

STA 431 Formulas

$$Pr\{Y \in A\} = \sum_x Pr\{Y \in A|X = x\}p_x(x) \quad Pr\{Y \in A\} = \int_{-\infty}^{\infty} Pr\{X \in A|X = x\}f_x(x) dx$$

$$Var(X) \stackrel{def}{=} E((X - \mu_x)^2)$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$Cov(X, Y) \stackrel{def}{=} E((X - \mu_x)(Y - \mu_y))$$

$$Cov(X, Y) = E(XY) - E(X)E(Y)$$

$$Cov(aX, bY) = abCov(X, Y)$$

$$Cov(x + a, Y + b) = Cov(X, Y)$$

$$Corr(X, Y) \stackrel{def}{=} \frac{Cov(X, Y)}{\sqrt{Var(X)Var(Y)}}$$

$$Cov\left(\sum_{i=1}^n a_i X_i, \sum_{j=1}^m b_j Y_j\right) = \sum_{i=1}^n \sum_{j=1}^m a_i b_j Cov(X_i, Y_j)$$

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$$

$$\mathbf{A} = \mathbf{C}\mathbf{D}\mathbf{C}^\top \text{ with } \mathbf{C}\mathbf{C}^\top = \mathbf{C}^\top\mathbf{C} = \mathbf{I}, \text{ for } \mathbf{A} \text{ symmetric.}$$

$$\mathbf{A}^{-1} = \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^\top$$

$$\mathbf{A}^{1/2} = \mathbf{C}\mathbf{D}^{1/2}\mathbf{C}^\top$$

$$\mathbf{A}^{-1/2} = \mathbf{C}\mathbf{D}^{-1/2}\mathbf{C}^\top$$

\mathbf{A} positive definite means $\mathbf{v}'\mathbf{A}\mathbf{v} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$$cov(\mathbf{x}) \stackrel{def}{=} E\{(\mathbf{x} - \boldsymbol{\mu}_x)(\mathbf{x} - \boldsymbol{\mu}_x)^\top\}$$

$$cov(\mathbf{x}, \mathbf{y}) \stackrel{def}{=} E\{(\mathbf{x} - \boldsymbol{\mu}_x)(\mathbf{y} - \boldsymbol{\mu}_y)^\top\}$$

$$cov(\mathbf{A}\mathbf{x}) = \mathbf{A} cov(\mathbf{x}) \mathbf{A}^\top$$

$$cov(\mathbf{A}\mathbf{x}, \mathbf{B}\mathbf{y}) = \mathbf{A} cov(\mathbf{x}, \mathbf{y}) \mathbf{B}^\top$$

$$\mathbf{L} = \mathbf{A}_1\mathbf{X}_1 + \cdots + \mathbf{A}_m\mathbf{X}_m + \mathbf{b}$$

$$cov(\mathbf{L}_1, \mathbf{L}_2) = \sum_{i=1}^m \sum_{j=1}^n \mathbf{A}_i cov(\mathbf{x}_i, \mathbf{y}_j) \mathbf{B}_j^\top$$

If $\mathbf{x} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $\mathbf{A}\mathbf{x} + \mathbf{b} \sim N_r(\mathbf{A}\boldsymbol{\mu} + \mathbf{b}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$ and $(\mathbf{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) \sim \chi^2(p)$

$$L(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2} (2\pi)^{-np/2} \exp -\frac{n}{2} \left\{ tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\bar{\mathbf{x}} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\bar{\mathbf{x}} - \boldsymbol{\mu}) \right\}$$

$$\widehat{\boldsymbol{\mu}} = \bar{\mathbf{x}}, \quad \widehat{\boldsymbol{\Sigma}} = \frac{1}{n} \sum_{i=1}^n (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^\top \quad \widehat{\sigma}_x^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2, \quad \widehat{\sigma}_{xy} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

If $\mathbf{t}_n \xrightarrow{p} \mathbf{c}$ and $g(\mathbf{x})$ is continuous at $\mathbf{x} = \mathbf{c}$, then $g(\mathbf{t}_n) \xrightarrow{p} g(\mathbf{c})$.

If $\mathbf{x}_1, \dots, \mathbf{x}_n \stackrel{iid}{\sim} (\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $\bar{\mathbf{x}}_n \xrightarrow{p} \boldsymbol{\mu}$ and $\bar{\mathbf{x}}_n \dot{\sim} N_p(\boldsymbol{\mu}, \frac{1}{n}\boldsymbol{\Sigma})$

