STA 312F07 Quiz 6

Let $\mathbf{X}_1, \ldots, \mathbf{X}_n$ be a random sample from a multivariate normal population with mean $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$. Recall that the MLEs are

$$\widehat{\boldsymbol{\mu}} = \overline{\mathbf{X}} \text{ and } \widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{X}_i - \overline{\mathbf{X}})(\mathbf{X}_i - \overline{\mathbf{X}})',$$

Show that

$$\frac{1}{n}\sum_{i=1}^{n}(\mathbf{X}_i - \overline{\mathbf{X}})'\boldsymbol{\Sigma}^{-1}(\mathbf{X}_i - \overline{\mathbf{X}}) = tr(\boldsymbol{\Sigma}^{-1}\widehat{\boldsymbol{\Sigma}}).$$

Total Marks = 10 Points