Name Jerry

Student Number

STA 431 Quiz 3

1. (5 points) Independently for $i = 1, \ldots, n$, let $y_i = \beta x_i + \epsilon_i$, where $x_i \sim N(\mu_x, \sigma_x^2)$, $\epsilon_i \sim N(0, \sigma_\epsilon^2)$, and x_i and ϵ_i are independent. Let $\widehat{\beta}_n = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$. Is $\widehat{\beta}_n$ a consistent estimator of β? Answer Yes or No and prove it.

$$\hat{\beta}_n = \frac{\frac{1}{n}\sum_{i=1}^n x_i y_i}{\frac{1}{n}\sum_{i=1}^n x_i^2} \xrightarrow{p} \frac{E(x_i y_i)}{E(x_i^2)}$$

By the Law of Large Numbers and continuous mapping

$$E(x_i y_i) = E\{x_i(\beta x_i + \varepsilon_i)\}$$

$$= E\{\beta x_i^2 + x_i \varepsilon_i\} = \beta E(x_i^2) + E(x_i y_i)$$

$$= \beta E(x_i^2) + E(x_i)E(y_i) \text{ by independence}$$

$$= \beta E(x_i^2) + 0 \text{, so}$$

$$\hat{\beta}_n \xrightarrow{p} \frac{E(x_i y_i)}{E(x_i^2)} = \frac{\beta \cancel{E(x_i^2)}}{\cancel{E(x_i^2)}} = \beta,$$

and $\hat{\beta}_n$ is consistent.

(There is more than one good way to do this one.)

2. (5 points) In Question 16 of this week's assignment, you estimated the parameters of the "mystery" distribution by maximum likelihood. In the space below, write the maximum likelihood estimate of μ. The answer is a number from your printout. On your printout, circle the number and write "Question 2" beside it. **Do not answer this question if you do not have a printout.**

$\hat{\mu} = 1.978$ (Number of decimal places does not matter.)

Please turn in your printout, showing your *complete* R input and output, with the quiz paper. Make sure your name and student number appear on the printout.

R work for Question 16

```
> # MLE for Mystery Distribution
> rm(list=ls()); options(scipen=999)
> mystery = scan("https://www.utstat.toronto.edu/brunner/openSEM/data/mystery2.data.txt")
Read 200 items
> # Mystery minus log likelihood function
> mmll = function(param,x)
+     {
+     theta = param[1]; mu = param[2]
+     n = length(x); xbar = mean(x)
+     value = 2 * sum(log(1 + exp(theta*(x-mu)))) -
+             n*log(theta) - n*theta*(xbar-mu)
+     return(value)
+ } # End of function mmll
>
> msearch = optim(par=c(1,0), fn = mmll,
+                 method = "L-BFGS-B", lower = c(0,-Inf), hessian=TRUE, x=mystery)
> msearch

$par
[1] 2.849444 1.978644

$value
[1] 192.4589

$counts
function gradient
      14       14

$convergence
[1] 0

$message
[1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"

$hessian
           [,1]        [,2]
[1,] 34.5927653   0.6156322
[2,]  0.6156322 549.5388601

> # (a) MLE
> thetahat = msearch$par[1]; muhat = msearch$par[2]
> c(thetahat,muhat)
[1] 2.849444 1.978644   Question 2

> # (b) Confidence interval for theta
> Vhat = solve(msearch$hessian)
> se = sqrt(diag(Vhat))
> se_thetahat = se[1]; se_muhat = se[2]
> low95 = thetahat - 1.96*se_thetahat
> upr95 = thetahat + 1.96*se_thetahat
> c(low95,upr95)
[1] 2.516196 3.182693
>
> # (c) Test H0: mu = 2.1. Reject if |z| > 1.96
> z = (muhat-2.1)/se_muhat; z
[1] -2.844819
```