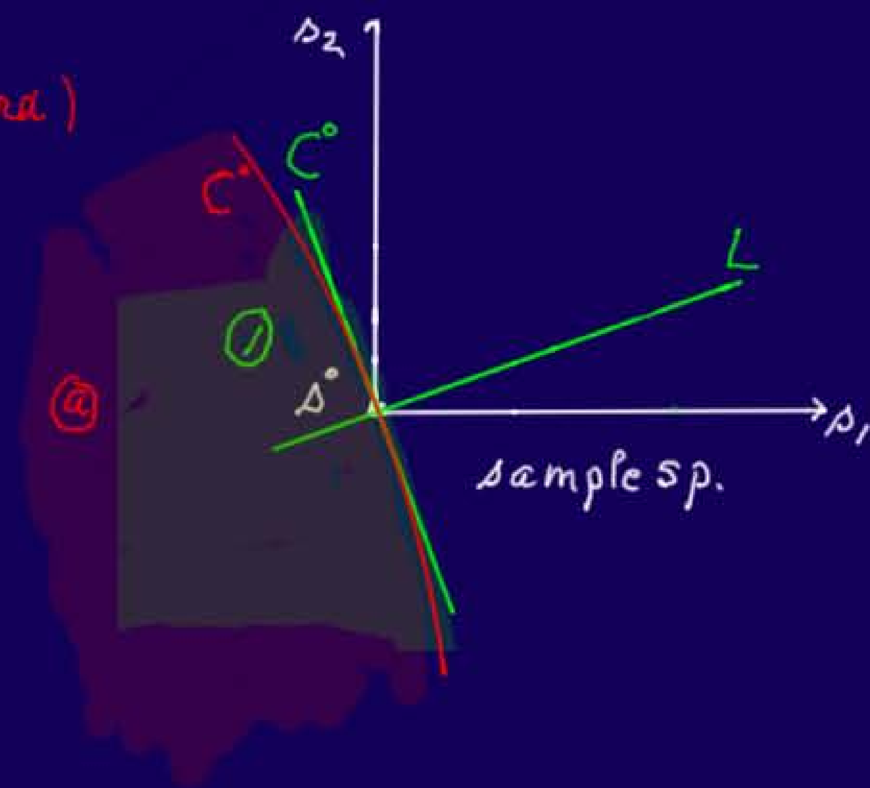

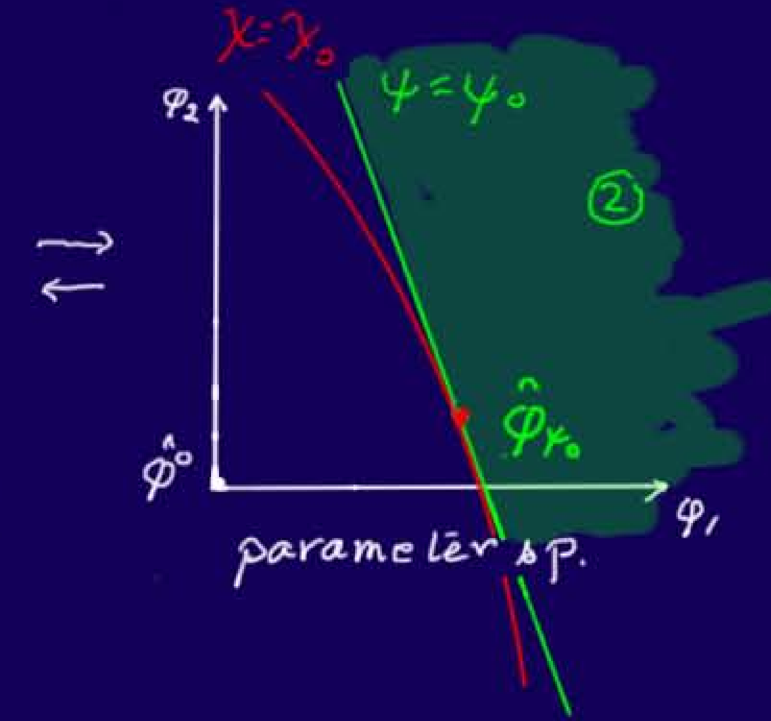

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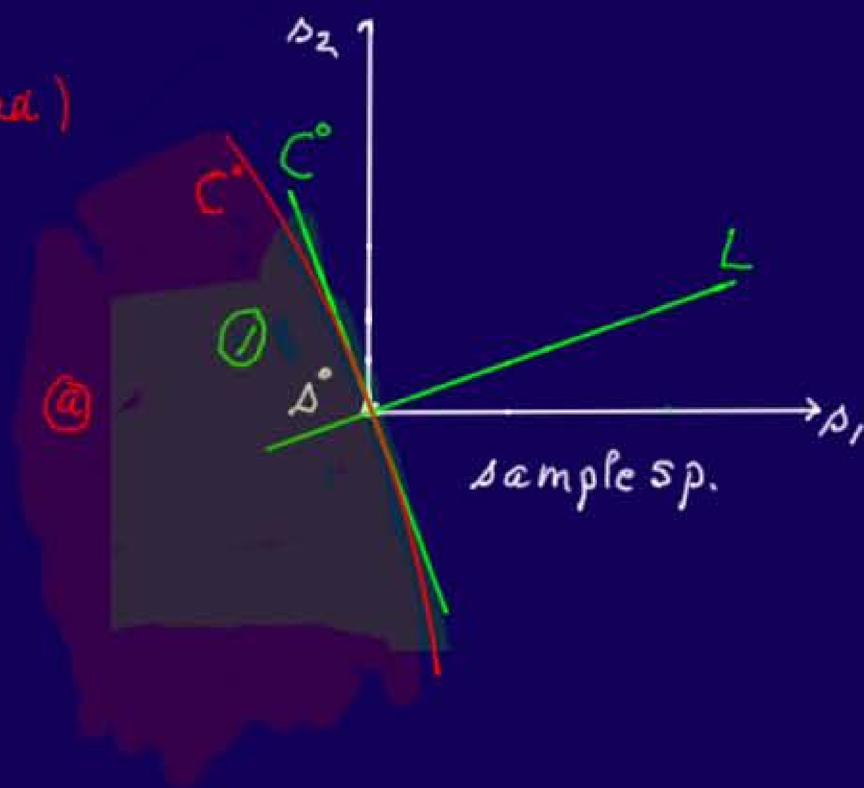
