

STA 312F2007 Solutions to Quiz 4

Yes, the model is identified.

$$V(\mathbf{X}) = \Phi$$

$$V(\mathbf{Y}) = V(\Gamma\mathbf{X} + \zeta) = \Gamma V(\mathbf{X})\Gamma' + V(\zeta) = \Gamma\Phi\Gamma' + \Psi$$

$$\begin{aligned} \text{Cov}(\mathbf{X}, \mathbf{Y}) &= E(\mathbf{X}\mathbf{Y}') = E[\mathbf{X}(\Gamma\mathbf{X} + \zeta)'] = E[\mathbf{X}(\mathbf{X}'\Gamma' + \zeta')] \\ &= E(\mathbf{X}\mathbf{X}'\Gamma' + \mathbf{X}\zeta') = E(\mathbf{X}\mathbf{X}')\Gamma' + E(\mathbf{X})E(\zeta)' \\ &= V(\mathbf{X})\Gamma' = \Phi\Gamma' \end{aligned}$$

$$\Sigma = \begin{pmatrix} \Phi & \Phi\Gamma' \\ \Gamma\Phi\Gamma' + \Psi & \end{pmatrix}$$

$$\begin{cases} \Sigma_{11} = \Phi & (1) \\ \Sigma_{12} = \Phi\Gamma' & (2) \\ \Sigma_{22} = \Gamma\Phi\Gamma' + \Psi & (3) \end{cases}$$

$$(1): \Phi = \Sigma_{11}$$

$$(2): \Gamma = (\Phi^{-1}\Sigma_{12})'$$

$$(3): \Psi = \Sigma_{22} - \Gamma\Phi\Gamma'$$

or

$$\begin{cases} \Sigma_{11} = \Phi & \Rightarrow \Phi = \Sigma_{11} \\ \Sigma_{12} = \Phi\Gamma' & \Rightarrow \Gamma = (\Sigma_{11}^{-1}\Sigma_{12})' = \Sigma_{12}'\Sigma_{11}^{-1} \\ \Sigma_{22} = \Gamma\Phi\Gamma' + \Psi & \Rightarrow \Psi = \Sigma_{22} - \Sigma_{12}'\Sigma_{11}^{-1}\Sigma_{12} \end{cases}$$

One solution is obtained therefore the model is identified.