Fuel additive paired differences: $n=64$, $\bar{x}=0.78$

From class notes. $s=4.1$

a. State the statistical model you adopt for these data.

Data are a random sample from a large population with mean μ & SD σ.

b. State the null hypothesis in symbols. $H_0: \mu = 0$

c. State the alternative hypothesis in symbols. $H_a: \mu \neq 0$

d. Calculate the test statistic; **show your work**. The answer is a number. **Circle your answer**.

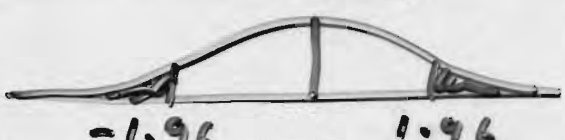
$$z = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{.78-0}{4.1/\sqrt{64}} = 1.52$$

e. Calculate the p-value (observed significance level). Show your work, including a picture of a curve with the tail area or areas shaded, and **circle your final answer**. (Ignore this question if it's a t-test)

$$p\text{-value} = 2(.5 - .4357) = .1286$$

f. State the rejection region and/or draw a picture of it. Use $\alpha = 0.05$.

$|z| > 1.96$



g. Do you reject the null hypothesis? Answer Yes or No.

No

h. Is $p<\alpha$? Answer Yes or No. (Ignore this question if it's a t-test)

No

i. Are the results statistically significant at the 0.05 level? Answer Yes or No.

No

j. Can we conclude that the additive affects fuel economy?

Answer Yes or No.

93

507

1. A machine is supposed to put an average of 500mg of peanut butter into a 500mg jar. The amount it actually dispenses is a normal random variable with mean that might equal 500 and unknown variance. A random sample of 9 jars of peanut butter yield a sample mean of ~~40~~ and a sample standard deviation of 1.2. Is there something wrong with the machine?

   a. State the statistical model you adopt for these data.

   The data are a random sample from a normal population with mean μ and standard deviation σ.

   b. State the null hypothesis in symbols. $H_0: \mu = 500$

   c. State the alternative hypothesis in symbols. $H_a: \mu \neq 500$

   d. Calculate the test statistic; **show your work**. The answer is a number. **Circle your answer**.

   $$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{507 - 500}{1.2/\sqrt{9}} = \frac{7}{.4} = 17.5$$

   e. State the rejection region and/or draw a picture of it. Use $\alpha = 0.05$.

   $|t| > 2.306$

   f. Do you reject the null hypothesis? Answer Yes or No. Yes

   g. ~~(Points)~~ Are the results statistically significant at the 0.05 level? Answer Yes or No. Yes

   h. ~~(Points)~~ Can we conclude that consumers are being cheated?

   NO, THEY are getting extra peanut butter

90

2. In a taste test, a random sample of 50 consumers tasted two unlabelled cups of cola beverage, and judged which one had a better flavour. Thirty chose Pepsi and 20 chose Cooke. We want to know whether, *in the population,* there is a difference in preference. Please proceed by answering these questions. The sample size is big enough; you don't need to check it.

   a. (~~Points~~) State the statistical model you adopt for these data.

   Data are a random sample from a large binary population with $P(X=1)=p$ and $P(X=0)=1-p$

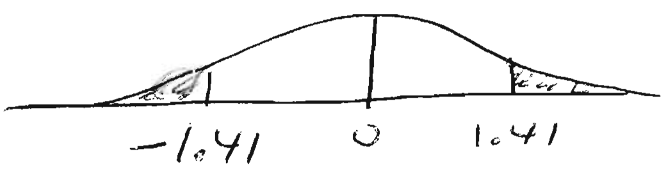
   b. (~~Points~~) State the null hypothesis in symbols. $H_0: p = \frac{1}{2}$

   c. (~~Points~~) State the alternative hypothesis in symbols. $H_a: p \neq \frac{1}{2}$

   d. (~~Points~~) Calculate the test statistic; **show your work**. The answer is a number. **Circle your answer**.

