Nested, Random Effects and Mixed Model ANOVA

```
1 1 25
1 1 29
1 2 14
1 2 11
2 1 11
2 1  6
2 2 22
2 2 18
3 1 17
3 1 20
3 2  5
3 2  2

/*************  train.sas ***********************
 Nested Example from NKNW Ch. 28 (Table 28.1)
*************************************************/
title 'Training school example: Nested design with fixed effects';
options linesize=79 noovp formdlim='_';

proc format;  /* value labels used in data step below */
     value cityfmt 1 = 'Atlanta'  2 = 'Chicago' 3 = 'San Francisco';

data school;
     infile 'training.data';
     input city teacher learning;
     format city cityfmt.;
     cityprof = 10*city + teacher;

proc freq;
     tables (city teacher) * cityprof / norow nocol nopercent missing;

proc glm;
     title2 'With Contrasts';
     class cityprof;
     model learning = cityprof;
     contrast 'City'
               cityprof  1  1 -1 -1  0  0,
               cityprof  1  1  0  0 -1 -1;
     contrast 'Prof within City'
               cityprof  1 -1  0  0  0  0,
               cityprof  0  0  1 -1  0  0,
               cityprof  0  0  0  0  1 -1;

proc glm;
     title2 'With proc glm syntax';
     class city teacher;
     model learning = city teacher(city) ;
     means city teacher(city);

proc glm;
     title2 'Teachers randomly selected within city';
     class city teacher;
     model learning = city teacher(city) ;
     random teacher(city) / test;
```

Training school example: Nested design with fixed effects 1

The FREQ Procedure

Table of city by cityprof

city / Frequency	cityprof 11	12	21	22	31	32	Total
Atlanta	2	2	0	0	0	0	4
Chicago	0	0	2	2	0	0	4
San Francisco	0	0	0	0	2	2	4
Total	2	2	2	2	2	2	12

Table of teacher by cityprof

teacher / Frequency	cityprof 11	12	21	22	31	32	Total
1	2	0	2	0	2	0	6
2	0	2	0	2	0	2	6
Total	2	2	2	2	2	2	12

Training school example: Nested design with fixed effects 2
With Contrasts

The GLM Procedure

Class Level Information

Class	Levels	Values
cityprof	6	11 12 21 22 31 32

Number of Observations Read 12
Number of Observations Used 12

With Contrasts

The GLM Procedure

Dependent Variable: learning

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	724.0000000	144.8000000	20.69	0.0010
Error	6	42.0000000	7.0000000		
Corrected Total	11	766.0000000			

R-Square	Coeff Var	Root MSE	learning Mean
0.945170	17.63834	2.645751	15.00000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
cityprof	5	724.0000000	144.8000000	20.69	0.0010

Source	DF	Type III SS	Mean Square	F Value	Pr > F
cityprof	5	724.0000000	144.8000000	20.69	0.0010

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
City	2	156.5000000	78.2500000	11.18	0.0095
Prof within City	3	567.5000000	189.1666667	27.02	0.0007

With proc glm syntax

The GLM Procedure

Class Level Information

Class	Levels	Values
city	3	Atlanta Chicago San Francisco
teacher	2	1 2

Number of Observations Read	12
Number of Observations Used	12

The GLM Procedure

Dependent Variable: learning

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	724.0000000	144.8000000	20.69	0.0010
Error	6	42.0000000	7.0000000		
Corrected Total	11	766.0000000			

R-Square	Coeff Var	Root MSE	learning Mean
0.945170	17.63834	2.645751	15.00000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
city	2	156.5000000	78.2500000	11.18	0.0095
teacher(city)	3	567.5000000	189.1666667	27.02	0.0007

Source	DF	Type III SS	Mean Square	F Value	Pr > F
city	2	156.5000000	78.2500000	11.18	0.0095
teacher(city)	3	567.5000000	189.1666667	27.02	0.0007