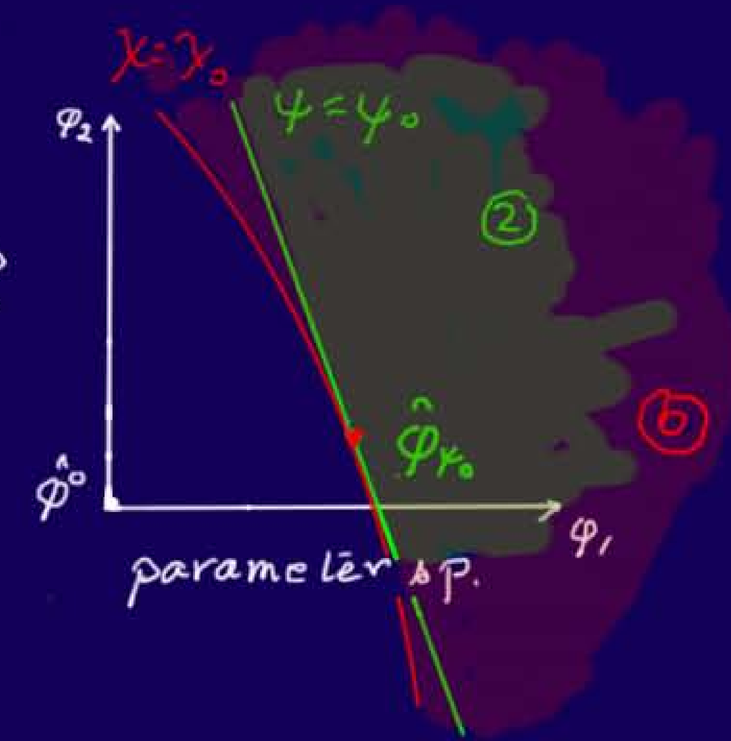
(b) (b)



$\longrightarrow$
$\longleftarrow$



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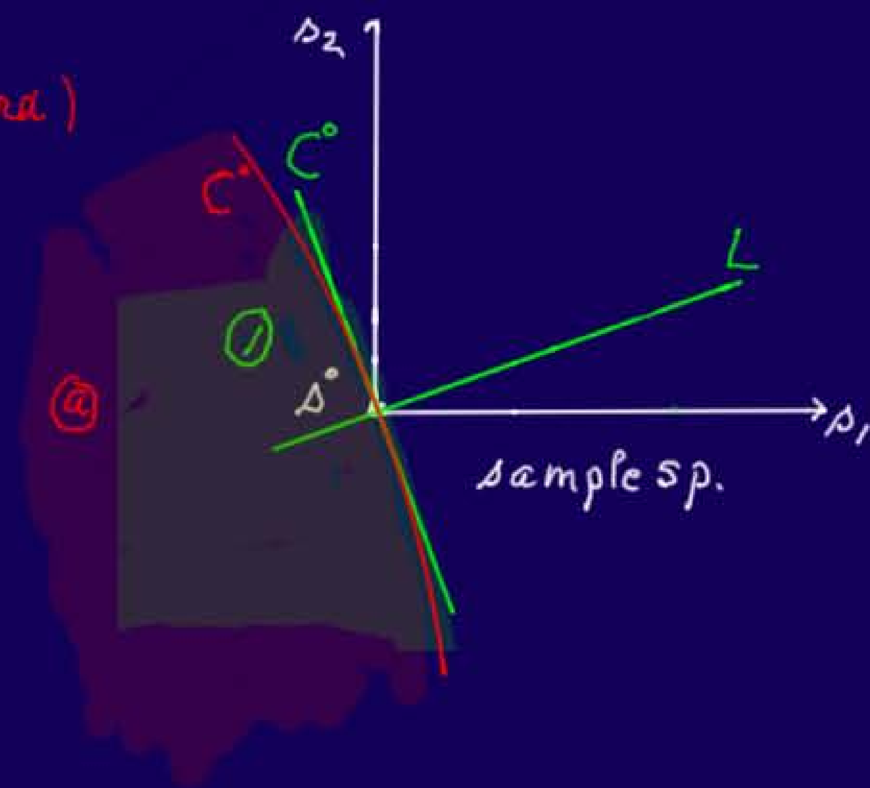

