Factorial ANOVA

```
/* potato.sas */
options linesize=79 noovp formdlim='_';
title 'Rotten potatoes';
title2 'Two-factor ANOVA several different ways';

proc format;
     value tfmt 1 = 'Cool'  2 = 'Warm';

data spud;
     infile 'potato2.data' firstobs=2; /* Skip the first line that R uses */
     input id bact temp rot;
     /* Cell means coding for all 6 treatment combinations */
     if temp=1 and bact=1 then mu11=1; else mu11=0;
     if temp=1 and bact=2 then mu12=1; else mu12=0;
     if temp=1 and bact=3 then mu13=1; else mu13=0;
     if temp=2 and bact=1 then mu21=1; else mu21=0;
     if temp=2 and bact=2 then mu22=1; else mu22=0;
     if temp=2 and bact=3 then mu23=1; else mu23=0;
     combo = 10*temp+bact;
     format temp tfmt.;

proc means;
     class bact temp;
     var rot;

/* Better looking output from proc tabulate */

proc tabulate;
     class bact temp;
     var rot;
     table (temp all),(bact all) * (mean*rot);

proc glm;
     title3 'Standard 2-way ANOVA with proc glm';
     class bact temp;
     model rot=temp|bact; /* Could have said   bact temp bact*temp */
     means temp|bact;

/* Need to plot it; SAS is not the tool. */
```

```
/* Now generate the tests for main effects and interaction using cell means
   coding.

               BACTERIA TYPE
TEMP      1         2         3
Cool    mu11      mu12      mu13
Warm    mu21      mu22      mu23                      */


/* The test statement of proc reg uses variable names to stand for the
corresponding regression coefficients. By naming the effect cell mean coding
dummy variables the same as the population cell means, I can just state the
null hypothesis. Isn't this a cute SAS trick? */

proc reg;
     title3 'Using the proc reg test statement and cell means coding';
     model rot = mu11--mu23 / noint;
     Overall:       test mu11=mu12=mu13=mu21=mu22=mu23;
     Temperature:   test mu11+mu12+mu13 = mu21+mu22+mu23;
     Bacteria:      test mu11+mu21 = mu12+mu22 = mu13+mu23;
     Bact_by_Temp1: test mu11-mu21 = mu12-mu22 = mu13-mu23;
     Bact_by_Temp2: test mu12-mu11 = mu22-mu21,
                         mu13-mu12 = mu23-mu22;

/* Bact_by_Temp1 checks equality of temperature effects.
   Bact_by_Temp2 checks parallel line segments. They are equivalent. */

proc glm;
     title3 'Proc glm: Using contrasts on the combination variable';
     class combo;      /* 11 12 13  21 22 23 */
     model rot=combo;
     contrast 'Main Effect for Temperature'
        combo  1  1  1  -1 -1 -1;
     contrast 'Main Effect for Bacteria'
        combo  1 -1  0   1 -1  0,
        combo  0  1 -1   0  1 -1;
     contrast 'Temperature by Bacteria Interaction'
        combo  1 -1  0  -1  1  0,
        combo  0  1 -1   0 -1  1;

/* Illustrate effect coding */

data mashed;
     set spud;
     /* Effect coding, with interactions */
        if bact = 1 then b1 =  1;
           else if bact = 2 then b1 =  0;
           else if bact = 3 then b1 = -1;
        if bact = 1 then b2 =  0;
           else if bact = 2 then b2 =  1;
           else if bact = 3 then b2 = -1;        if temp = 1 then t =  1;
           else if temp = 2 then t = -1;
     /* Interaction terms */
        tb1 = t*b1; tb2 = t*b2;
```

```
proc reg;
     title3 'Effect coding';
     model rot = b1 b2  t  tb1 tb2;
     Temperature:   test t=0;
     Bacteria:      test b1=b2=0;
     Bact_by_Temp:  test tb1=tb2=0;

/* Do some exploration to follow up the interaction. The regression
   with cell means coding is easiest. The final product of several runs
   is shown below. For reference, here is the table of population means again.

               BACTERIA TYPE
TEMP      1         2        3
Cool    mu11      mu12      mu13
Warm    mu21      mu22      mu23                      */

proc reg;
     title3 'Further exploration using cell means coding';
     model rot = mu11--mu23 / noint;
     /* Pairwise comparisons of marginal means for Bacteria Type */
     Bact1vs2: test mu11+mu21=mu12+mu22;
     Bact1vs3: test mu11+mu21=mu13+mu23;
     Bact2vs3: test mu12+mu22=mu13+mu23;
     /* Effect of temperature for each bacteria type */
     Temp_for_Bac1: test mu11=mu21;
     Temp_for_Bac2: test mu12=mu22;
     Temp_for_Bac3: test mu13=mu23;
     /* Effect of bacteria type for each temperature */
     Bact_for_CoolTemp: test mu11=mu12=mu13;
     Bact_for_WarmTemp: test mu21=mu22=mu23;
     /* Pairwise comparisons of bacteria types at warm temperature */
     Bact1vs2_for_WarmTemp: test mu21=mu22;
     Bact1vs3_for_WarmTemp: test mu21=mu23;
     Bact2vs3_for_WarmTemp: test mu22=mu23;

/* We have done a lot of tests. Concerned about buildup of Type I
error? We can make ALL the tests into Scheffe follow-ups of the
initial test for equality of the 6 cell means. The Scheffe family
includes all COLLECTIONS of contrasts, not just all contrasts. */

proc iml;
     title3 'Table of critical values for all possible Scheffe tests';
     numdf = 5;   /* Numerator degrees of freedom for initial test */
     dendf =48;   /* Denominator degrees of freedom for initial test */
     alpha = 0.05;
     critval = finv(1-alpha,numdf,dendf);
     zero = {0 0}; S_table = repeat(zero,numdf,1);  /* Make empty matrix */
     /* Label the columns */
     namz = {"Number of Contrasts in followup test"
             "    Scheffe Critical Value"}; mattrib S_table colname=namz;
     do i = 1 to numdf;
        s_table(|i,1|) = i;
        s_table(|i,2|) = numdf/i *  critval;
     end;
     reset noname; /* Makes output look nicer in this case */
     print "Initial test has"  numdf " and " dendf "degrees of freedom."
           "Using significance level alpha = " alpha;
     print s_table;
```

