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K. Schreiner

Fiducial and Structural Statistical Inference

Fiducial inference is a statistical approach to interval estimation first advocated by R. A. Fisher as an alternative to the then dominant method of inverse probability, i.e., using Bayes' Theorem. Considerable effort has gone into formalizing Fisher's notions using such concepts as statistical invariance and pivotal quantities. This entry describes elements of the fiducial approach and relates them to other currently more widely used statistical approaches to inference. Section 1 introduces some basic inferential ideas via a simple example.

1. A Simple Example

Consider a variable y that is directly available or has arisen by some preliminary reduction process and suppose that y measures θ in an unbiased manner and has error that is normal with known variance σ_0^2. Then we can say for example that $P(y \leqslant \theta + 1.64\sigma_0; \theta) = 95$ percent.

Parenthetically, we note that the preliminary reduction could have occurred as part of the underlying investigation or as part of some subsequent simplification of the statistical model by one of the common statistical reduction methods, sufficiency or conditionality. If the reduction is to a sufficient statistic, then the conditional distribution describing possible antecedent data has no dependence on the parameter and the model for the sufficient statistic is used in place of the original model. If the reduction is by conditionality then there is typically an ancillary variable with a distribution free of the parameter and the given model for the possible original data is replaced by the conditional model given the ancillary (supportive) variable; see also *Statistical Sufficiency*.

With a data value y^0 the fiducial methodology would take the above probability expression and substitute the value y^0 and then treat θ as the variable for the probability statement, thus giving a 95 percent fiducial probability statement $P(\theta \geqslant y^0 - 1.64\sigma_0; y^0) = 95$ percent. The structural approach would consider the normal $(0; \sigma_0^2)$ distribution for the error $e = y - \theta$ and might record for example the probability $P(e \leqslant 1.64\sigma_0) = 95$ percent; then with data value y^0 the probability statement would be applied to the error $e = y^0 - \theta$ giving the structural probability $P(y^0 - \theta \leqslant 1.64\sigma_0;\ y^0) = 95$ percent or equivalently $P(\theta \geqslant y^0 - 1.64\sigma_0; y^0) = 95$ percent. In a somewhat related manner the Bayesian methodology might use a uniform prior $cd\theta$ and obtain a posterior distribution for θ that would have $P(\theta \geqslant y^0 - 1.64\sigma_0; y^0) = 95$ percent. For this simple example the three methods give the same result at the 95 percent level and also at other levels, thus saying essentially that with data value y^0 the probability distribution describing the unknown θ is normal $(y^0; \sigma_0^2)$.

With more complicated models the results from the three methodologies can differ and philosophical arguments concerning substance and relative merits arise. However, for one straightforward generalization the methods remain in agreement: the normal distribution can be replaced by some alternative distribution form; this is discussed in some detail in Sect. 2.

2. Fiducial Probability

Fisher (1922, 1925; see also *Fisher, Ronald A (1890–1962)*) had already introduced most of the fundamental concepts of statistical theory, such as sufficiency, likelihood, efficiency, exhaustiveness (minimal sufficiency), when he chose (Fisher 1930) to address directly the aspiration mentioned above. He took Laplace and Gauss to task for 'fall(ing) into error on a question of prime theoretical importance' by adopting the Bayesian approach that 'Bayes (had) tentatively wished to postulate in a special case' and which was published posthumously (Bayes, ibid). Be then proposed in a restricted context the fiducial method, as discussed.

Neyman and Pearson (1933) then gave a mathematical formulation of fiducial probability that became known as confidence intervals. Fisher (1956) however treated Neyman and Pearson's formulation as a 'misconception having some troubling consequences …'; logical and philosophical arguments between the two sides were intense for many years. In particular the slight to Laplace and Gauss may well have affected the views of the more mathematical participants.

