STA 442/2101 Formulas

Columns of $\mathbf{A}$ *linearly dependent* means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $\mathbf{Av} = \mathbf{0}$.

Columns of $\mathbf{A}$ *linearly independent* means that $\mathbf{Av} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

$\mathbf{A}$ *positive definite* means $\mathbf{v}^\top \mathbf{A} \mathbf{v} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$\boldsymbol{\Sigma} = \mathbf{P} \boldsymbol{\Lambda} \mathbf{P}^\top$

$\boldsymbol{\Sigma}^{-1} = \mathbf{P} \boldsymbol{\Lambda}^{-1} \mathbf{P}^\top$

$\boldsymbol{\Sigma}^{1/2} = \mathbf{P} \boldsymbol{\Lambda}^{1/2} \mathbf{P}^\top$

$\boldsymbol{\Sigma}^{-1/2} = \mathbf{P} \boldsymbol{\Lambda}^{-1/2} \mathbf{P}^\top$

If $\lim_{n\rightarrow\infty} E(T_n) = \theta$ and $\lim_{n\rightarrow\infty} Var(T_n) = 0$, then $T_n \stackrel{p}{\rightarrow} \theta$

If $\sqrt{n}(T_n - \mu) \stackrel{d}{\rightarrow} T \sim N(0, \sigma^2)$, then $\sqrt{n}\left(g(T_n) - g(\mu)\right) \stackrel{d}{\rightarrow} g'(\mu) T \sim N(0, g'(\mu)^2 \sigma^2)$

If $\mathbf{T}_n \stackrel{d}{\rightarrow} \mathbf{T}$ and $\mathbf{Y}_n \stackrel{p}{\rightarrow} \mathbf{c}$, then $\left(\begin{array}{c} \mathbf{T}_n \\ \mathbf{Y}_n \end{array} \right) \stackrel{d}{\rightarrow} \left(\begin{array}{c} \mathbf{T} \\ \mathbf{c} \end{array} \right)$

$\sqrt{n}(\overline{\mathbf{x}}_n - \boldsymbol{\mu}) \stackrel{d}{\rightarrow} \mathbf{x} \sim N(\mathbf{0}, \boldsymbol{\Sigma})$

Let $g: \mathbb{R}^d \rightarrow \mathbb{R}^k$ etc. If $\sqrt{n}(\mathbf{T}_n - \boldsymbol{\theta}) \stackrel{d}{\rightarrow} \mathbf{T}$, then $\sqrt{n}(g(\mathbf{T}_n) - g(\boldsymbol{\theta})) \stackrel{d}{\rightarrow} \dot{\mbox{g}}(\boldsymbol{\theta}) \mathbf{T}$, where $\dot{\mbox{g}}(\boldsymbol{\theta}) = \left[\frac{\partial g_i}{\partial \theta_j} \right]_{k \times d}$

$G^2 = -2 \log \left(\frac{\max_{\theta \in \Theta_0} L(\theta)}{\max_{\theta \in \Theta} L(\theta)} \right) = -2 \log \left(\frac{L(\widehat{\theta}_0)}{L(\widehat{\theta})} \right)$

$W_n = (\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h})^\top \left(\mathbf{L} \widehat{\mathbf{V}}_n \mathbf{L}^\top \right)^{-1} (\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h})$

$cov(\mathbf{w}) = E\left\{ (\mathbf{w} - \boldsymbol{\mu}_w)(\mathbf{w} - \boldsymbol{\mu}_w)^\top \right\}$

$cov(\mathbf{w}, \mathbf{t}) = E\left\{ (\mathbf{w} - \boldsymbol{\mu}_w)(\mathbf{t} - \boldsymbol{\mu}_t)^\top \right\}$

$cov(\mathbf{w}) = E\{\mathbf{w}\mathbf{w}^\top\} - \boldsymbol{\mu}_w \boldsymbol{\mu}_w^\top$

$cov(\mathbf{Aw}) = \mathbf{A} cov(\mathbf{w}) \mathbf{A}^\top$

If $\mathbf{w} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $\mathbf{Aw} + \mathbf{c} \sim N_r(\mathbf{A}\boldsymbol{\mu} + \mathbf{c}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$

and $(\mathbf{w} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1} (\mathbf{w} - \boldsymbol{\mu}) \sim \chi^2(p)$

$L(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2} (2\pi)^{-np/2} \exp -\frac{n}{2} \left\{ tr(\widehat{\boldsymbol{\Sigma}} \boldsymbol{\Sigma}^{-1}) + (\overline{\mathbf{y}} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1} (\overline{\mathbf{y}} - \boldsymbol{\mu}) \right\}$, where $\widehat{\boldsymbol{\Sigma}} = \frac{1}{n} \sum_{i=1}^n (\mathbf{y}_i - \overline{\mathbf{y}})(\mathbf{y}_i - \overline{\mathbf{y}})^\top$

$M_y(t) = E(e^{yt})$

$M_{ay}(t) = M_y(at)$

$M_{y+a}(t) = e^{at} M_y(t)$

$M_{\sum_{i=1}^n y_i}(t) = \prod_{i=1}^n M_{y_i}(t)$

$y \sim N(\mu, \sigma^2)$ means $M_y(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

$y \sim \chi^2(\nu)$ means $M_y(t) = (1 - 2t)^{-\nu/2}$

If $W = W_1 + W_2$ with W_1 and W_2 independent, $W \sim \chi^2(\nu_1 + \nu_2)$, $W_2 \sim \chi^2(\nu_2)$ then $W_1 \sim \chi^2(\nu_1)$