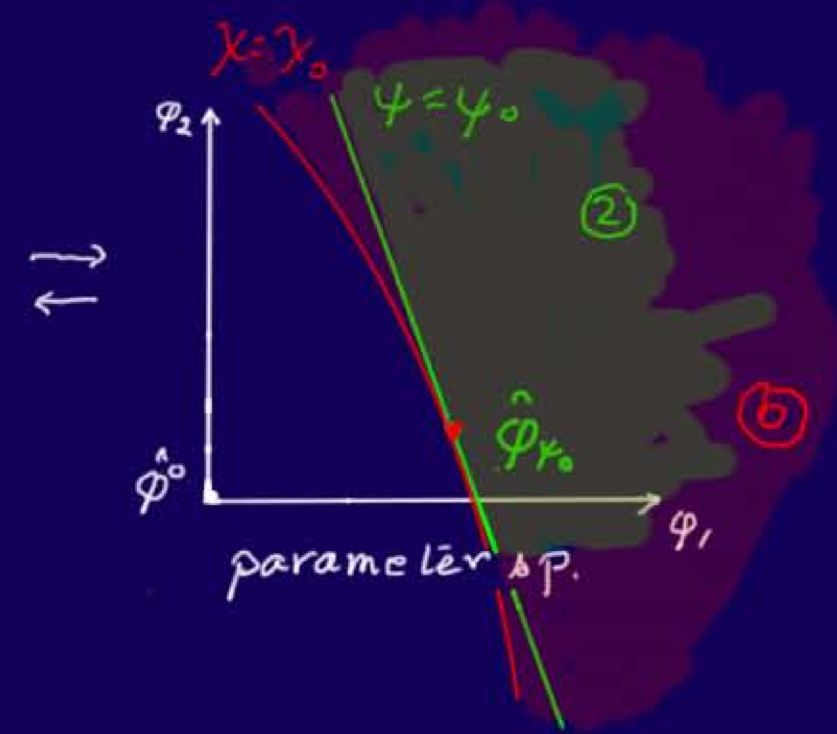

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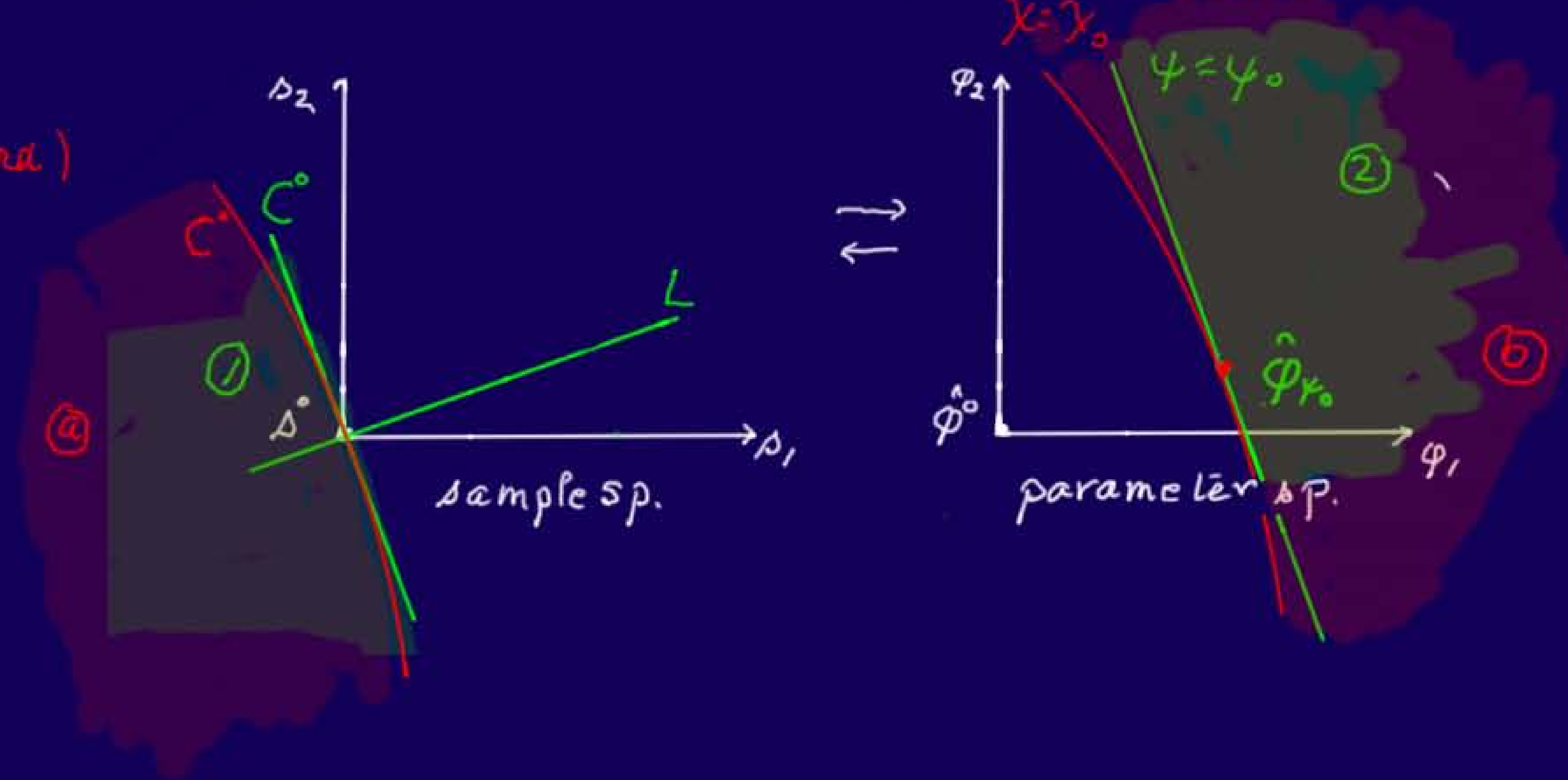