Fisher (1930) entitled his paper 'Inverse Probability' and examined a statistic $t(y)$ whose distribution depended on a single parameter θ. Let $P = F(t; \theta)$ be the distribution function of t, and let P itself be what we might now call a p-value for assessing θ; of course in the usual continuous case P has the uniform distribution on (0, 1). 'If now we give P any particular value such as 0.95, we have … the perfectly objective fact that in 5 percent of samples' t 'will exceed the 95 percent value corresponding to the actual value of θ … .' Then to 'any value of' t 'there will moreover be usually a particular value of θ to which it bears this relationship; we may call this the 'fiducial 5 percent value of θ' corresponding to' the given t. This led to Neyman and Pearson's (ibid) confidence methodology but Fisher treated this as a misconception and he followed different directions and interpretations for the fiducial methodology; for a view on related approaches see *Estimation: Point and Interval*. For this present simple case with scalar t and scalar θ, there seems little difference between the fiducial and the confidence approaches and interpretations.

This discussion effectively ascribes a distribution to θ based on an observed t; this is called the fiducial distribution for θ. Just as the density of t for given θ is obtained as $(\partial/\partial t)F(t; \theta)$ so also the fiducial density is obtained as $(-\partial/\partial\theta)F(t;\theta)$; the negative sign is inconsequential and is merely the result of $F(t; \theta)$ being examined typically for the case that is increasing with θ. The fiducial distribution for cases like the present can also be called the confidence distribution, which recently has a variety of uses in statistical inference. This alternative name emphasizes its more conventional role and attempts to avoid the inappropriate stigma that surrounds the fiducial concept.

For the fiducial approach in more general contexts, Fisher (1935) recommended a maximum reduction to a statistic $t(y)$ by the use of sufficiency. For example, with a sample $y_1,\dots, y_n$ from the normal $(\mu; \sigma_0^2)$ distribution, the reduction would be to $t(y) = (\bar{y}, s_y^2)$. The methodology then suggests the use of a pivotal quantity $p = p(t; \theta)$ with a fixed distribution and a one-one relationship between any two of p, t, θ; recall more generally that a pivotal quantity is a function of the variable and parameter that provides a measure of departure of variable value from parameter value, and has a fixed distribution which allows an assessment of an observed departure. Thus for the example a natural pivotal is

$$\left\{ z = \frac{\bar{y}-\mu}{\sigma/\sqrt{n}}, \quad \chi^2_{n-1} = \frac{(n-1)\, s_y^2}{\sigma^2} \right\}$$

which has independent components, normal (0, 1) and chi-square with $n-1$ degrees of freedom. Fisher (1956, p. 172), however, rather deviously rejected this as a legitimate part of the fiducial methodology, but he was somewhat less explicit about what would be legitimate. This pivotal reduction procedure is now however a rather familiar component of standard inference theory and in particular of confidence theory.

The final step is to invert the pivotal quantity, that is, to insert the observed values for the variables and then transform the distribution of the pivotal quantity to the parameter. For the example this gives

$$\mu = \bar{y} - \{z/(n-1)^{-1/2}\chi_{n-1}\} s_y/\sqrt{n}$$
$$\sigma^2 = (n-1)\, s_y^2/\chi^2_{n-1}$$

where the $\bar{y}$, s_y^2 have their observed values. We can then write

$$\mu = \bar{y} - t s_y/\sqrt{n}$$

where t is Student with $n-1$ degrees of freedom. This fiducial calculation closely parallels that for the confidence approach, except that the limits here are calculated from the Student distribution for μ rather than from the Student distribution of the pivotal t. In quite wide generality fiducial regions can correspond to confidence regions; it is just a matter of whether the limits are calculated before or after the data are observed, a non-issue from the Bayesian viewpoint.

