Name Jerry

Student Number _____

STA 431S 2017 Quiz 7

1. In the measurement part of the double measurement model,

$$\begin{aligned} \mathbf{D}_{i,1} &= \mathbf{F}_i + \mathbf{e}_{i,1} \\ \mathbf{D}_{i,2} &= \mathbf{F}_i + \mathbf{e}_{i,2}, \end{aligned}$$

where $\mathbf{F}_i \sim N_k(\mathbf{0}, \mathbf{\Phi})$, $\mathbf{e}_{i,1} \sim N_k(\mathbf{0}, \mathbf{\Omega}_1)$ and $\mathbf{e}_{i,2} \sim N_k(\mathbf{0}, \mathbf{\Omega}_2)$. The random vectors $\mathbf{F}_i$, $\mathbf{e}_{i,1}$, and $\mathbf{e}_{i,2}$ are independent.

(a) (2 points) Calculate the covariance matrix of the observable data $\mathbf{\Sigma} = cov\begin{pmatrix} \mathbf{D}_{i,1} \\ \mathbf{D}_{i,2} \end{pmatrix}$ as a function of the model parameters. Write it as a partitioned matrix.

$cov(D_{i1}, D_{i2}) = E\{D_{i1} D_{i2}^T\} = E\{(F_i + e_{i1})(F_i + e_{i2})^T\}$

$= E\{F_i F_i^T\} + 0 + 0 + 0 = \Phi$, so

$$\Sigma = \left(\begin{array}{c|c} \Phi + \Omega_1 & \Phi \\ \hline \Phi & \Phi + \Omega_2 \end{array}\right)$$

(b) (2 points) How many equality constraints on the *unique* elements of $\mathbf{\Sigma}$ are implied by the model? ~~Show your work or justify your answer~~. Your answer is an expression in k. Remember that of course $\mathbf{\Sigma}$ is symmetric; you are being asked to count the contraints on its *unique* elements. Circle your answer

Not required: Φ is $k \times k$ symmetric so off-diagonal elements are equal and there are

$\frac{k(k-1)}{2}$ constraints

(c) (2 points) Suppose there are $k = 3$ latent variables, so that $\mathbf{F}_i$ is 3×1. Then

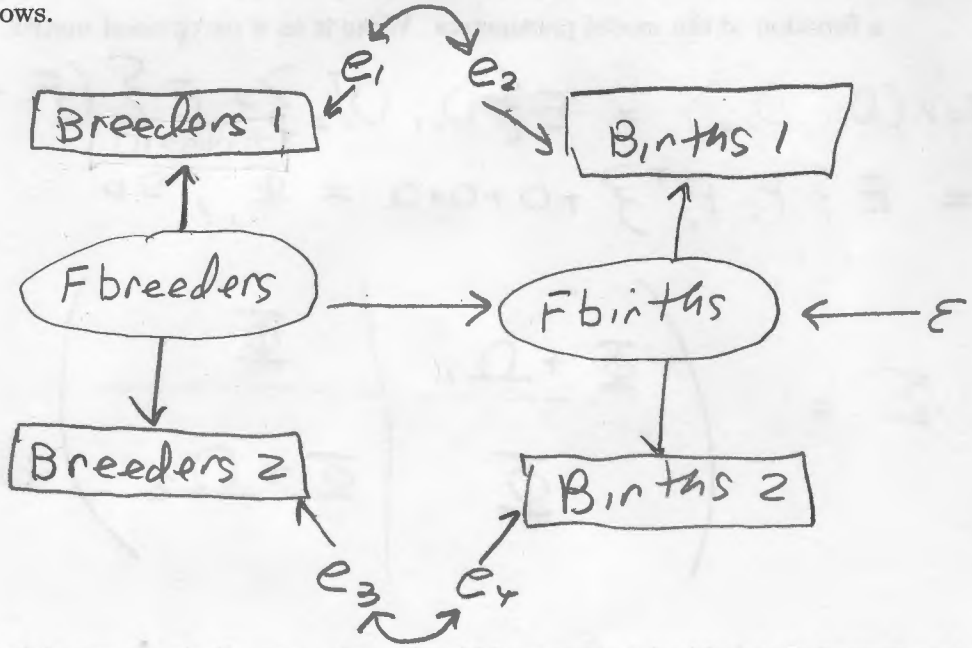
$$\boldsymbol{\Sigma} = \left(\begin{array}{ccc|ccc} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} & \sigma_{16} \\ & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} & \sigma_{26} \\ & & \sigma_{33} & \sigma_{34} & \sigma_{35} & \sigma_{36} \\ \hline & & & \sigma_{44} & \sigma_{45} & \sigma_{46} \\ & & & & \sigma_{55} & \sigma_{56} \\ & & & & & \sigma_{66} \end{array}\right)$$

What are the equality constraints? Your answer is a set of equations involving σ_{ij} symbols.

$\sigma_{15} = \sigma_{24}$, $\sigma_{16} = \sigma_{34}$, $\sigma_{26} = \sigma_{35}$

2. In the SAS part of the assignment, you analyzed the Pig Birth data.

(a) (2 points) Make a path diagram of the model. There is no need to write coefficients on the arrows.



(b) (2 points) In your path diagram, there should be a curved double-headed arrow connecting the measurement error terms for Questionnaire One. To get marks on this question, your answers to both questions below must be correct.

i. What is the value of the test statistic for testing whether the arrow is present? The answer is a number on your printout. Write the number in the space below. On your printout, circle the number and write "Question 2" beside it.

$t = 5.3894$ or $X^2 = 29.04613$

ii. Using the usual $\alpha = 0.05$ significance level, is there evidence that the error terms for Questionnaire One are correlated? Answer Yes or No.

Yes

Please attach *both* your log file and your results file. Make sure your name appears on both documents. Attach the *log file*, not just a listing of the SAS program.