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.6 - .5}{\sqrt{\frac{(.5)(.5)}{50}}} = \frac{.1}{.0707} = 1.41$$

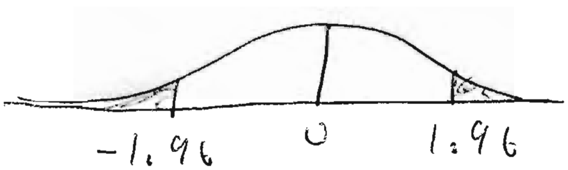
   e. (~~Points~~) Calculate the p-value (observed significance level). Show your work, including a picture of a curve with the tail area or areas shaded, and **circle your final answer**.



$$2(.5 - .4207) = .1586$$

91

f. (Points) State the rejection region and/or draw a picture of it. Use $\alpha = 0.05$.



$|z| > 1.96$

g. (Points) Do you reject the null hypothesis? Answer Yes or No.

h. (Points) Is p<a? Answer Yes or No.

i. (Points) Are the results statistically significant at the 0.05 level? Answer Yes or No.

j. (Points) Can we conclude that Pepsi is favoured over Coke in the population? Answer Yes or No.

95

3. Who retires earlier, men or women? Independent random samples of 75 men and 75 women yielded a mean retirement age of 66.2 with a standard deviation of 4.5 for men, and a mean retirement age of 64.7 with a standard deviation of 3.2 for women.

   a. State the statistical model you adopt for these data.

   Data are independent random samples from large populations with means μ_1 & μ_2, and standard deviations σ_1 & σ_2.

   b. State the null hypothesis in symbols. $H_0: \mu_1 = \mu_2$

   c. State the alternative hypothesis in symbols. $H_a: \mu_1 \neq \mu_2$

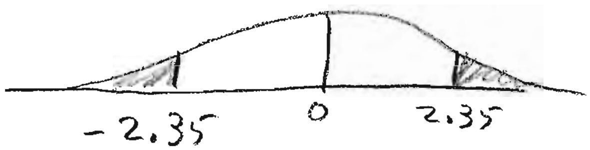
   d. Calculate the test statistic; **show your work**. The answer is a number. **Circle your answer**.

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{66.2 - 64.7}{\sqrt{\frac{4.5^2}{75} + \frac{3.2^2}{75}}}$$

$$= \frac{1.5}{\sqrt{.27 + .1365}} = \frac{1.5}{.6376} = 2.35$$

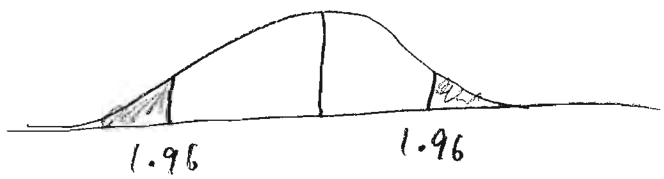
96

e. Calculate the p-value (observed significance level). Show your work, including a picture of a curve with the tail area or areas shaded, and **circle your final answer**.



p-value is 2(.5-.4906) = .0188

f. (~~Points~~) State the rejection region and/or draw a picture of it. Use $\alpha = 0.05$.



$|z| > 1.96$

g. (~~Points~~) Do you reject the null hypothesis? Answer Yes or No.

h. (~~Points~~) Is $p<\alpha$? Answer Yes or No.

i. (~~Points~~) Are the results statistically significant at the 0.05 level? Answer Yes or No.

j. (~~Points~~) Can we conclude that men retire later on average? Yes or No.

4. Weights of livestock are known to be normally distributed. Five beef cattle are randomly assigned to an experimental growth hormone, and four are not. In other ways, all the cattle are treated the same. At maturity, the group that received the hormone had a mean weight of 840 kg with a standard deviation of 34, while those that did not get the hormone had a mean weight of 790 kg with a standard deviation of 33.

   a. State the statistical model you adopt for these data.

