Name Jerry

Student Number

STA 431s2017 Quiz 3

The following model looks similar to one of the homework problems, but it is not identical. **Please be alert**. Independently for $i = 1, \ldots, n$, let

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i,$$

where $X_i \sim N(\mu_x, \sigma_x^2)$, $\epsilon_i \sim N(0, \sigma_\epsilon^2)$, and ϵ_i is independent of X_i.

1. (2 points) What is the parameter space for this model?

$\Theta = \{(\beta_0, \beta_1, \mu_x, \sigma_x^2, \sigma_\epsilon^2): -\infty < \beta_0 < \infty, -\infty < \beta_1 < \infty, -\infty < \mu_x < \infty, \sigma_x^2 > 0, \sigma_\epsilon^2 > 0\}$

2. (1 point) Calculate $E(X_i^2)$. Show your work and **Circle your answer**.

$\sigma_x^2 = E(X_i^2) - \mu_x^2$, so $E(X_i^2) = \sigma_x^2 + \mu_x^2$

3. (1 point) Calculate $E(X_iY_i)$. Show your work and **Circle your answer**.

$E(X_iY_i) = E\left(X_i(\beta_0 + \beta_1 X_i + \epsilon_i)\right)$

$= E\left(\beta_0 X_i + \beta_1 X_i^2 + X_i\epsilon_i\right)$

$= \beta_0 E(X_i) + \beta_1 E(X_i^2) + E(X_i)E(\epsilon_i)$

$= \beta_0 \mu_x + \beta_1(\sigma_x^2 + \mu_x^2)$

4. (6 points) Let $\widehat{\beta}_1 = \frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2}$. Is $\widehat{\beta}_1$ a consistent estimator of β_1? Answer Yes or No and prove your answer.

By the Law of Large Numbers and continuous mapping (mention both),

$$\hat{\beta}_1 = \frac{\frac{1}{n}\sum_{i=1}^{n} X_i Y_i}{\frac{1}{n}\sum_{i=1}^{n} X_i^2} \xrightarrow{p} \frac{E(X_i Y_i)}{E(X_i^2)}$$

$$= \frac{\beta_0 \mu_x + \beta_1 (\sigma_x^2 + \mu_x^2)}{\sigma_x^2 + \mu_x^2}$$

$$= \frac{\beta_0 \mu_x}{\sigma_x^2 + \mu_x^2} + \beta_1 \neq \beta_1$$

(At least not in the whole parameter space, but they need not say this)

So the answer is No, not consistent.