STA 313F2004 Assignment 5

Do this assignment in preparation for the quiz on Friday, Nov. 19th. The problems are not to be handed in, except possibly the log and list files for the last question.

1. Denoting the likelihood function of the multivariate normal by $L(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, simplify $-2\log L(\overline{\mathbf{x}}, \boldsymbol{\Sigma})$ so that it depends on the sample data only through $\overline{\mathbf{x}}$ and

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{x}_i - \overline{\mathbf{x}})(\mathbf{x}_i - \overline{\mathbf{x}})'.$$

   As a check, evaluate your answer at $\boldsymbol{\Sigma} = \widehat{\boldsymbol{\Sigma}}$. Does it equal $np[1+\log(2\pi)]+n\log|\widehat{\boldsymbol{\Sigma}}|$?

2. Consider this model:

$$\begin{aligned} y_1 &= b_1x_1 + b_2x_2 + e_1 \\ y_2 &= b_3x_2 + b_4x_3 + e_2 \\ y_3 &= b_5y_1 + b_6y_2 + e_3, \end{aligned}$$

   where all random variables have expected value zero, the vector $\mathbf{x} = (x_1, x_2, x_3)'$ is multivariate normal, and the error terms e_1, e_2 and e_3 are normal, and independent of $\mathbf{x}$ and of each other. As usual, the manifest variables are the x and y variables.

   (a) Make up symbols for all the non-zero variances and covariances of the exogenous variables in this model. I count nine variances and covariances.

   (b) Draw the path diagram for this model.

   (c) Express the model in the notation of the Bentler and Weeks EQS model; consult the formula sheet if necessary. Give the matrices $\boldsymbol{\eta}$, $\boldsymbol{\xi}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ and $\boldsymbol{\phi}$. Also give the selection matrix $\mathbf{J}$ for this example.

3. For the Bentler and Weeks EQS model,

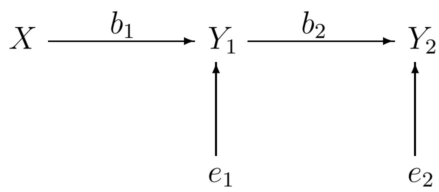
   (a) Derive $V(\boldsymbol{\eta})$. Show your work.

   (b) Derive $C(\boldsymbol{\eta}, \boldsymbol{\xi})$. Show your work.

   (c) Give the variance-covariance matrix of the entire collection of random variables $\binom{\boldsymbol{\xi}}{\boldsymbol{\eta}}$. You may express it as a partitioned matrix.

   (d) How would you get the variance-covariance matrix of just the manifest variables?

4. Consider this model:

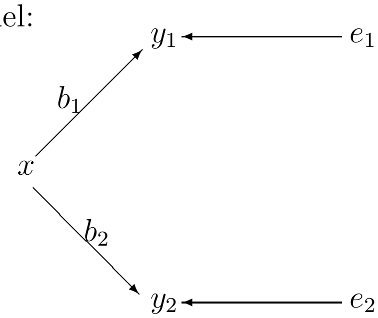


   (a) Write the model as a set of simultaneous equations.

   (b) Make up symbols for all the non-zero variances and covariances of the exogenous variables.

   (c) Express the model in the notation of the Bentler and Weeks EQS model. Consult the formula sheet if necessary. Give the matrices $\boldsymbol{\eta}$, $\boldsymbol{\xi}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ and $\boldsymbol{\phi}$. Also give the selection matrix $\mathbf{J}$ for this example.

5. Consider this model:



   (a) Write this model as a set of simultaneous equations.

   (b) Make up symbols for all the non-zero variances and covariances in the model.

   (c) Express this model in the notation of the Bentler and Weeks EQS model. Consult the formula sheet if necessary. Give the matrices $\boldsymbol{\eta}$, $\boldsymbol{\xi}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ and $\boldsymbol{\phi}$. Also give the selection matrix $\mathbf{J}$ for this example.

6. For the model of Question 5, and using the data in `wishbone.dat` (see course Web page for link), test $H_0 : b_1 = b_2$ and $V(e_1) = V(e_2)$ using SAS. Fit the full and the reduced model with `proc calis`, and compute G, and the p-value with `proc iml`. Bring both your log file and your list file to the quiz. Note: This is a single null hypothesis that imposes two restrictions on the parameter vector.