STA 312F07 Formulas

Here are some useful formulas. You may use any them without proof, unless you are explicitly asked to derive it. You will not need them all.

- If $\mathbf{A}$ is $n \times r$ and $\mathbf{B}$ is $r \times m$, then $\mathbf{AB} = [\sum_{=1}^{r} a_{ik}b_{kj}]$.
- If $\mathbf{A}$ is $n \times r$ and $\mathbf{B}$ is $r \times n$, then $tr(\mathbf{AB}) = tr(\mathbf{BA})$.
- The $r \times r$ matrix $\mathbf{A}$ is said to be positive definite if $\mathbf{b}'\mathbf{Ab} > 0$ for every nonzero $r \times 1$ vector $\mathbf{b}$.
- The inverse of a square symmetric matrix exists if and only if it is positive definite.
- $E(\mathbf{AXB}) = \mathbf{A}E(\mathbf{X})\mathbf{B}$
- $V(\mathbf{X}) = E\left((\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{X} - \boldsymbol{\mu}_x)'\right)$
- $C(\mathbf{X}, \mathbf{Y}) = E\left((\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{Y} - \boldsymbol{\mu}_y)'\right)$
- If $\mathbf{X}$ and $\mathbf{Y}$ are independent, $E(\mathbf{XY}) = E(\mathbf{X})E(\mathbf{Y})$.
- If $\mathbf{X}$ and $\mathbf{Y}$ are independent, $V(\mathbf{X} + \mathbf{Y}) = V(\mathbf{X}) + V(\mathbf{Y})$.
- The multivariate normal density is $f(\mathbf{x}) = \frac{1}{|\boldsymbol{\Sigma}|^{\frac{1}{2}}(2\pi)^{\frac{p}{2}}} \exp\left[-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu})\right]$.
- If $\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $\mathbf{A}$ is a matrix of constants, $\mathbf{AX} \sim N(\mathbf{A}\boldsymbol{\mu}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}')$.
- For the multivariate normal, $\widehat{\boldsymbol{\mu}} = \overline{\mathbf{x}}$ and $\widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{x}_i - \overline{\mathbf{x}})(\mathbf{x}_i - \overline{\mathbf{x}})'$
- For the multivariate normal, $-2\log L(\widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}) = n \log|\widehat{\boldsymbol{\Sigma}}| + np[1 + \log(2\pi)]$.