There are however cases where routinely obtained confidence and fiducial regions can differ. For example, consider a sample $(y_{11}, y_{12}),\dots, (y_{n1}, y_{n2})$ from a bivariate normal distribution with means μ_1, μ_2, variances σ_1^2, σ_2^2, and correlation ρ, and suppose primary interest centers on the regression coefficient $\beta = \rho\sigma_2/\sigma_1$ of the second variable on the first variable. Standard regression analysis of the second variable on the first variable can produce an interval that differs from that obtained from the joint fiducial or structural distribution for all the parameters by integrating out the unwanted parameters; see for example, Fraser (1979, pp. 189, 204, 280, and 293). Also there are other familiar cases where fiducial, structural and Bayesian methods can differ; see for example, the Behrens-Fisher and the Fieller-Creasy problems, and for some discussion see Wallace (1980).

Now suppose we generalize and examine a sample $y_1, \ldots, y_n$ from some distribution with location μ and scale σ. For example the basic form $f(z)$ of the distribution might be, say, the longer-tailed logistic, or the Student with 6 degrees of freedom often cited as a realistic error pattern. The distribution for the underlying errors is then $f(z_1) \cdots f(z_n)$ and for the sample is

$$\sigma^{-n} \prod f\{\sigma^{-1}(y_i - \mu)\}$$

With a typical general error shape we do not have a simple minimal sufficient statistic as with the normal error pattern above. Fisher (1934) then recommended conditioning on an ancillary statistic and suggested the statistic

$$d_1 = \frac{y_1 - \hat{\mu}}{\hat{\sigma}}, \ldots, d_n = \frac{y_n - \hat{\mu}}{\hat{\sigma}}$$

called the configuration statistic, whose coordinates are standardized residuals, standardized here with respect to maximum likelihood values. The conditional distribution of $(\hat{\mu}, \hat{\sigma})$ is fairly easily expressed in terms of likelihood. Let

$$L^0(\mu, \sigma) = c\sigma^{-n} \prod f\{\sigma^{-1}(y_i^0 - \mu)\}$$

be the observed likelihood function from data $(y_1^0, \ldots, y_n^0)$, and $(\hat{\mu}^0, \hat{\sigma}^0)$ be the observed maximum likelihood values. Then the conditional distribution given the observed configuration statistic $(d_1^0, \ldots, d_n^0)$ is

$$g(\hat{\mu}, \hat{\sigma} | \mathbf{d}^0; \mu, \sigma) \frac{d\hat{\mu} d\hat{\sigma}}{\hat{\sigma}^2} = cL^0(\hat{\mu}^0 + (\mu - \hat{\mu})\hat{\sigma}^0/\hat{\sigma}, \sigma\hat{\sigma}^0/\hat{\sigma}) \frac{d\hat{\mu} d\hat{\sigma}}{\hat{\sigma}^2}$$

which can be used to give confidence intervals; also a straightforward fiducial argument following that for the normal case gives

$$L^0(\mu, \sigma) \frac{d\mu d\sigma}{\sigma}$$

for the fiduicial distribution. If f is replaced by the standard normal we get the result discussed earlier and we get it without any reference to sufficiency. This with some other results suggests (Fraser and Reid 2000) that the primary concept sufficiency can be replaced quite widely by appropriate conditioning, and with a major gain in generality of viewpoint.

Fisher (1935) did not use maximum likelihood notation for this location scale model. The notation here adapts to more general contexts (Fraser 1968, 1979) but needs to be modified if the error distribution involves a shape parameter which would then make the maximum likelihood estimates parameter-dependent.

For more general problems the fiducial can have non-uniqueness difficulties, and these tend to arise from the choice of what pivotal to use. For example with the bivariate normal discussed, different pivotals can produce different fiducial distributions (Fisher 1956, p. 172, where he elusively denigrates the pivotal approach).

There are also issues connected with finding a marginal fiducial for a component parameter. For a general discussion of marginalization paradoxes connected with methodologies concerning distributions for parameters, including the Bayesian methodology, see David et al. (1973).

For a mathematical analysis that avoids some of the difficulties with the fiducial methodology, see the transformation group framework in Fraser (1961a, 1961b); this mathematical analysis led to the development of the structural approach described below. The marginalization issues still remain, however, and this is related to the common assessment technique of examining a procedure with many repetitions from the same distribution with fixed parameter value. Alternatives to this are suggested in Fraser and Reid (2000).

