Name Jerry

Student Number

STA 442/2101 f2012 Quiz 2

1. (3 points) Food researchers ~~select a random~~ sample ~~of~~ 200 half-kilogram boxes of rice from local grocery stores. One hundred boxes are randomly ~~selected~~ chosen, and the rice in ~~them~~ those boxes is rinsed with water before cooking. Rice from the other 100 boxes is not rinsed. Then, all the rice is cooked in standard rice cookers for the same length of time and using the same amount of water. A laboratory tests the concentration of arsenic (a poison used in some pesticides) in each batch of rice.

   (a) State a reasonable model for these data. It would not be too surprising if arsenic concentration had a distribution that was close to normal. If some of the random variables in your model are independent of each other, be sure to mention it.

   Washed Rice: $X_1 \ldots X_{n_1} \stackrel{iid}{\sim} N(\mu_1, \sigma^2)$

   Unwashed Rice: $Y_1 \ldots Y_{n_2} \stackrel{iid}{\sim} N(\mu_2, \sigma^2)$

   X_i, Y_j independent for all i, j

   Or one could assume $\sigma_1^2 \neq \sigma_2^2$

   Of course $n_1 = n_2 = 100$ is okay

   (b) The point of the study is to see whether rinsing the rice reduces the average arsenic concentration. Using symbols from your model, what is the null hypothesis?

   $H_0: \mu_1 = \mu_2$ or $\mu_1 \geq \mu_2$

   I like this one more, but no points off for the other one.

2. (7 points) Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with parameter $\lambda > 0$, so that $E(X_i) = Var(X_i) = \lambda$. Estimate λ with

$$\widehat{\lambda}_n = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X}_n)^2$$

Is $\widehat{\lambda}_n$ a consistent estimator of λ? Answer Yes or No. **Circle the word Yes or the word No**. Show your work.

$$\hat{\lambda}_n = \frac{1}{n}\sum_{i=1}^{n}(X_i^2 - 2\bar{X}X_i + \bar{X}^2) = \frac{1}{n}\left(\sum_{i=1}^{n}X_i^2 - 2\bar{X}_n\sum_{i=1}^{n}X_i + n\bar{X}_n^2\right)$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n}X_i^2 - 2n\bar{X}_n^2 + n\bar{X}_n^2\right) = \frac{1}{n}\sum_{i=1}^{n}X_i^2 - \bar{X}_n^2$$

By Law of Large Numbers, $\frac{1}{n}\sum_{i=1}^{n}X_i^2 \xrightarrow{P} E(X_i^2) = \lambda + \lambda^2$

By Law of Large Numbers and continuous mapping (Slutsky),

$\bar{X}_n^2 \xrightarrow{P} (E(X_i))^2 = \lambda^2$, and by continuous mapping again,

$\frac{1}{n}\sum_{i=1}^{n}X_i^2 - \bar{X}_n^2 \xrightarrow{P} (\lambda + \lambda^2) - \lambda^2 = \lambda$ consistent

$\boxed{\text{YES}}$