$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$

$\widehat{\beta}_0 = \overline{y} - \widehat{\beta}_1 \overline{x}$

$\widehat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^n (x_i - \overline{x})^2} = \frac{\sum_{i=1}^n x_i y_i - n\,\overline{x}\,\overline{y}}{\sum_{i=1}^n x_i^2 - n\overline{x}^2}$

$r = \frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^n (x_i - \overline{x})^2} \sqrt{\sum_{i=1}^n (y_i - \overline{y})^2}}$

$y_i = \beta_0 + \beta_1 x_{i,1} + \cdots + \beta_{p-1} x_{i,p-1} + \epsilon_i$ $\epsilon_1, \ldots, \epsilon_n$ independent $N(0, \sigma^2)$

$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ $\boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \sigma^2 \mathbf{I}_n)$

$\widehat{\boldsymbol{\beta}} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{y} \sim N_p\left(\boldsymbol{\beta}, \sigma^2(\mathbf{X}^\top\mathbf{X})^{-1}\right)$ $\widehat{\mathbf{y}} = \mathbf{X}\widehat{\boldsymbol{\beta}} = \mathbf{H}\mathbf{y}$, where $\mathbf{H} = \mathbf{X}(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top$

$\mathbf{e} = \mathbf{y} - \widehat{\mathbf{y}} = (\mathbf{I} - \mathbf{H})\mathbf{y}, \quad \mathbf{X}^\top\mathbf{e} = \mathbf{0}$ $\widehat{\boldsymbol{\beta}}$ and $\mathbf{e}$ are independent under normality.

$\sum_{i=1}^n (y_i - \overline{y})^2 = \sum_{i=1}^n (y_i - \widehat{y}_i)^2 + \sum_{i=1}^n (\widehat{y}_i - \overline{y})^2$ $SST = SSE + SSR$ and $R^2 = \frac{SSR}{SST}$

$\frac{SSE}{\sigma^2} = \frac{\mathbf{e}^\top\mathbf{e}}{\sigma^2} \sim \chi^2(n-p)$ $MSE = \frac{SSE}{n-p}$

$T = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$ $F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1, \nu_2)$

Under $H_0 : \mathbf{L}\boldsymbol{\beta} = \mathbf{h}$, $F^* = \frac{(\mathbf{L}\widehat{\boldsymbol{\beta}}-\mathbf{h})^\top(\mathbf{L}(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{L}^\top)^{-1}(\mathbf{L}\widehat{\boldsymbol{\beta}}-\mathbf{h})}{r\,MSE} = \frac{SSR_F - SSR_R}{r\,MSE_F} \sim F(r, n-p)$

$\pi(\theta|x) = \frac{f(x|\theta)\pi(\theta)}{\int f(x|t)\pi(t)\,dt}$ $E(L(d,x)|X=x) = \int L\left(d(x), \theta\right)\pi(\theta|x)\,d\theta$

Distribution	**Density or pmf**	**Mean**	**Variance**
Exponential	$f(x) = \theta e^{-\theta x}$ for $x > 0$	$1/\theta$	$1/\theta^2$
Gamma	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} e^{-\beta x}\, x^{\alpha-1}$ for $x > 0$	α/β	α/β^2
Exponential	$f(x) = \theta e^{-\theta x}$ for $x > 0$	$1/\theta$	$1/\theta^2$
Normal	$f(x) = \frac{\tau^{1/2}}{\sqrt{2\pi}} e^{-\frac{\tau}{2}(x-\mu)^2}$	μ	$1/\tau$
Beta	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\, x^{\alpha-1}(1-x)^{\beta-1}$ for $0 < x < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Binomial	$p(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$ for $x = 0, 1, \ldots, n$	$n\theta$	$n\theta(1-\theta)$
Poisson	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, \ldots$	λ	λ

$\log\left(\frac{\pi_i}{1-\pi_i}\right) = \beta_0 + \beta_1 x_{i,1} + \ldots + \beta_{p-1} x_{i,p-1}$ $\pi_i = \frac{e^{\beta_0 + \beta_1 x_{i,1} + \ldots + \beta_{p-1} x_{i,p-1}}}{1 + e^{\beta_0 + \beta_1 x_{i,1} + \ldots + \beta_{p-1} x_{i,p-1}}}$

$\log(\lambda_i) = \beta_0 + \beta_1 x_{i,1} + \ldots + \beta_{p-1} x_{i,p-1}$

$$\log\left(\frac{\pi_1}{\pi_3}\right) = \beta_{0,1} + \beta_{1,1} x_1 + \ldots + \beta_{p-1,1} x_{p-1} = L_1$$

$$\log\left(\frac{\pi_2}{\pi_3}\right) = \beta_{0,2} + \beta_{1,2} x_1 + \ldots + \beta_{p-1,2} x_{p-1} = L_2$$

$$\pi_1 = \frac{e^{L_1}}{1 + e^{L_1} + e^{L_2}}$$

$$\pi_2 = \frac{e^{L_2}}{1 + e^{L_1} + e^{L_2}}$$

$$\pi_3 = \frac{1}{1 + e^{L_1} + e^{L_2}}$$

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> # Chi-squared critical values
> df = 1:6
> Critical_Value = qchisq(0.95,df)
> cbind(df,Critical_Value)
     df Critical_Value
[1,]  1       3.841459
[2,]  2       5.991465
[3,]  3       7.814728
[4,]  4       9.487729
[5,]  5      11.070498
[6,]  6      12.591587
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