Likelihood methods on the Gamma data with R

```
> # Could do:  rm(list=ls())

> set.seed(3201); alpha=2; beta=3
> D <- round(rgamma(50,shape=alpha, scale=beta),2); D
 [1] 20.87 13.74  5.13  2.76  4.73  2.66 11.74  0.75 22.07 10.49  7.26  5.82 13.08
[14]  1.79  4.57  1.40  1.13  6.84  3.21  0.38 11.24  1.72  4.69  1.96  7.87  8.49
[27]  5.31  3.40  5.24  1.64  7.17  9.60  6.97 10.87  5.23  5.53 15.80  6.40 11.25
[40]  4.91 12.05  5.44 12.62  1.81  2.70  3.03  4.09 12.29  3.23 10.94
> # Gamma minus log likelihood: alpha=a, beta=b
> gmll <- function(theta,datta)
+       {
+        a <- theta[1]; b <- theta[2]
+        n <- length(datta); sumd <- sum(datta); sumlogd <- sum(log(datta))
+        gmll <- n*a*log(b) + n*lgamma(a) + sumd/b - (a-1)*sumlogd
+       gmll
+       } # End function gmll
>
> momalpha <- mean(D)^2/var(D); momalpha
[1] 1.899754
> mombeta <- var(D)/mean(D); mombeta
[1] 3.620574
> gammasearch = nlm(gmll,c(momalpha,mombeta),hessian=T,datta=D); gammasearch
$minimum
[1] 142.0316

$estimate
[1] 1.805930 3.808674

$gradient
[1] 2.847002e-05 9.133932e-06

$hessian
         [,1]      [,2]
[1,] 36.68932 13.127271
[2,] 13.12727  6.222282

$code
[1] 1

$iterations
[1] 6

>
> ################################################
>
```

```
> V_hat = solve(gammasearch$hessian); V_hat
           [,1]       [,2]
[1,]  0.1111796 -0.2345577
[2,] -0.2345577  0.6555638

> SEalphahat = sqrt(V_hat[1,1]); SEbetahat = sqrt(V_hat[2,2])
> alphahat = gammasearch$estimate[1]; betahat = gammasearch$estimate[2]
> Lalpha = alphahat - 1.96*SEalphahat; Ualpha = alphahat + 1.96*SEalphahat
> Lbeta = betahat - 1.96*SEbetahat; Ubeta = betahat + 1.96*SEbetahat
> cat("\nEstimated alpha = ",round(alphahat,2),"  95 percent CI from ",
+      round(Lalpha,2)," to ",round(Ualpha,2), "\n\n")

Estimated alpha =  1.81   95 percent CI from  1.15  to  2.46
>
> WaldTest = function(L,thetahat,Vhat,h=0) # H0: L theta = h
+     {
+     WaldTest = numeric(2)
+     names(WaldTest) = c("W","p-value")
+     dfree = dim(L)[1]
+     W = t(L%*%thetahat-h) %*% solve(L%*%Vhat%*%t(L)) %*% (L%*%thetahat-h)
+     W = as.numeric(W)
+     pval = 1-pchisq(W,dfree)
+     WaldTest[1] = W; WaldTest[2] = pval
+     WaldTest
+     } # End function WaldTest

>
> # H0: alpha = 1
> WaldTest(t(c(1,0)),as.matrix(c(alphahat,betahat)),V_hat,1)

         W    p-value
5.84210986 0.01564705

>
> #  H0: C theta = 0 is that alpha = beta <=> alpha-beta=0
> # Name C is used by R
> CC = rbind(c(1,-1)); is.matrix(CC); dim(CC)
[1] TRUE
[1] 1 2
> WaldTest(CC,as.matrix(c(alphahat,betahat)),V_hat)

        W   p-value
3.2455032 0.0716197

>
> # H0: alpha=2, beta=3
> C2 = rbind(c(1,0),
+           c(0,1) )
> WaldTest(C2,as.matrix(c(alphahat,betahat)),V_hat,c(2,3))

        W   p-value
1.3305501 0.5141321
```

```
>
> # LR alpha=1
> gmll1 <- function(b,datta) # Restricted gamma minus LL with alpha=1
+     { gmll1 <- gmll(c(1,b),datta)
+       gmll1
+     } # End of function gmll1
> mean(D) # Restricted MLE of beta, just to check
[1] 6.8782

> gsearch1 <- nlm(gmll1,betahat,datta=D); gsearch1

$minimum
[1] 146.4178

$estimate
[1] 6.878195

$gradient
[1] -1.772691e-06

$code
[1] 1
$iterations
[1] 7

> G = 2 * (gsearch1$minimum-gammasearch$minimum); pval = 1-pchisq(G,df=1)
> G; pval
[1] 8.772448
[1] 0.003058146
>
> # LR alpha=beta
> gmll2 <- function(ab,datta) # Restricted gamma minus LL with alpha=1
+     { gmll2 <- gmll(c(ab,ab),datta)
+       gmll2
+     } # End of function gmll2
> abstart = (alphahat+betahat)/2; abstart
[1] 2.807302
> gsearch2 <- nlm(gmll2,abstart,datta=D); gsearch2
Warning messages:
1: NaNs produced in: log(x)
2: NA/Inf replaced by maximum positive value
$minimum
[1] 144.1704

$estimate
[1] 2.562369

$gradient
[1] -4.769545e-07

$code
[1] 1

$iterations
[1] 4
```

```
> G = 2 * (gsearch2$minimum-gammasearch$minimum); pval = 1-pchisq(G,df=1)
> G; pval
[1] 4.277603
[1] 0.03861777
>
> gsearch2b <- nlminb(start=abstart,objective=gmll2,lower=0,datta=D); gsearch2b

$par
[1] 2.562371

$objective
[1] 144.1704

$convergence
[1] 0

$message
[1] "relative convergence (4)"

$iterations
[1] 5

$evaluations
function gradient
       7        8
>
> # LR alpha=2, beta=3
> G = 2 * (gmll(c(2,3),D)-gammasearch$minimum); pval = 1-pchisq(G,df=1)
> G; pval
[1] 2.269162
[1] 0.1319713
>
```

Table A.2: Tests on data from a gamma distribution with $\alpha = 2$ and $\beta = 3$

$n = 50$				
	Wald		Likelihood Ratio	
H_0	χ^2	p-value	χ^2	p-value
$\alpha = 1$	5.8421	0.0156	8.7724	0.0031
$\alpha = \beta$	3.2455	0.0762	4.2776	0.0386
$\alpha = 2, \beta = 3$	1.3305	0.5141	2.2692	0.1320
$n = 200$				
$\alpha = 1$	34.1847	5.01e-09	58.2194	2.34e-14
$\alpha = \beta$	0.9197	0.3376	0.9664	0.3256
$\alpha = 2, \beta = 3$	1.5286	0.4657	1.2724	0.2593