Name Jerry

Student Number ____________

STA 431s2017 Quiz 6

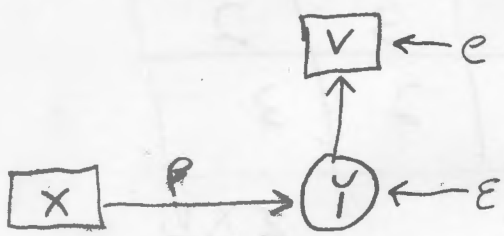
Independently for $i = 1, \ldots, n$, let

$$
\begin{aligned}
Y_i &= \beta X_i + \epsilon_i \\
V_i &= Y_i + e_i
\end{aligned}
$$

where $X_i \sim N(\mu_x, \phi)$ with $\phi > 0$, $\epsilon_i \sim N(0, \psi)$ with $\psi > 0$, $e_i \sim N(0, \omega)$ with $\omega > 0$, and X_i, ϵ_i and e_i are independent of one another. Y_i is a latent variable, while V_i and X_i are observable variables.

maybe 2

1. (1 point) Make a path diagram of this model.



2. (1 point) What is the parameter vector $\boldsymbol{\theta}$ for this model? Don't forget the expected values.

$\theta = (\beta, \mu_x, \phi, \psi, \omega)$

3. (2 points) Give the expected value and variance-covariance matrix of the observable variables as a function of the model parameters.

$$E\begin{pmatrix} X_i \\ V_i \end{pmatrix} = \begin{pmatrix} \mu_x \\ \beta\mu_x \end{pmatrix}, \quad cov\begin{pmatrix} X_i \\ V_i \end{pmatrix} = \begin{pmatrix} \phi & \beta\phi \\ \beta\phi & \beta^2\phi + \psi + \omega \end{pmatrix}$$

$$E(\overset{c}{X}\overset{c}{V}) = E(\overset{c}{X}_i \beta \overset{c}{X}_i) = \beta\phi$$

4. (1 point) Does this problem pass the test of the Parameter Count Rule? Answer Yes or No and give the numbers.

Yes, 5 parameters and 5 moment structure equations

5. **(3 points) Is the parameter vector identifiable at every point in the parameter space? Answer Yes or No and prove your answer.**

No. For any value of β, μ_x and ϕ, the infinitely many (ψ, ω) pairs with $\psi + \omega = c > 0$ will all produce the same $E(D_i)$ & $cov(D_i)$ and hence the same distribution of the observable data.

<u>or, another answer</u>

No. Here are two distinct parameter vectors that produce the same distribution of the observable data:

	β	μ_x	ϕ	ψ	ω
θ_1	1	1	1	3	2
θ_2	1	1	1	2	3

6. (2 points) The naive estimator of β is $\widehat{\beta}_n = \cancel{\frac{\sum_{i=1}^n W_iY_i}{\sum_{i=1}^n W_i^2}}$ $\frac{\sum X_iV_i}{\sum X_i^2}$. Is $\widehat{\beta}_n$ a consistent estimator of β? Answer Yes or No and prove your answer.

By LLN and continuous mapping

$$\hat{\beta}_n = \frac{\frac{1}{n}\sum_{i=1}^n X_iV_i}{\frac{1}{n}\sum_{i=1}^n X_i^2} \xrightarrow{p} \frac{E(X_iV_i)}{E(X_i^2)}$$

$$= \frac{E(X_i(\beta X_i + \epsilon_i + e_i))}{E(X_i^2)} = \frac{\beta E(X_i^2) + 0 + 0}{E(X_i^2)}$$

$$= \beta$$

Yes