The GLM Procedure

Level of city	N	learning Mean	learning Std Dev
Atlanta	4	19.7500000	8.61684397
Chicago	4	14.2500000	7.13559154
San Francisco	4	11.0000000	8.83176087

Level of teacher	Level of city	N	learning Mean	learning Std Dev
1	Atlanta	2	27.0000000	2.82842712
2	Atlanta	2	12.5000000	2.12132034
1	Chicago	2	8.5000000	3.53553391
2	Chicago	2	20.0000000	2.82842712
1	San Francisco	2	18.5000000	2.12132034
2	San Francisco	2	3.5000000	2.12132034

Teachers randomly selected within city

The GLM Procedure

Class Level Information

Class	Levels	Values
city	3	Atlanta Chicago San Francisco
teacher	2	1 2

Number of Observations Read	12
Number of Observations Used	12

Teachers randomly selected within city

The GLM Procedure

Dependent Variable: learning

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	724.0000000	144.8000000	20.69	0.0010
Error	6	42.0000000	7.0000000		
Corrected Total	11	766.0000000			

R-Square	Coeff Var	Root MSE	learning Mean
0.945170	17.63834	2.645751	15.00000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
city	2	156.5000000	78.2500000	11.18	0.0095
teacher(city)	3	567.5000000	189.1666667	27.02	0.0007

Source	DF	Type III SS	Mean Square	F Value	Pr > F
city	2	156.5000000	78.2500000	11.18	0.0095
teacher(city)	3	567.5000000	189.1666667	27.02	0.0007

The GLM Procedure

Source	Type III Expected Mean Square
city	Var(Error) + 2 Var(teacher(city)) + Q(city)
teacher(city)	Var(Error) + 2 Var(teacher(city))

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Dependent Variable: learning

Source	DF	Type III SS	Mean Square	F Value	Pr > F
city	2	156.500000	78.250000	0.41	0.6940
Error	3	567.500000	189.166667		

Error: MS(teacher(city))

Source	DF	Type III SS	Mean Square	F Value	Pr > F
teacher(city)	3	567.500000	189.166667	27.02	0.0007
Error: MS(Error)	6	42.000000	7.000000		

Level of city	N	learning Mean	learning Std Dev
Atlanta	4	19.7500000	8.61684397
Chicago	4	14.2500000	7.13559154
San Francisco	4	11.0000000	8.83176087

Level of teacher	Level of city	N	learning Mean	learning Std Dev
1	Atlanta	2	27.0000000	2.82842712
2	Atlanta	2	12.5000000	2.12132034
1	Chicago	2	8.5000000	3.53553391
2	Chicago	2	20.0000000	2.82842712
1	San Francisco	2	18.5000000	2.12132034

```
      2            San Francisco     2        3.5000000       2.12132034
```

```
/************************  mpg.sas  **************************************
* A random sample of 4 drivers tests a random sample of 5 cars of the same *
* make and model. Each driver drover each car twice over a set course, and *
* fuel consumption in miles per gallon was recorded.                        *
***************************************************************************/

title 'MPG: Two-factor random effects model with proc glm';
options linesize=79 noovp formdlim=' ';

data gas;
     infile 'Prob25.15.data' firstobs=2;
     input Mpg Driver Car Replicate;

proc glm;
     class driver car;
     model mpg = driver|car;
     random driver|car / test;
     means driver car; /* Just main effects */
```

```
              MPG: Two-factor random effects model with proc glm              1

                               The GLM Procedure

                           Class Level Information

                      Class         Levels    Values

                      Driver             4    1 2 3 4

                      Car                5    1 2 3 4 5


                    Number of Observations Read          40
                    Number of Observations Used          40
```

```
              MPG: Two-factor random effects model with proc glm              2

                               The GLM Procedure

Dependent Variable: Mpg

                                        Sum of
Source                      DF         Squares    Mean Square   F Value   Pr > F

Model                       19     377.4447500     19.8655132    113.03   <.0001

Error                       20       3.5150000      0.1757500

Corrected Total             39     380.9597500
```

