# Multivariate Analysis

STA441 Spring 2018

## Multiple (quantitative) Response Variables

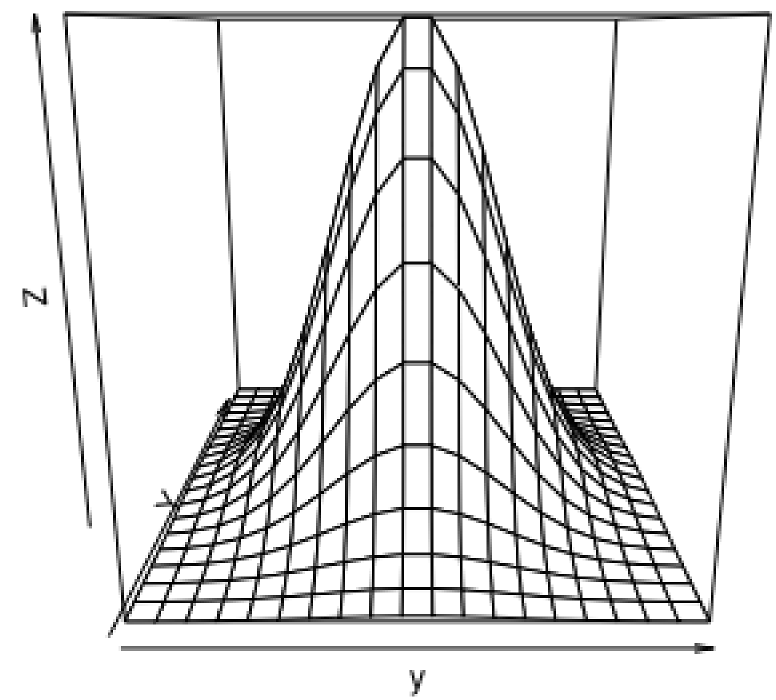
# More than one response variable at once: Why do it?

- Control Type I error rate.
- More powerful than a set of Bonferroni-corrected univariate tests.
- In principle, could detect an effect that is not significant in any of the univariate tests, even without correction.

# Model Assumptions

- There are *k* response variables: $\mathbf{Y}=(Y_1, \ldots Y_k)$
- At each combination of explanatory variable values, there is a conditional distribution of **Y**.
- Each conditional distribution is multivariate normal, with
  - The same variance-covariance matrix
  - A linear regression structure for the set of means

# Multivariate Normal



# Multivariate Normal Parameters

- Vector of means $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3, \mu_4)$

- Variance-covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \sigma_{1,3} & \sigma_{1,4} \\ \sigma_{2,1} & \sigma_2^2 & \sigma_{2,3} & \sigma_{2,4} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_3^2 & \sigma_{3,4} \\ \sigma_{4,1} & \sigma_{4,2} & \sigma_{4,3} & \sigma_4^2 \end{bmatrix}$$

# Multivariate Regression

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_k \end{bmatrix} = \begin{bmatrix} E[Y_1|\mathbf{X}=\mathbf{x}] \\ E[Y_2|\mathbf{X}=\mathbf{x}] \\ \vdots \\ E[Y_k|\mathbf{X}=\mathbf{x}] \end{bmatrix} = \begin{bmatrix} \beta_{0,1} + \beta_{1,1}x_1 + & \cdots & +\beta_{p-1,1}x_{p-1} \\ \beta_{0,2} + \beta_{1,2}x_1 + & \cdots & +\beta_{p-1,2}x_{p-1} \\ \vdots & \vdots & \vdots \\ \beta_{0,k} + \beta_{1,k}x_1 + & \cdots & +\beta_{p-1,k}x_{p-1} \end{bmatrix}$$

- There are *k* regression equations, one for each response variable.
- Second subscript on the betas says which response variable.
- Same explanatory variables in each equation
- Estimate betas by least squares - same as univariate regression.
- Dummy variables, etc.
- Only the significance tests are different.

# Multivariate test statistics

- Wilks' Lambda
- Pillai's Trace
- Hotelling-Lawley Trace
- Roy's Greatest Root

# The four multivariate test statistics

- All control Type I error properly.
- Differ somewhat in power, sometimes, but none is most powerful all the time.
- Distributions under $H_0$ are known
  - Tables of critical values are available.
  - Exact p-values are nasty to compute.
  - There are F *approximations*, sometimes exact.

# I like Wilks' Lambda

- F approximations are best (p-values are more often exactly right).
- Based most directly on the likelihood ratio, so I understand it most easily.
- Scheffé tests are relatively easy to construct.

- So let’s use Wilks' Lambda.

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