Rotten potatoes
Two-factor ANOVA several different ways

The MEANS Procedure

Analysis Variable : rot

bact	temp	N Obs	N	Mean	Std Dev	Minimum
1	Cool	9	9	3.5555556	4.2752518	0
	Warm	9	9	7.0000000	3.5355339	0
2	Cool	9	9	4.7777778	3.1135903	0
	Warm	9	9	13.5555556	6.3267510	3.0000000
3	Cool	9	9	8.0000000	4.5552168	2.0000000
	Warm	9	9	19.5555556	5.5251948	8.0000000

Analysis Variable : rot

bact	temp	N Obs	Maximum
1	Cool	9	9.0000000
	Warm	9	10.0000000
2	Cool	9	10.0000000
	Warm	9	23.0000000
3	Cool	9	15.0000000
	Warm	9	26.0000000

Two-factor ANOVA several different ways

	bact 1	bact 2	bact 3	All
	Mean	Mean	Mean	Mean
	rot	rot	rot	rot
temp				
Cool	3.56	4.78	8.00	5.44
Warm	7.00	13.56	19.56	13.37
All	5.28	9.17	13.78	9.41

Two-factor ANOVA several different ways
Standard 2-way ANOVA with proc glm

The GLM Procedure

Class Level Information

Class	Levels	Values
bact	3	1 2 3
temp	2	Cool Warm

Number of Observations Read	54
Number of Observations Used	54

Two-factor ANOVA several different ways
Standard 2-way ANOVA with proc glm

The GLM Procedure

Dependent Variable: rot

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	1652.814815	330.562963	15.05	<.0001
Error	48	1054.222222	21.962963		
Corrected Total	53	2707.037037			