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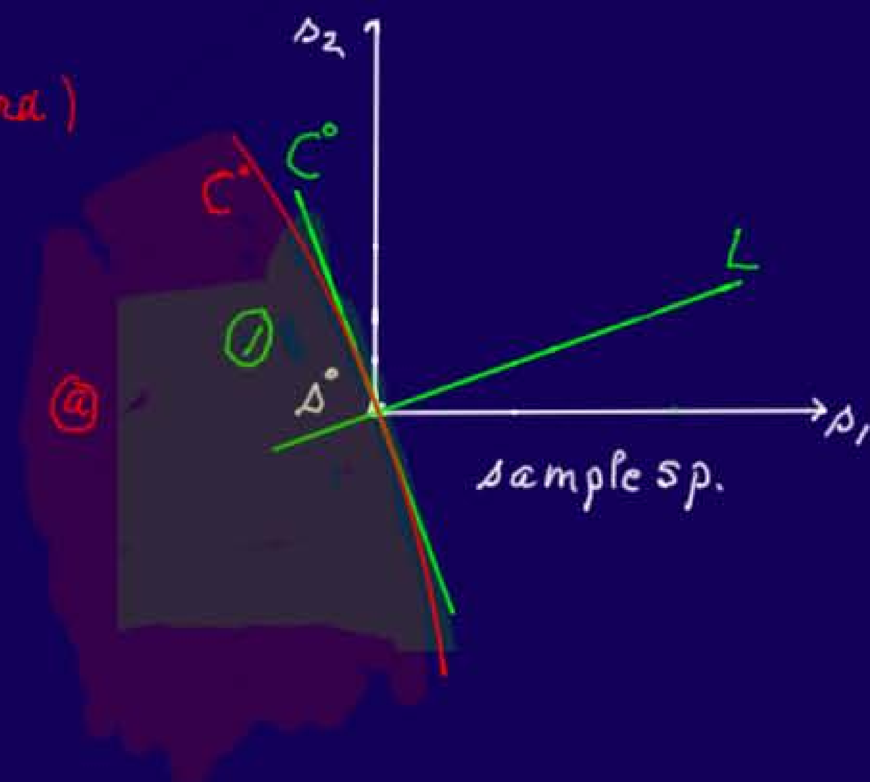

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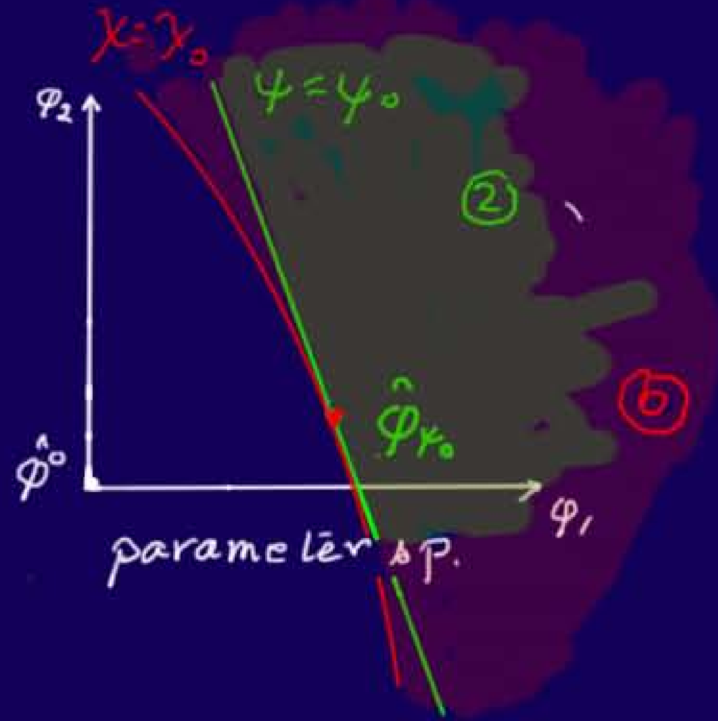

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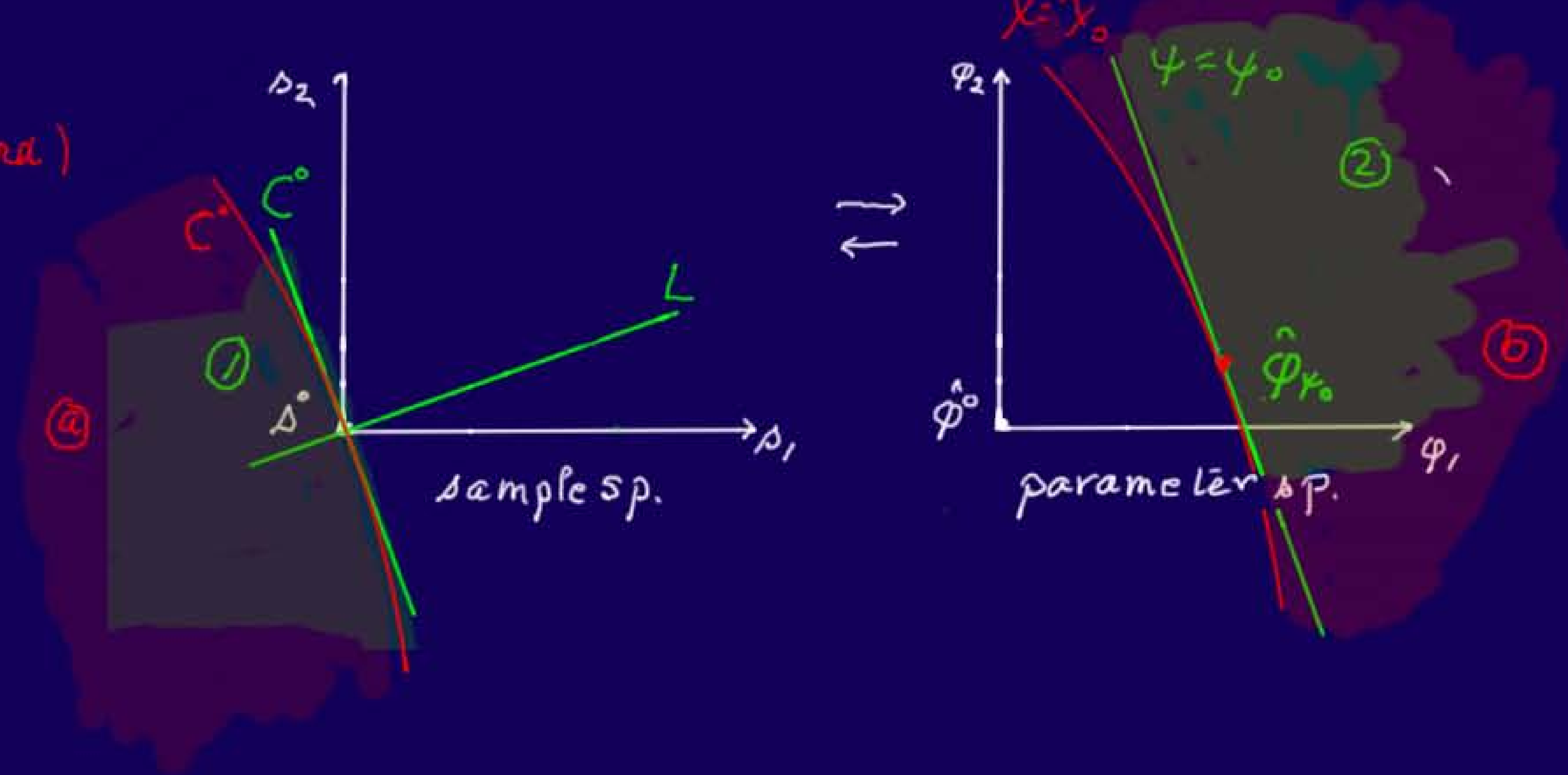