R-Square	Coeff Var	Root MSE	Mpg Mean
0.990773	1.395209	0.419225	30.04750

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Driver	3	280.2847500	93.4282500	531.60	<.0001
Car	4	94.7135000	23.6783750	134.73	<.0001
Driver*Car	12	2.4465000	0.2038750	1.16	0.3715

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Driver	3	280.2847500	93.4282500	531.60	<.0001
Car	4	94.7135000	23.6783750	134.73	<.0001
Driver*Car	12	2.4465000	0.2038750	1.16	0.3715

MPG: Two-factor random effects model with proc glm 3

The GLM Procedure

Source	Type III Expected Mean Square
Driver	Var(Error) + 2 Var(Driver*Car) + 10 Var(Driver)
Car	Var(Error) + 2 Var(Driver*Car) + 8 Var(Car)
Driver*Car	Var(Error) + 2 Var(Driver*Car)

MPG: Two-factor random effects model with proc glm 4

The GLM Procedure
Tests of Hypotheses for Random Model Analysis of Variance

Dependent Variable: Mpg

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Driver	3	280.284750	93.428250	458.26	<.0001
Car	4	94.713500	23.678375	116.14	<.0001
Error	12	2.446500	0.203875		

Error: MS(Driver*Car)

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Driver*Car	12	2.446500	0.203875	1.16	0.3715
Error: MS(Error)	20	3.515000	0.175750		

Level of Driver	N	Mpg Mean	Mpg Std Dev
1	10	26.9300000	1.82820495
2	10	34.1500000	1.76525101
3	10	28.8500000	1.49981480
4	10	30.2600000	1.57423703

Level of Car	N	Mpg Mean	Mpg Std Dev
1	8	28.9625000	3.08356820
2	8	32.2500000	2.92818813
3	8	27.9125000	2.73831935
4	8	31.1625000	2.83797589
5	8	29.9500000	2.69390847

```
/************************* pearls2.sas ********************************
Plastic beads are produced in batches of 12, and coated with several layers of
lacquer to make false pearls. Four batches of 12 beads are randomly selected,
and four beads from each batch are randomly selected to receive either 6, 8 or
10 coats of lacquer. The factor number of coats (6, 8 or 10) is fixed, and
batch is a random effect. The dependent variable is assessed value of the
pearl. This is an improvement on pearls.sas, because we specify the
denominator MS for the multiple comparisons instead of doing calculations by
hand.
*************************************************************************/

title 'Pearls: Two-factor mixed model with proc glm';
options linesize=79 noovp formdlim=' ';

data gas;
     infile 'Prob25.17.data' firstobs=2;
     input Value Coatings Batch Replicate;
     if coatings=1 then coatings=6;
        else if coatings=2 then coatings=8;
        else if coatings=3 then coatings=10;

proc glm;
     class coatings batch;
     model value = coatings|batch;
     random batch coatings*batch / test;
     means coatings batch;
     contrast 'Marginal 6 vs 8 Coatings'
               coatings 1 -1  0 / e = coatings*batch;
     contrast 'Marginal 6 vs 10 Coatings'
               coatings 1  0 -1 / e = coatings*batch;
     contrast 'Marginal 8 vs 10 Coatings'
               coatings 0  1 -1 / e = coatings*batch;
     means coatings / scheffe e = coatings*batch;

/* Unless all effects are fixed, you have to tell contrast and means what MS
Error term to use in the denominator of the F-ratio. Use the denominator Mean
Square from the mixed model test for the effect you are following up -- in
this case, MS for interaction.  Here, the initial test is for main effect of
number of coatings.  */

proc iml;
     title3 'Table of critical values for all possible Scheffe tests';
     numdf = 2;   /* Numerator degrees of freedom for initial test */
     dendf = 6;  /* Denominator degrees of freedom for initial test */
     alpha = 0.05;
     critval = finv(1-alpha,numdf,dendf);
     zero = {0 0}; S_table = repeat(zero,numdf,1);  /* Make empty matrix */
     /* Label the columns */
     namz = {"Number of Contrasts in followup test"
             "    Scheffe Critical Value"}; mattrib S_table colname=namz;
     do i = 1 to numdf;
        s_table(|i,1|) = i;
        s_table(|i,2|) = numdf/i *  critval;
     end;
     reset noname; /* Makes output look nicer in this case */
     print "Initial test has"  numdf " and " dendf "degrees of freedom."
           "Using significance level alpha = " alpha;
     print s_table;
```