R-Square	Coeff Var	Root MSE	rot Mean
0.610562	49.81676	4.686466	9.407407

Source	DF	Type I SS	Mean Square	F Value	Pr > F
temp	1	848.0740741	848.0740741	38.61	<.0001
bact	2	651.8148148	325.9074074	14.84	<.0001
bact*temp	2	152.9259259	76.4629630	3.48	0.0387

Source	DF	Type III SS	Mean Square	F Value	Pr > F
temp	1	848.0740741	848.0740741	38.61	<.0001
bact	2	651.8148148	325.9074074	14.84	<.0001
bact*temp	2	152.9259259	76.4629630	3.48	0.0387

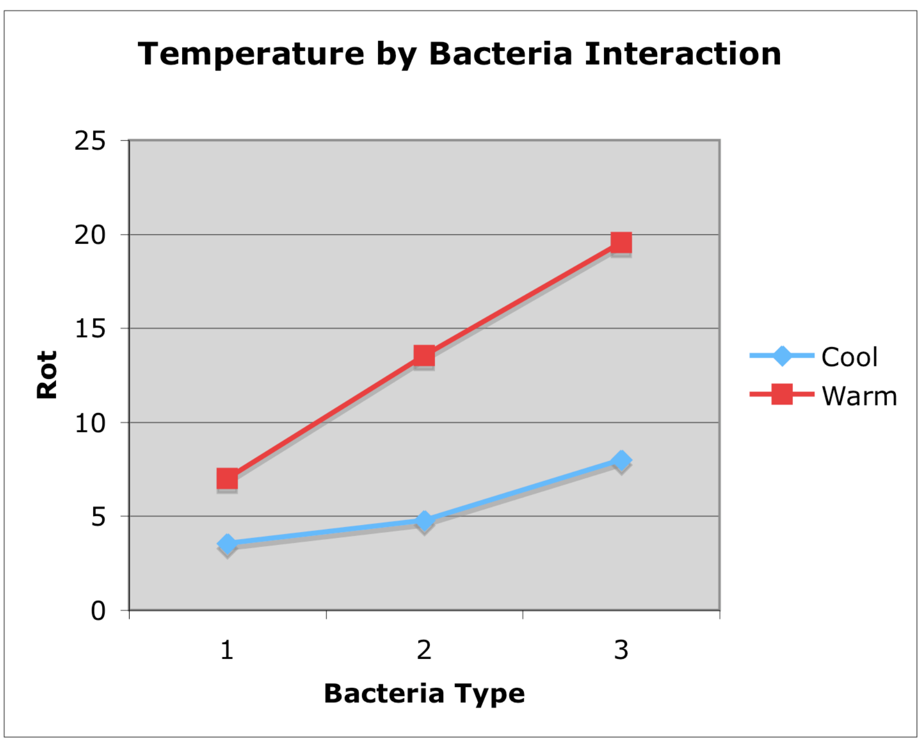
Rotten potatoes
Two-factor ANOVA several different ways
Standard 2-way ANOVA with proc glm

The GLM Procedure

Level of temp	N	rot Mean	rot Std Dev
Cool	27	5.4444444	4.31752541
Warm	27	13.3703704	7.27031979

Level of bact	N	rot Mean	rot Std Dev
1	18	5.2777778	4.19811660
2	18	9.1666667	6.61771242
3	18	13.7777778	7.71214135

Level of bact	Level of temp	N	rot Mean	rot Std Dev
1	Cool	9	3.5555556	4.27525178
1	Warm	9	7.0000000	3.53553391
2	Cool	9	4.7777778	3.11359028
2	Warm	9	13.5555556	6.32675097
3	Cool	9	8.0000000	4.55521679
3	Warm	9	19.5555556	5.52519482



Two-factor ANOVA several different ways
Using the proc reg test statement and cell means coding

The REG Procedure
Model: MODEL1
Dependent Variable: rot

Number of Observations Read	54
Number of Observations Used	54

NOTE: No intercept in model. R-Square is redefined.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	6431.77778	1071.96296	48.81	<.0001
Error	48	1054.22222	21.96296		
Uncorrected Total	54	7486.00000			