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Bayes is just first order

$\Rightarrow$ Use mle ?

Dawid Stone Zidek — Marginalization
JRSS B 1973

F Reid Marras Yi — Curvature
JRSS B 2010

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Have observed Likelihood $L(\theta)$
Want info re θ

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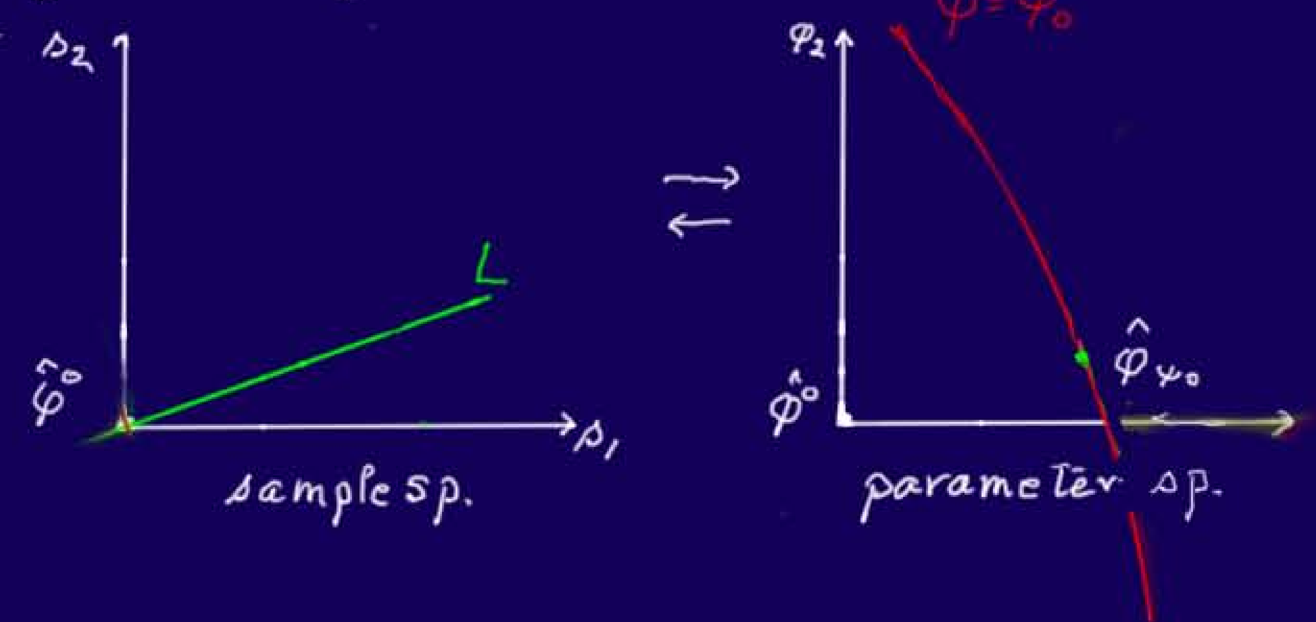