Pearls: Two-factor mixed model with proc glm 1

The GLM Procedure

Class Level Information

Class	Levels	Values
Coatings	3	6 8 10
Batch	4	1 2 3 4

Number of Observations Read	48
Number of Observations Used	48

Pearls: Two-factor mixed model with proc glm 2

The GLM Procedure

Dependent Variable: Value

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	305.0916667	27.7356061	5.75	<.0001
Error	36	173.6250000	4.8229167		
Corrected Total	47	478.7166667			

R-Square	Coeff Var	Root MSE	Value Mean
0.637312	2.904593	2.196114	75.60833

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Coatings	2	150.3879167	75.1939583	15.59	<.0001
Batch	3	152.8516667	50.9505556	10.56	<.0001
Coatings*Batch	6	1.8520833	0.3086806	0.06	0.9988

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Coatings	2	150.3879167	75.1939583	15.59	<.0001
Batch	3	152.8516667	50.9505556	10.56	<.0001
Coatings*Batch	6	1.8520833	0.3086806	0.06	0.9988

Pearls: Two-factor mixed model with proc glm 3

The GLM Procedure

Source	Type III Expected Mean Square
Coatings	Var(Error) + 4 Var(Coatings*Batch) + Q(Coatings)
Batch	Var(Error) + 4 Var(Coatings*Batch) + 12 Var(Batch)
Coatings*Batch	Var(Error) + 4 Var(Coatings*Batch)

Pearls: Two-factor mixed model with proc glm 4

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance

Dependent Variable: Value

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Coatings	2	150.387917	75.193958	243.60	<.0001
Batch	3	152.851667	50.950556	165.06	<.0001
Error	6	1.852083	0.308681		
Error: MS(Coatings*Batch)					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Coatings*Batch	6	1.852083	0.308681	0.06	0.9988
Error: MS(Error)	36	173.625000	4.822917		

Pearls: Two-factor mixed model with proc glm 5

Level of Coatings	N	Value Mean	Value Std Dev
6	16	73.1062500	2.84522260
8	16	76.7937500	2.47237234
10	16	76.9250000	2.77140157

Level of Batch	N	Value Mean	Value Std Dev
1	12	74.2833333	2.87270330
2	12	76.9916667	2.98220734
3	12	77.7083333	2.18526401
4	12	73.4500000	2.77537876

Scheffe's Test for Value

NOTE: This test controls the Type I experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	6
Error Mean Square	0.308681
Critical Value of F	5.14325
Minimum Significant Difference	0.63

Means with the same letter are not significantly different.

Scheffe Grouping	Mean	N	Coatings
A	76.9250	16	10
A			
A	76.7938	16	8
B	73.1063	16	6

Dependent Variable: Value

Tests of Hypotheses Using the Type III MS for Coatings*Batch as an Error Term

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Marginal 6 vs 8 Coatings	1	108.7812500	108.7812500	352.41	<.0001
Marginal 6 vs 10 Coating	1	116.6628125	116.6628125	377.94	<.0001
Marginal 8 vs 10 Coating	1	0.1378125	0.1378125	0.45	0.5289

Table of critical values for all possible Scheffe tests

Initial test has 2 and 6 degrees of freedom.
Using significance level alpha = 0.05

Number of Contrasts in followup test	Scheffe Critical Value
1	10.286506
2	5.1432528