Name Jerry

Student Number

STA 442/2101 f2013 Quiz 3

1. (4 points) One form of the delta method says that if $\sqrt{n}\,(T_n - \mu) \stackrel{d}{\rightarrow} T$, then $\sqrt{n}\,(g(T_n) - g(\mu)) \stackrel{d}{\rightarrow} g'(\mu)\,T$. Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with mean $\mu \neq 0$ and variance μ^2. What is the limiting distribution of $Y_n = \sqrt{n}\left(\log(\overline{X}_n) - \log(\mu)\right)$? That's the natural log, of course. Show your work.

Central Limit Theorem says

$$\sqrt{n}\,(\overline{X}_n - \mu) \xrightarrow{d} T \sim N(0, \mu^2), \text{ so}$$

$$\sqrt{n}\,(\log(\overline{X}_n) - \log(\mu)) \xrightarrow{d} g'(\mu)\,T$$

$$= \frac{1}{\mu}\,T \sim N(0,1)$$

So the limiting distribution of Y_n is standard normal.

2. (5 points) The standard multiple regression model is $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, where $\mathbf{X}$ is an $n \times p$ matrix of known constants with linearly independent columns, $\boldsymbol{\beta}$ is a $p \times 1$ vector of unknown constants, and $\boldsymbol{\epsilon}$ is multivariate normal with mean zero and covariance matrix $\sigma^2\mathbf{I}_n$, where $\sigma^2 > 0$ is an unknown constant. The maximum likelihood estimator (MLE) of $\boldsymbol{\beta}$ is $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$. What is the distribution of $\hat{\boldsymbol{\beta}}$? Show the calculations. You may use $V(\mathbf{AZ}) = \mathbf{A}\boldsymbol{\Sigma}_z\mathbf{A}'$ without proof. **End your answer with** "So the distribution of $\hat{\boldsymbol{\beta}}$ is ...

$Y \sim N_n(X\beta, \sigma^2 I_n)$, and

$\hat{\beta} = (X'X)^{-1}X'Y = AY$, so

$\hat{\beta} \sim N_p(AX\beta, A\sigma^2 I_n A')$

$AX\beta = (X'X)^{-1}X'X\beta = \beta$, and

$A\sigma^2 I_n A' = (X'X)^{-1}X'\sigma^2 I_n ((X'X)^{-1}X')'$

$= \sigma^2 (X'X)^{-1}X'X(X'X)^{-1}$ Because $(X'X)^{-1}$ is symmetric

$= \sigma^2 (X'X)^{-1}$

So the distribution of $\hat{\beta}$ is $N_p(\beta, \sigma^2(X'X)^{-1})$

(No marks off for not mentioning symmetry of $(X'X)^{-1}$ explicitly, but something off for leaving it as $(X'X)^{-1\prime}$.)