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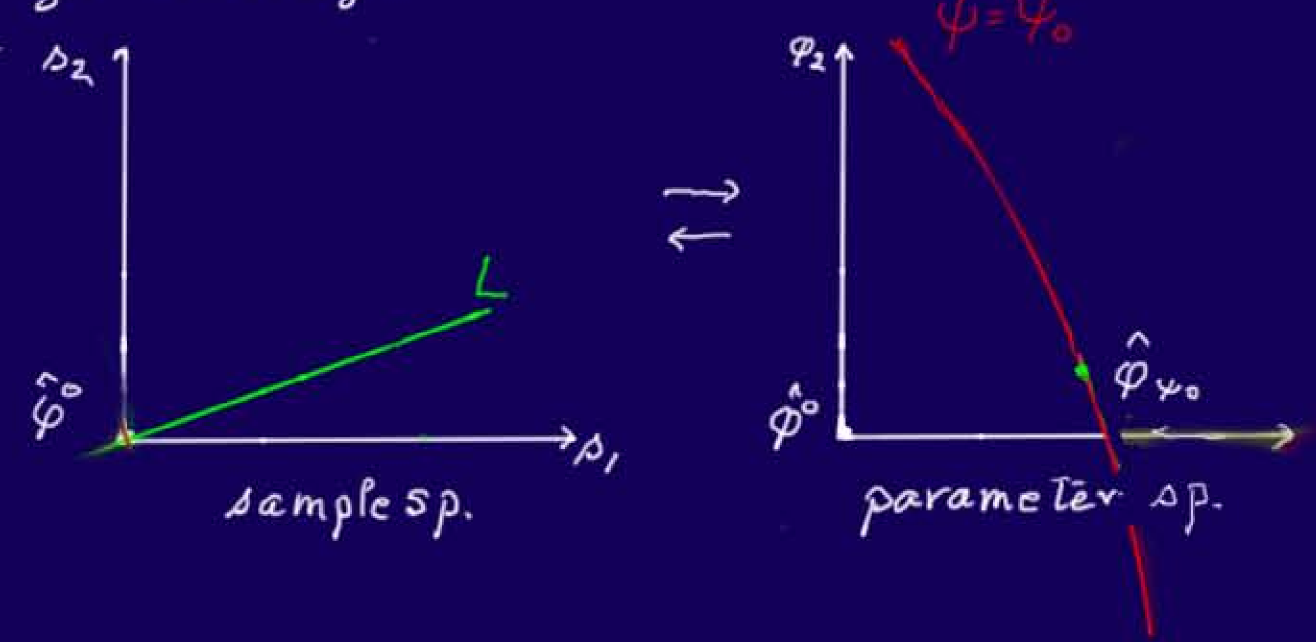

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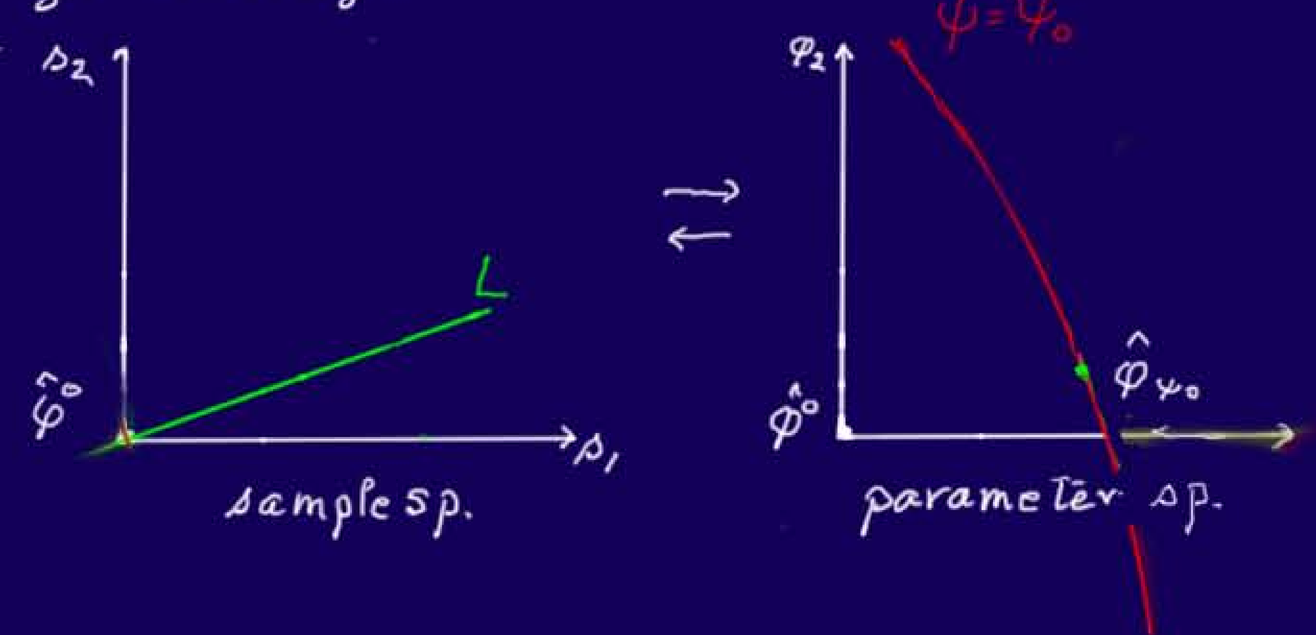

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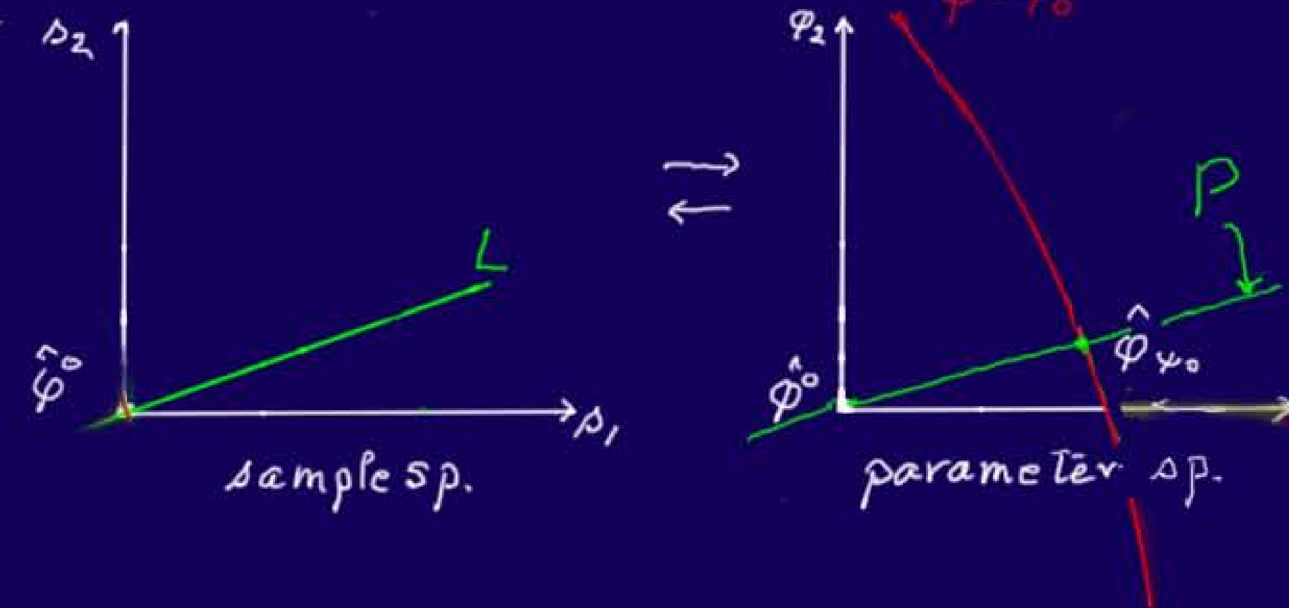