$\widehat{\boldsymbol{\theta}}_n \xrightarrow{p} \boldsymbol{\theta}$ and $\widehat{\boldsymbol{\theta}}_n \dot{\sim} N_m(\boldsymbol{\theta}, \mathbf{V}_n)$, with $\widehat{\mathbf{V}}_n = \mathbf{H}^{-1}$, where $\mathbf{H} = \left[\frac{\partial^2(-\ell)}{\partial \theta_i \partial \theta_j} \right]_{\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}}$

$$G^2 = -2 \ln \left(\frac{\max_{\boldsymbol{\theta} \in \Theta_0} L(\boldsymbol{\theta})}{\max_{\boldsymbol{\theta} \in \Theta} L(\boldsymbol{\theta})} \right) = -2 \ln \left(\frac{L(\widehat{\boldsymbol{\theta}}_0)}{L(\widehat{\boldsymbol{\theta}})} \right) \sim \chi^2(r) \text{ if } H_0 : \boldsymbol{\theta} \in \Theta_0 \text{ is true.}$$

$$W_n = (\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h})^\top \left(\mathbf{L}\widehat{\mathbf{V}}_n\mathbf{L}^\top \right)^{-1} (\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h}) \dot{\sim} \chi^2(r) \text{ if } H_0 : \mathbf{L}\boldsymbol{\theta} = \mathbf{h} \text{ is true.}$$

The reliability of an observable variable d as a measurement of a latent variable F is $\rho^2 \stackrel{def}{=} (Corr(F, d))^2$.

If $d = F + e$ with $Var(F) = \phi$ and $Var(e) = \omega$, then $\rho^2 = \frac{\phi}{\phi + \omega}$.

For symmetric $\boldsymbol{\Sigma}_{k \times k}$, there are $k(k+1)/2$ unique elements and $k(k-1)/2$ unique off-diagonal elements.

The Double Measurement Model in centered form:

$$\begin{aligned}
 \mathbf{y}_i &= \beta \mathbf{x}_i + \boldsymbol{\epsilon}_i & \text{cov}(\mathbf{x}_i) &= \boldsymbol{\Phi}_x, \text{cov}(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi} \\
 \mathbf{F}_i &= \begin{pmatrix} \mathbf{x}_i \\ \mathbf{y}_i \end{pmatrix} & \mathbf{x}_i \text{ is } p \times 1, \mathbf{y}_i \text{ is } q \times 1, \mathbf{F}_i \text{ is } (p+q) \times 1 \\
 & & \text{cov}(\mathbf{F}_i) &= \boldsymbol{\Phi} \\
 \mathbf{d}_{i,1} &= \mathbf{F}_i + \mathbf{e}_{i,1} & \text{cov}(\mathbf{e}_{i,1}) &= \boldsymbol{\Omega}_1, \text{cov}(\mathbf{e}_{i,2}) = \boldsymbol{\Omega}_2 \\
 \mathbf{d}_{i,2} &= \mathbf{F}_i + \mathbf{e}_{i,2} & \mathbf{x}_i, \boldsymbol{\epsilon}_i, \mathbf{e}_{i,1} \text{ and } \mathbf{e}_{i,2} & \text{ are independent.}
 \end{aligned}$$

The General Structural Equation Model in centered form:

$$\begin{aligned}
 \mathbf{y}_i &= \beta \mathbf{y}_i + \boldsymbol{\Gamma} \mathbf{x}_i + \boldsymbol{\epsilon}_i & \text{cov}(\mathbf{x}_i) &= \boldsymbol{\Phi}_x \text{ and } \text{cov}(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi} \\
 \mathbf{F}_i &= \begin{pmatrix} \mathbf{x}_i \\ \mathbf{y}_i \end{pmatrix} & \text{cov}(\mathbf{F}_i) &= \boldsymbol{\Phi} = \begin{pmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \boldsymbol{\Phi}_{12}^\top & \boldsymbol{\Phi}_{22} \end{pmatrix} \\
 \mathbf{d}_i &= \boldsymbol{\Lambda} \mathbf{F}_i + \mathbf{e}_i & \text{cov}(\mathbf{e}_i) &= \boldsymbol{\Omega} \\
 \mathbf{x}_i, \boldsymbol{\epsilon}_i \text{ and } \mathbf{e}_i & \text{ are independent.} & \mathbf{x}_i \text{ is } p \times 1, \mathbf{y}_i \text{ is } q \times 1, \mathbf{d}_i \text{ is } k \times 1.
 \end{aligned}$$

$\boldsymbol{\Phi}_x$ and $\boldsymbol{\Psi}$ are positive definite.

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> df = 1:8; CriticalValue = qchisq(0.95,df)
> round(rbind(df,CriticalValue),3)
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	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]
df	1.000	2.000	3.000	4.000	5.00	6.000	7.000	8.000
CriticalValue	3.841	5.991	7.815	9.488	11.07	12.592	14.067	15.507

This formula sheet was prepared by [Jerry Brunner](#), Department of Statistics, University of Toronto. It is licensed under a [Creative Commons Attribution - ShareAlike 3.0 Unported License](#). Use any part of it as you like and share the result freely. The \LaTeX source code is available from the course website:

<http://www.utstat.toronto.edu/brunner/oldclass/431s23>