Root MSE	4.68647	R-Square	0.8592
Dependent Mean	9.40741	Adj R-Sq	0.8416
Coeff Var	49.81676		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
mu11	1	3.55556	1.56216	2.28	0.0273
mu12	1	4.77778	1.56216	3.06	0.0036
mu13	1	8.00000	1.56216	5.12	<.0001
mu21	1	7.00000	1.56216	4.48	<.0001
mu22	1	13.55556	1.56216	8.68	<.0001
mu23	1	19.55556	1.56216	12.52	<.0001

The REG Procedure
Model: MODEL1

Test Overall Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	5	330.56296	15.05	<.0001
Denominator	48	21.96296		

Test Temperature Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	848.07407	38.61	<.0001
Denominator	48	21.96296		

Test Bacteria Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	2	325.90741	14.84	<.0001
Denominator	48	21.96296		

Test Bact_by_Temp1 Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	2	76.46296	3.48	0.0387
Denominator	48	21.96296		

Test Bact_by_Temp2 Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	2	76.46296	3.48	0.0387
Denominator	48	21.96296		

The GLM Procedure

Class Level Information

Class	Levels	Values
combo	6	11 12 13 21 22 23

Number of Observations Read 54
Number of Observations Used 54

Dependent Variable: rot

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	1652.814815	330.562963	15.05	<.0001
Error	48	1054.222222	21.962963		
Corrected Total	53	2707.037037			

R-Square	Coeff Var	Root MSE	rot Mean
0.610562	49.81676	4.686466	9.407407

Source	DF	Type I SS	Mean Square	F Value	Pr > F
combo	5	1652.814815	330.562963	15.05	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
combo	5	1652.814815	330.562963	15.05	<.0001

Contrast	DF	Contrast SS	Mean Square
Main Effect for Temperature	1	848.0740741	848.0740741
Main Effect for Bacteria	2	651.8148148	325.9074074
Temperature by Bacteria Interaction	2	152.9259259	76.4629630

Contrast	F Value	Pr > F
Main Effect for Temperature	38.61	<.0001
Main Effect for Bacteria	14.84	<.0001
Temperature by Bacteria Interaction	3.48	0.0387

The REG Procedure
Dependent Variable: rot

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	1652.81481	330.56296	15.05	<.0001
Error	48	1054.22222	21.96296		
Corrected Total	53	2707.03704			

Root MSE	4.68647	R-Square	0.6106
Dependent Mean	9.40741	Adj R-Sq	0.5700

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	9.40741	0.63775	14.75	<.0001
b1	1	-4.12963	0.90191	-4.58	<.0001
b2	1	-0.24074	0.90191	-0.27	0.7907
t	1	-3.96296	0.63775	-6.21	<.0001
tb1	1	2.24074	0.90191	2.48	0.0165
tb2	1	-0.42593	0.90191	-0.47	0.6389

Test Temperature Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	848.07407	38.61	<.0001
Denominator	48	21.96296		

Test Bacteria Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	2	325.90741	14.84	<.0001
Denominator	48	21.96296		

Test Bact_by_Temp Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	2	76.46296	3.48	0.0387
Denominator	48	21.96296		