3. *Structural Probability*

In many applied problems it is possible to ascribe an underlying error distribution to the model. For example with measurements that are normal with unknown error scaling we might write

$$y_i = \mu + \sigma z_i \quad i = 1, \ldots, n$$

where the $z_1, \ldots, z_n$, are independent standard normal variables. If we then examine (Fraser 1966) these expressions with observed data we can note rather easily that many characteristics of the underlying realized errors can be numerically evaluated. Specifically, let

$$d_1 = \frac{y_1 - \bar{y}}{s_y}, \ldots, d_n = \frac{y_n - \bar{y}}{s_y}$$

be standardized residuals; then the observed $y_1, \ldots, y_n$ and the underlying realized $(z_1, \ldots, z_n)$ have the same standardized residuals. In fact, $n-2$ characteristics as presented by the residuals are available concerning the underlying errors. Most theory concerning applications of probability would then say the analysis should be conditional on these observed characteristics, that is, on what you know concerning the realized error (Fraser 1976, Chap. 4, pp. 161–2 and Chap. 11, pp. 456–66). This for quite general problems automatically gives the conditioning used by Fisher and avoids the need to invoke a conditioning principle and seek an ancillary such as the configuration statistic.

This present type of analysis carries through equally (Fraser 1966) if the normal distribution is replaced by some other error pattern. The conditional error distribution can be viewed from an invariance argument as being valid after the full data are observed. This distribution then gives the structural distribution for μ and σ. The mechanics follow closely what is presented for the fiducial case and the resulting structural distribution agrees with the fiducial distribution, $L^0(\mu, \sigma)\, d\mu d\sigma/\sigma$. The difference here is that a rather strict invariance structure is assumed as part of the argument. This is summarized in general notation.

The structural approach is possible in cases where the underlying error can be directly modeled and the observed response is obtained as some transformation of this. Let Z designate a general error variable with known density $f(Z)$ on say R^n. Let Y designate the response as obtained by a transformation or re-expression θ in some space Ω. Then we have

$$Y = \theta Z, \quad f(Z)\, dZ,$$

For the invariance we assume that a transformation θ carries R^n into R^n and that the collection Ω of such transformations is closed under composition and inversion. Also for details here we assume that the transformations are smooth and at most one transformation carries a point Z into a point Y. For some general background on transformations and invariance, see *Causal Inference and Statistical Fallacies*.

If we consider all the transformation Ω and apply them to a data point Y^0 we obtain a set ΩY^0 of images, of possible antecedent error values. If Z^0 is the unknown realization of error that produced Y^0 then we have $\Omega Z^0 = \Omega Y^0$ or equivalently we have that Z^0 lies in the set ΩY_0. This means that a probability assessment should be made conditional on the observed set ΩY^0. Let $D(Z)$ be a reference point in the set ΩZ and let $[Z]$ in Ω be the transformation in Ω that carries $D(Z)$ to Z. Then we have that the model $Y = \theta Z$ with data Y^0 can be rewritten as

$$[Y] = \theta[Z], \quad D(Y) = D^0,$$

giving the conditioning $D(Y) = D(Y^0) = D^0$ that appears as an observed ancillary in the fiducial framework. The conditional distribution of $[Z]$ given $D(Z) = D^0$ then describes the reduced model $[Y] = \theta[Z]$ with data $[Y^0]$. The structural distribution of θ is then given as $\theta = [Y^0][Z]^{-1}$ using the conditional distribution $[Z]$ just described.

In most contexts the support measure dZ can be replaced by an invariant measure $dM(Z)$

$$dM(Z) = \frac{dZ}{J(Z)}$$

where $J(Z)$ is an appropriate Jacobian (Fraser 1979). Expressions are simpler if we use density $\bar{f}(Z)$ with respect to the measure $dM(Z)$: $\bar{f}(Z) = f(Z)J(Z)$.