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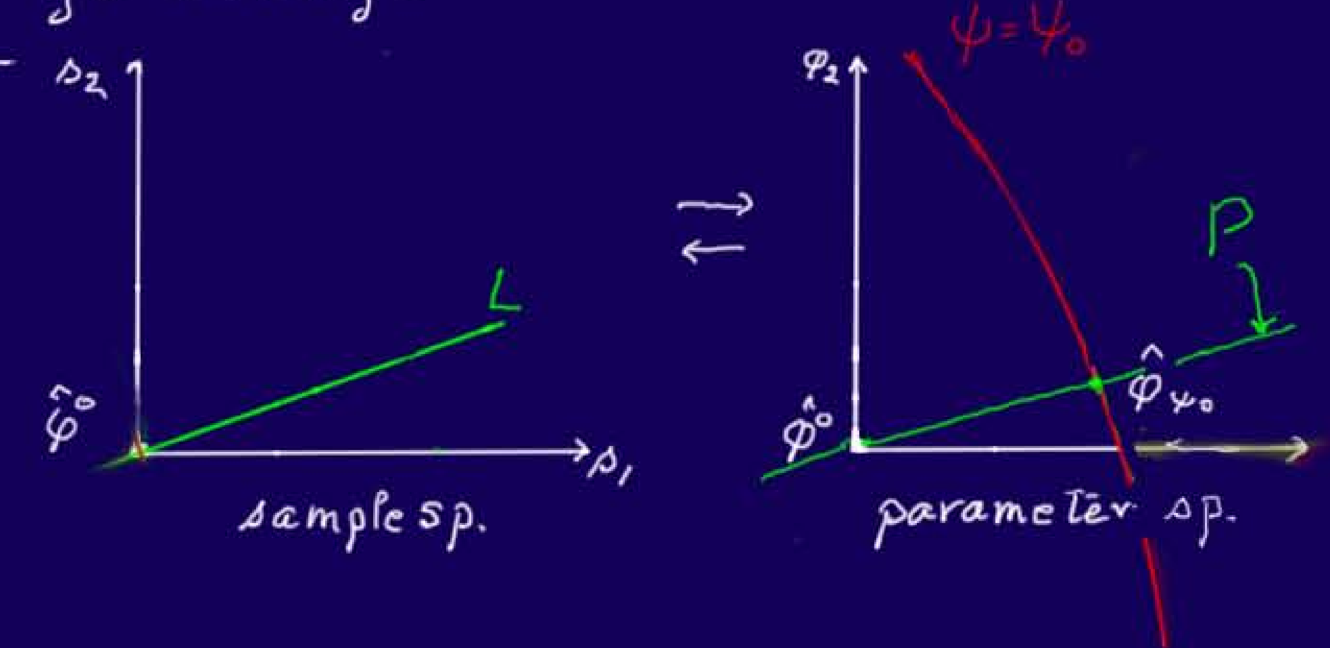

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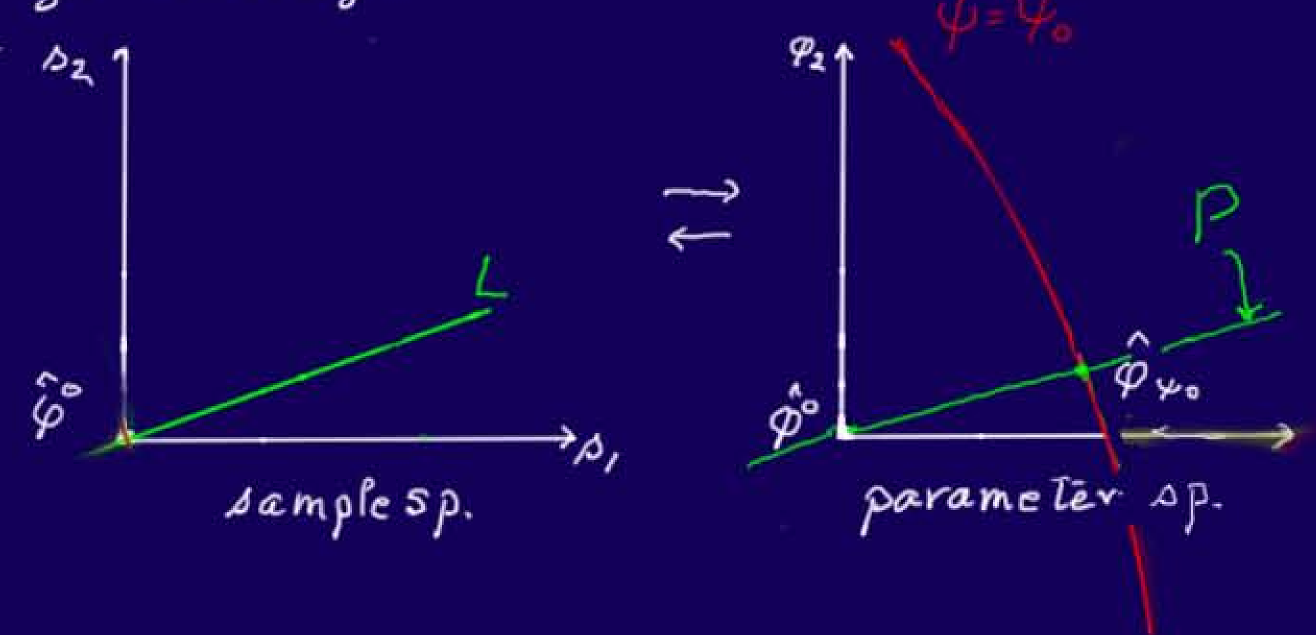

