

Tables of $E(Y|\mathbf{x})$ for SAS Example Five¹
STA441 Spring 2024

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Just Country

- $c_1 = 1$ if Country = U.S., zero otherwise
- $c_2 = 1$ if Country = Europe, zero otherwise
- $E[Y|\mathbf{X} = \mathbf{x}] = \beta_0 + \beta_1 c_1 + \beta_2 c_2$

| Country | c_1 | c_2 | $E(Y \mathbf{x}) = \beta_0 + \beta_1 c_1 + \beta_2 c_2$ |
|---------------|-------|-------|---|
| United States | 1 | 0 | $\mu_1 = \beta_0 + \beta_1$ |
| Europe | 0 | 1 | $\mu_2 = \beta_0 + \beta_2$ |
| Japan | 0 | 0 | $\mu_3 = \beta_0$ |

With weight and length

- $c_1 = 1$ if Country = U.S., zero otherwise
- $c_2 = 1$ if Country = Europe, zero otherwise
- $x_1 = \text{weight}$
- $x_2 = \text{length}$
- $E[Y|\mathbf{X} = \mathbf{x}] = \beta_0 + \beta_1 c_1 + \beta_2 c_2 + \beta_3 x_1 + \beta_4 x_2$.

| Country | c_1 | c_2 | $E(Y \mathbf{x})$ |
|---------------|-------|-------|---|
| United States | 1 | 0 | $\mu_1 = (\beta_0 + \beta_1) + \beta_3 x_1 + \beta_4 x_2$ |
| Europe | 0 | 1 | $\mu_2 = (\beta_0 + \beta_2) + \beta_3 x_1 + \beta_4 x_2$ |
| Japan | 0 | 0 | $\mu_3 = \beta_0 + \beta_3 x_1 + \beta_4 x_2$ |

Parallel regression planes.

With product terms for interactions

- $c_1 = 1$ if Country = U.S., zero otherwise
- $c_2 = 1$ if Country = Europe, zero otherwise
- $x_1 = \text{weight}$
- $x_2 = \text{length}$
- $E[Y|\mathbf{X} = \mathbf{x}] = \beta_0 + \beta_1 c_1 + \beta_2 c_2 + \beta_3 x_1 + \beta_4 x_2$
 $+ \beta_5 c_1 x_1 + \beta_6 c_2 x_1 + \beta_7 c_1 x_2 + \beta_8 c_2 x_2.$

| Country | c_1 | c_2 | $E(Y \mathbf{x})$ |
|---------------|-------|-------|---|
| United States | 1 | 0 | $\mu_1 = (\beta_0 + \beta_1) + (\beta_3 + \beta_5)x_1 + (\beta_4 + \beta_7)x_2$ |
| Europe | 0 | 1 | $\mu_2 = (\beta_0 + \beta_2) + (\beta_3 + \beta_6)x_1 + (\beta_4 + \beta_8)x_2$ |
| Japan | 0 | 0 | $\mu_3 = \beta_0 + \beta_3 x_1 + \beta_4 x_2$ |

Non-parallel regression planes.

Drop length and use cell means coding

- $c_1 = 1$ if Country = U.S., zero otherwise
- $c_2 = 1$ if Country = Europe, zero otherwise
- $c_3 = 1$ if Country = Japan, zero otherwise
- $x = \text{weight}$
- Bad model: $E(Y|\mathbf{X} = \mathbf{x}) = \beta_1 c_1 + \beta_2 c_2 + \beta_3 c_3 + \beta_4 x + \beta_5 c_1 x + \beta_6 c_2 x + \beta_7 c_3 x.$
- Better model:
 $E(Y|\mathbf{X} = \mathbf{x}) = \beta_1 c_1 + \beta_2 c_2 + \beta_3 c_3 + \beta_4 c_1 x + \beta_5 c_2 x + \beta_6 c_3 x.$

| Country | c_1 | c_2 | c_3 | $E(Y \mathbf{x})$ |
|---------------|-------|-------|-------|-------------------------------|
| United States | 1 | 0 | 0 | $\mu_1 = \beta_1 + \beta_4 x$ |
| Europe | 0 | 1 | 0 | $\mu_2 = \beta_2 + \beta_5 x$ |
| Japan | 0 | 0 | 0 | $\mu_3 = \beta_3 + \beta_6 x$ |

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<http://www.utstat.toronto.edu/brunner/oldclass/441s24>