STA 312f07 Assignment 5

Do this assignment in preparation for the quiz on Friday, Oct. 17th. The questions are practice for the quiz, and are not to be handed in.

1. Let

$$\begin{aligned} Y &= \gamma\xi + \zeta \\ X &= \xi + \delta, \end{aligned}$$

   where ξ, ζ and δ are independent normal random variables with expected value zero, $Var(\xi) = \phi$, $Var(\zeta) = \psi$, and $Var(\delta) = \theta_\delta$.

   (a) Give the covariance matrix of the observed variables X and Y.

   (b) What are the parameters of this model? That is, give the parameter vector θ.

   (c) Is this model identified? Answer Yes or No and prove your answer.

2. In the reading for this section, we had an example of instrumental variables in which we added two more dependent variables, and the model became identified. This question will explore whether adding just one more dependent variable will do the job. Let

$$\begin{aligned} Y_1 &= \gamma_1\xi + \zeta_1 \\ Y_2 &= \gamma_2\xi + \zeta_2 \\ X &= \xi + \delta, \end{aligned}$$

   where δ, ξ, ζ_1 and ζ_2 are all independent normals with expected value zero, $Var(\xi) = \phi$, $Var(\zeta_1) = \psi_1$, $Var(\zeta_2) = \psi_2$, $Var(\delta) = \theta_\delta$, and the regression coefficients γ_1 and γ_2 are fixed constants. ξ is a latent variable.

   (a) Give the covariance matrix of the observed variables X, Y_1 and Y_2.

   (b) What are the parameters of this model? That is, give the parameter vector θ.

   (c) Is this model identified? Answer Yes or No and prove your answer.

3. Consider the following simple regression through the origin with measurement error in both the independent and dependent variables.

$$\begin{aligned} \eta &= \gamma\xi + \zeta \\ X_1 &= \xi + \delta_1, \\ X_2 &= \xi + \delta_2, \\ Y_1 &= \eta + \epsilon_1, \\ Y_2 &= \eta + \epsilon_2, \end{aligned}$$

where ξ and η are latent variables, ζ, δ_1, δ_2, ϵ_1, ϵ_2 and ξ are normal random variables with expected value zero, $Var(\xi) = \phi$, $Var(\zeta) = \psi$, $Var(\delta_1) = \theta_{\delta_1}$, $Var(\delta_2) = \theta_{\delta_2}$, $Var(\epsilon_1) = \theta_{\epsilon_1}$, $Var(\epsilon_2) = \theta_{\epsilon_2}$, $Cov(\delta_1, \epsilon_1) = \theta_{\delta\epsilon_1}$, and $Cov(\delta_2, \epsilon_2) = \theta_{\delta\epsilon_2}$. If the covariance for a pair of random variables is not explicitly given, then they are independent. The regression coefficient γ is a fixed constant. The observed variables are X_1, X_2, Y_1 and Y_2.

(a) Give the covariance matrix of the observed variables.

(b) What are the parameters of this model? That is, give the parameter vector θ.

(c) Is this model identified? Answer Yes or No and prove your answer.

4. Let

$$\begin{aligned} \boldsymbol{\eta} &= \boldsymbol{\Gamma\xi} + \boldsymbol{\zeta} \\ \mathbf{X} &= \boldsymbol{\xi} + \boldsymbol{\delta} \\ \mathbf{Y} &= \boldsymbol{\eta} + \boldsymbol{\epsilon}, \end{aligned}$$

where $\boldsymbol{\eta}$ is an $m \times 1$ vector of latent dependent variables, $\boldsymbol{\xi}$ is a $p \times 1$ vector of latent independent variables with $V(\boldsymbol{\xi}) = \boldsymbol{\Phi}$, $\boldsymbol{\zeta}$ is an $m \times 1$ vector of error terms with $V(\boldsymbol{\zeta}) = \boldsymbol{\Psi}$, $\boldsymbol{\Gamma}$ is an $m \times p$ matrix of regression coefficients, $\mathbf{X}$ is a $p \times 1$ vector of observable variables, $\boldsymbol{\delta}$ is the measurement error in $\mathbf{X}$ with $V(\boldsymbol{\delta}) = \boldsymbol{\Theta}_\delta$, $\mathbf{Y}$ is an $m \times 1$ vector of observable variables, and $\boldsymbol{\epsilon}$ is the measurement error in $\mathbf{Y}$ with $V(\boldsymbol{\epsilon}) = \boldsymbol{\Theta}_\epsilon$. The exogenous variables $\boldsymbol{\xi}$, $\boldsymbol{\zeta}$, $\boldsymbol{\delta}$ and $\boldsymbol{\epsilon}$ are all independent multivariate normals with expected value zero.

(a) Write the covariance matrix $\boldsymbol{\Sigma}$ of the observed variables as a partitioned matrix. What are the dimensions of $\boldsymbol{\Sigma}$?

(b) The parameter vector θ consists of all the unique elements of the matrices mentioned above. How many elements does θ have?

(c) Is this model identified in general, that is, without any further restrictions on the parameter matrices? Answer Yes or No and prove your answer.