   The data are independent random samples from normal populations with means μ_1 & μ_2, and the same variance σ^2.

   b. State the null hypothesis in symbols. $H_0: \mu_1 = \mu_2$

   c. State the alternative hypothesis in symbols. $H_a: \mu_1 \neq \mu_2$

   d. Calculate the test statistic; **show your work**. The answer is a number. **Circle your answer**.

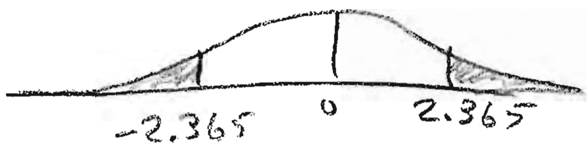
$$S_p^2 = \frac{(5-1)34^2 + (4-1)33^2}{5+4-2} = \frac{4624 + 3267}{7} = 1127.2857$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{840 - 790}{(33.5751)\sqrt{\frac{1}{5} + \frac{1}{4}}}$$

$$= \frac{50}{(33.5751)(.6708)} = 2.22$$

102

e. (~~Points~~) State the rejection region and/or draw a picture of it. Use $\alpha = 0.05$.



$|t| > 2.365$

f. (~~Points~~) Do you reject the null hypothesis? Answer Yes or No.

g. (~~Points~~) Is $p<\alpha$? Answer Yes or No.

h. ([redacted]) Are the results statistically significant at the 0.05 level? Answer Yes or No.

i. ([redacted]) Can we conclude that the growth hormone makes the cattle heavier? NO

5. Every quarter, a different random sample of chewing gum and bubble gum users are asked about gum advertisements they recall seeing or hearing. This is called a "tracking study." Last quarter, 550 consumers were interviewed, and 64% recalled at least one commercial for Wrigley's gum. This quarter, 500 consumers were interviewed, and 62% recalled at least one Wrigley's advertisement. We want to know if this is a real change. Please proceed by answering these questions. The sample sizes are more than big enough; you don't need to check.

   a. State the statistical model you adopt for these data.

   Data are independent random samples from large binary populations w/ the proportions of ones P_1 & P_2

   b. State the null hypothesis in symbols. $H_0: p_1 = p_2$

   c. State the alternative hypothesis in symbols. $H_a: p_1 \neq p_2$

   d. Calculate the test statistic; **show your work**. The answer is a number. **Circle your answer.**

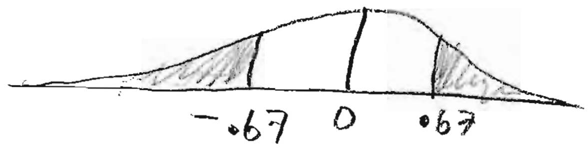
$$\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2} = \frac{(550)(.64) + (500)(.62)}{1050}$$

$$= \frac{352 + 310}{1050} = .6305$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{.64 - .62}{\sqrt{(.6305)(.3695)(.0038)}}$$

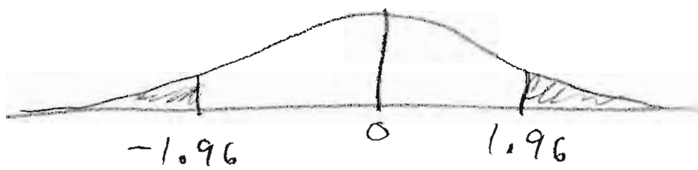
$$= \frac{.02}{.0298} = 0.67$$

e. Calculate the p-value (observed significance level). Show your work, including a picture of a curve with the tail area or areas shaded, and **circle your final answer**.



P-value is $2(.5 - .2486) = .5028 \approx .50$

f. (Points) State the rejection region and/or draw a picture of it. Use $\alpha = 0.05$.



$|z| > 1.96$

g. (Points) Do you reject the null hypothesis? Answer Yes or No.

h. (Points) Is $p<\alpha$? Answer Yes or No.

i. (Points) Are the results statistically significant at the 0.05 level? Answer Yes or No.

j. (Points) Can we conclude that advertising awareness really went down in the population? Answer Yes or No.