

Background for the Personality and Job
Satisfaction Example¹
STA431 Winter/Spring 2017

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Multivariate Normal Likelihood $L(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

$$\begin{aligned} & |\boldsymbol{\Sigma}|^{-n/2} (2\pi)^{-np/2} \exp -\frac{n}{2} \left\{ \text{tr}(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\bar{\mathbf{d}} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1} (\bar{\mathbf{d}} - \boldsymbol{\mu}) \right\} \\ = & |\boldsymbol{\Sigma}|^{-n/2} (2\pi)^{-np/2} \exp -\frac{n}{2} \left\{ \text{tr}(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) \right\} \end{aligned}$$

- All you need is $\widehat{\boldsymbol{\Sigma}}$ and more rarely, $\bar{\mathbf{d}}$.
- You don't need the raw data.
- $(\bar{\mathbf{d}}, \widehat{\boldsymbol{\Sigma}})$ are *sufficient statistics* for $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.
- Software should be able to fit a model based only on $\widehat{\boldsymbol{\Sigma}}$ and n .

Auxiliary Information

- Auxiliary means extra, supporting.
- Mostly used to identify parameters of the measurement model.
- For example, there might be double or gold standard measurement on just a subset of the sample.
- In a multi-group analysis, the groups can have overlapping sets of variables and overlapping sets of parameters.
- Or you can sometimes use published information from other studies.
- Like (estimated) reliabilities.

Typical Example

$$D_1 = F_1 + e_1$$

$$D_2 = F_2 + e_2$$

$$D_3 = F_3 + e_3$$

$$\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{12} & \phi_{22} & \phi_{23} \\ \phi_{13} & \phi_{23} & \phi_{33} \end{pmatrix} \quad \Sigma = \begin{pmatrix} \phi_{11} + \omega_1 & \phi_{12} & \phi_{13} \\ \phi_{12} & \phi_{22} + \omega_2 & \phi_{23} \\ \phi_{13} & \phi_{23} & \phi_{33} + \omega_3 \end{pmatrix}$$

What if you knew the reliabilities?

$$\rho_1^2 = \frac{\phi_{11}}{\phi_{11} + \omega_1} \quad \rho_2^2 = \frac{\phi_{22}}{\phi_{22} + \omega_2} \quad \rho_3^2 = \frac{\phi_{33}}{\phi_{33} + \omega_3}$$

Solve for Φ : Compute $\phi_{jj} = \sigma_{jj}\rho_j^2$

$$\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{12} & \phi_{22} & \phi_{23} \\ \phi_{13} & \phi_{23} & \phi_{33} \end{pmatrix} \quad \Sigma = \begin{pmatrix} \phi_{11} + \omega_1 & \phi_{12} & \phi_{13} \\ \phi_{12} & \phi_{22} + \omega_2 & \phi_{23} \\ \phi_{13} & \phi_{23} & \phi_{33} + \omega_3 \end{pmatrix}$$

$$\rho_1^2 = \frac{\phi_{11}}{\phi_{11} + \omega_1} \quad \rho_2^2 = \frac{\phi_{22}}{\phi_{22} + \omega_2} \quad \rho_3^2 = \frac{\phi_{33}}{\phi_{33} + \omega_3}$$

- Do it using the estimates: $\hat{\phi}_{jj} = \hat{\sigma}_{jj}\hat{\rho}_j^2$,
- Converting $\hat{\Sigma}$ to $\hat{\Phi}$.
- Operate on $\hat{\Phi}$ as if it's a $\hat{\Sigma}$ and the variables are measured without error.

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<http://www.utstat.toronto.edu/~brunner/oldclass/431s17>