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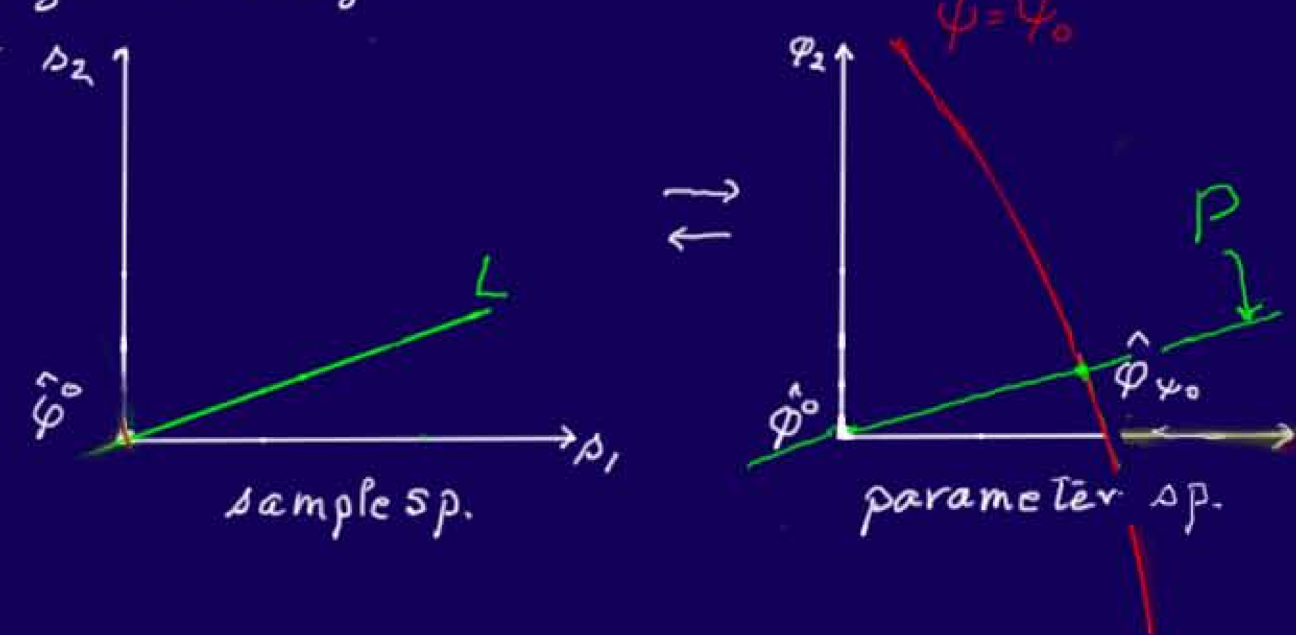

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But even easier: just use frequentist results

2016 Statistical Science.
F Bédard Wong Lin Fraser

Summary

(a) For a scalar interest parameter $\psi = \psi(\theta)$ Model $f(y;\theta)$ $\left|\begin{array}{l}\text{regular} \\ \text{continuity} \\ \text{quantile fn for sim}\end{array}\right.$

Summary

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$\Phi(r^*_\psi) = p(\psi)$ "where data is located (%) re parameter" $(xxx = 171)$

$= p$-value function for ψ HOL or Bootstrap r_ψ at $\hat{\theta}^0_\psi$ $(xxx = 229, 266)$

Summary

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(c) Even for scalar ψ, say for Normal(μ, σ_0^2) ... ?

$C_1 = (\bar{y} - 1.64\sigma_0, \infty)$ is 95% interval for μ frequentist! Bayes!

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"where data is located (%) re parameter" ($xxx = 171$)

HOL or Bootstrap r_ψ ($xxx = 229, 266$)

Also: $=$ right-tail dist'n f'n for ψ Needs targetted, data dependent prior! (conditional ?)

(b) For vector parameters say θ, or for a vector ψ

Can't marginalize... !

... get non-reproducible results ! Can go doubly wrong !

(c) Even for scalar ψ, say for Normal(μ, σ_0^2) ... ?

$C_1 = (\bar{y} - 1.64\sigma_0, \infty)$ is 95% interval for μ frequentist! Bayes!

but

"Interval away from 0"

$$C_2 = \begin{cases} (\bar{y} - 1.64\sigma_0, \infty) & \text{if } \bar{y} < 0 \\ (-\infty, \bar{y} + 1.64\sigma_0) & \text{if } \bar{y} > 0 \end{cases}$$ ... is 95% Bayes

is NOT confidence

Summary

(a) For a scalar interest parameter $\psi = \psi(\theta)$ Model $f(y;\theta)$ | regular | continuity | quantile f'n for sim

$\Phi(r_\psi^*) = p(\psi)$ ← marginal result "where data is located (%) re parameter" ($xxx = 171$)

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Thank you ! $\begin{cases} \text{Bayes needs - scalar interest parameter} \\ \text{- targetted, data dependent prior} \\ \text{- Can't marginalize vector posterior} \end{cases}$