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                       Rotten potatoes                                 18
                Two-factor ANOVA several different ways
              Further exploration using cell means coding
```

Showing only the output from the test statements ...

Test Bact1vs2 Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	136.11111	6.20	0.0163
Denominator	48	21.96296		

Test Bact1vs3 Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	650.25000	**29.61**	<.0001
Denominator	48	21.96296		

Test Bact2vs3 Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	191.36111	8.71	0.0049
Denominator	48	21.96296		

Test Temp_for_Bac1 Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	53.38889	2.43	0.1255
Denominator	48	21.96296		

Test Temp_for_Bac2 Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	346.72222	**15.79**	0.0002
Denominator	48	21.96296		

Test Temp_for_Bac3 Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	600.88889	**27.36**	<.0001
Denominator	48	21.96296		

Test Bact_for_CoolTemp Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	2	47.44444	2.16	0.1264
Denominator	48	21.96296		

Test Bact_for_WarmTemp Results for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	2	354.92593	**16.16**	<.0001
Denominator	48	21.96296		

Test Bact1vs2_for_WarmTemp Results
for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	193.38889	8.81	0.0047
Denominator	48	21.96296		

Test Bact1vs3_for_WarmTemp Results
for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	709.38889	**32.30**	<.0001
Denominator	48	21.96296		

Test Bact2vs3_for_WarmTemp Results
for Dependent Variable rot

Source	DF	Mean Square	F Value	Pr > F
Numerator	1	162.00000	7.38	0.0092
Denominator	48	21.96296		

```
                              Rotten potatoes                            30
                Two-factor ANOVA several different ways
        Table of critical values for all possible Scheffe tests

        Initial test has         5  and         48 degrees of freedom.
                  Using significance level alpha =       0.05


        Number of Contrasts in followup test     Scheffe Critical Value

                                            1                 12.042571
                                            2                 6.0212853
                                            3                 4.0141902
                                            4                 3.0106426
                                            5                 2.4085141
```

First, note that the interaction is not significant as a Scheffé test. For the custom tests, the ones that are significant as Scheffé follow-ups are in boldface. Go back and check.

```
/* Comments:

         Conclusions without Scheffe correction

Version One

"Bacteria types 2 and 3 caused more rot at the warmer temperature than
at the cooler temperature.  At the warmer temperature, bacteria type 3
caused more rot than type 2, and type 2 caused more rot than type 1."

Version Two

"Bacteria types 2 and 3 caused more rot at the warmer temperature than
at the cooler temperature, but clear evidence of a temperature effect
was not present for bacteria type 1. At the warmer temperature,
bacteria type 3 caused more rot than type 2, and type 2 caused more
rot than type 1. But at the cool temperature, there was no convincing
evidence of a bacteria effect."


I like version one more. From now on we won't mention what's NOT significant.

              Conclusions with Scheffe correction

Averaging across bacteria types, there was more rot at the warmer
temperature. In particular, bacteria types 2 and 3 caused more rot at
the warmer temperature than at the cooler temperature.  Averaging
across temperatures, bacteria type 3 caused more rot than bacteria
type 1; this arose from a substantial difference between bacteria
types 1 and 3 at the warmer temperature.
```

Notice that with the Scheffe correction, the interaction was not significant, so I discussed the main effects. SOMETIMES, it still makes sense to discuss main effects in the presence of an interaction. For example, if the temperature effect had been significant for bacteria type 1, one might say something like "In general, there was more rot at the warmer temperature."

Two more comments (in case I forgot to say this):

1. For any factorial ANOVA, saying that all main effects and interactions are zero is the same as saying that all cell means are equal. A simultaneous test of all main effects and interactions is the same as a simple one-way ANOVA on a combination variable whose values are all combinations of the factors. The two tests yield the same value of F the same p-value, everything.

2. As in this data set, it is often helpful to explore an interaction between IV1 and IV2 by testing for the effect of IV1 separately for each value of IV2 (or the other way around). Now, IN THE POPULATION, if an effect of IV1 is present for one value of IV2 and absent for another value of IV2, there is definitely an interaction. However, an effect for IV1 might be present for all values of IV2, and it might even be of the same general form, but larger for one of the IV2 values. In this case there is still an interaction.

Furthermore, there is not necessarily any consistency between tests for interaction and tests for one of the independent variables separately for the values of another. For example, suppose that IV1 and IV2 each have 2 levels, so it is a 2-by-2 design. The difference in the mean of Y between IV1=1 and IV1=2 might be just barely significant when IV2=1, and just barely non-significant (but in the same direction) when IV2=2. In this situation the interaction could easily be non-significant. On the other hand, the differences between means might be just barely non-significant for both IV2=1 and IV2=2, but in opposite directions. In this case, the interaction could well be significant.