The distributions of $[Y]$ and $[Z]$ conditional on $D(Y) = D(Z) = D^0$ are

$$c\bar{f}(\theta^{-1}[Y]D^0)\, d\mu[Y], \quad c\bar{f}([Z]D^0)\, d\mu[Z]$$

where $d\mu[Z]$ is the corresponding invariant measure on the group Ω. The inverted distribution, called the structural distribution for θ, is

$$c\bar{f}(\theta^{-1}[Y^0]D^0)\, dv(\theta) = cL^0(\theta)\, dv(\theta)$$

using the right invariant measure v on the group Ω. In the Bayesian context the typically preferred noninformative prior is the right invariant measure $dv(\theta)$. Thus the structural distribution coincides with this preferred Bayesian posterior.

This group type model covers a wide range of regression multivariate and spherical distribution models but not in the simplified notation used here; for further details see Fraser (1968, 1979).

Recent likelihood asymptotics has been able to obtain many aspects of these results in a general asymptotic context; see Fraser et al. (1999) and Fraser et al. (1999).

4. *Comment*

Fiducial inference led to confidence intervals, with both having evolved from the Bayesian procedures initiated in the eighteenth century. In terms of freedom of usage, we have Bayesian inference as the more liberal. The foundational issues center on validity and interpretation of the probabilities for the parameters. Here the conditions are strictest for the structural and minimal for the Bayesian. In the nice cases they agree; and in the less standard cases they pressure the proponents to clarify their assumptions.

For some recent discussion of confidence and fiducial interconnections plus references see Barnard (1995), with complementing views in Fraser (1996). Also see Fraser and Reid (2000) for a discussion of the interplay with objective Bayesian methods.

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D. A. S. Fraser

Field Observational Research in Anthropology and Sociology

Field observational research in anthropology and sociology—or the process of doing ethnography—brings the social science investigator directly to the scene of the human behavior she or he describes. It involves 'being there,' or 'I-witnessing.' So also, of course, do other forms of documentation: journalism, film, travel writing, explorer or missionary's report, even background research by a novelist. What makes ethnography distinctive as a research practice is that it forms one point of a triangular process of knowledge construction which also includes comparative social theory and documentary contextualization.

The fieldnotes that result from ethnographic discovery procedures are not merely 'written up,' but are filtered and interpreted against theoretical propositions and the comparative record of other field observational studies; they are also grounded and enhanced by historical, ecological, demographic, economic, and other documentary sources that provide background and context. As the products of this mode of research are read—monographs and articles also referred to as ethnography—they stimulate new comparative theoretical thinking, which in turn suggests further problems and interpretations to be resolved through more field observational research. Ethnographies also regularly lead to new demands and rising standards for documentary contextualization—more history, more ecological or demographic backgrounding, more attention to state policy, economic trends or the world system. This triangle of ethnography, comparison, and contextualization is, in essence, the process by which field observational research in anthropology and sociology is utilized to explain and interpret human cultures and social life.

1. History and Scope in Anthropology and Sociology

In 1851 Morgan published his *League of the Ho-de-no-sau-nee, or Iroquois*; combining fieldwork and comparative and theoretical interests in political organization, this book was the first anthropological ethnography. It depicted the structure and operation of Iroquois society, detailing matrilineal kinship, political and ceremonial life, material culture and religion. After formation of the US government Bureau of American Ethnology in 1879, a stream of ethnographic accounts of native American societies began, including landmark field observational studies of the Zuni by Cushing, the Baffin Island Inuit by Boas, and, after 1900, ethnographies of Plains, Eastern Woodland, and California groups by Boas' students trained at Columbia University. During the 1930s the scope broadened to include rural and urban studies in Latin America, the Caribbean, Africa, and Europe, and also in contemporary US settings.

In Great Britain's colonies, local government official or missionary fieldworkers followed the topical guidebook *Notes and Queries on Anthropology*, drafted by Oxford University theorist Tylor and others, and first published in 1874. This armchair scholar/man-on-the-

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