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Student Number

STA 431s2017 Quiz 5

1. (7 points) Let $\mathbf{D}_1, \ldots, \mathbf{D}_n$ be a random sample from a p-dimensional multivariate normal population with mean $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$. We want to test $H_0 : \boldsymbol{\Sigma} = \mathbf{I}$.

   (a) Using the formula sheet, write down and simplify $L(\widehat{\theta})$. ~~Circle your answer~~.

$$L(\bar{D}, \hat{\Sigma}) = |\hat{\Sigma}|^{-\frac{n}{2}} (2\pi)^{-\frac{np}{2}} \exp -\frac{n}{2}\{ tr(\hat{\Sigma}\hat{\Sigma}^{-1}) + 0 \}$$
$$= |\hat{\Sigma}|^{-\frac{n}{2}} (2\pi)^{-\frac{np}{2}} e^{-np/2}$$

   (b) Using the formula sheet, give $L(\widehat{\theta}_0)$. ~~Circle your answer~~.

$$L(\bar{D}, I) = 1 \cdot (2\pi)^{-\frac{np}{2}} \exp -\frac{n}{2}\{ tr(\hat{\Sigma}) + 0 \}$$
$$= (2\pi)^{-\frac{np}{2}} \exp -\frac{n}{2} tr(\hat{\Sigma})$$

   (c) Give a formula for the large-sample likelihood ratio test statistic. For full marks, simplify. **Circle your final answer.**

$$G^2 = -2\ln \left(\frac{\cancel{(2\pi)^{-\frac{np}{2}}} \exp -\frac{n}{2} tr(\hat{\Sigma})}{|\hat{\Sigma}|^{-n/2} \cancel{(2\pi)^{-np/2}} e^{-np/2}} \right)$$
$$= -2\left(-\frac{n}{2} tr(\hat{\Sigma}) - \left(-\frac{n}{2} \ln|\hat{\Sigma}| - \frac{np}{2} \right) \right)$$
$$= n\, tr(\hat{\Sigma}) - n \ln|\hat{\Sigma}| - np$$
$$= n\left(tr(\hat{\Sigma}) - \ln|\hat{\Sigma}| - p \right)$$

$$G^2 = n\left(tr(\hat{\Sigma}) - \ln|\hat{\Sigma}| - p\right)$$

(d) Suppose the data vectors have length $p = 3$. A sample of size $n = 200$ yields $tr(\widehat{\Sigma}) = 2.91$ and $\ln|\widehat{\Sigma}| = -0.134$. Calculate the test statistic. The answer is a number. **Circle the number.**

$$G^2 = 200(2.91 - -0.134 - 3) = 200\,(0.044)$$

$$= 8.8$$

at $\alpha = 0.05$

(e) What is the critical value of the test statistic? The answer is a number.

12.59

(f) Do you reject the null hypothesis? Answer Yes or No.

No

(g) Are you able to conclude that Σ is different from the identity matrix? Answer Yes or No.

No

2. (3 points) Let $W = X + e$, where $E(X) = \mu_x$, $E(e) = 0$, $Var(X) = \sigma_x^2$, $Var(e) = \sigma_e^2$, and $Cov(X, e) = 0$. As the notation suggests, X is a latent variable and W is observable. Prove the expression for the reliability of W on the formula sheet.

$$\left(Corr(W,X)\right)^2 = \left(\frac{Cov(W,X)}{\sqrt{\sigma_x^2+\sigma_e^2}\,\sqrt{\sigma_x^2}}\right)^2 = \frac{\left(E(\overset{c}{X}(\overset{c}{X}+e))\right)^2}{(\sigma_x^2+\sigma_e^2)\,\sigma_x^2}$$

$$= \frac{(\sigma_x^2)^2}{(\sigma_x^2+\sigma_e^2)\,\sigma_x^2} = \frac{\sigma_x^2}{\sigma_x^2+\sigma_e^2}$$