Background for the Personality and Job Satisfaction Example[1]

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[1] See last slide for copyright information.

Multivariate Normal Likelihood $L(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

$$
\begin{aligned}
& |\boldsymbol{\Sigma}|^{-n/2}(2\pi)^{-np/2} \exp -\frac{n}{2}\left\{tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\overline{\mathbf{d}}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\overline{\mathbf{d}}-\boldsymbol{\mu})\right\} \\
= \; & |\boldsymbol{\Sigma}|^{-n/2}(2\pi)^{-np/2} \exp -\frac{n}{2}\left\{tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1})\right\}
\end{aligned}
$$

- All you need is $\widehat{\boldsymbol{\Sigma}}$ and more rarely, $\overline{\mathbf{d}}$.
- You don't need the raw data.
- $(\overline{\mathbf{d}}, \widehat{\boldsymbol{\Sigma}})$ are *sufficient statistics* for $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.
- Software should be able to fit a model based only on $\widehat{\boldsymbol{\Sigma}}$ and n.

Auxiliary Information

- Auxiliary means extra, supporting.
- Mostly used to identify parameters of the measurement model.
- For example, there might be double or gold standard measurement on just a subset of the sample.
- In a multi-group analysis, the groups can have overlapping sets of variables and overlapping sets of parameters.
- Or you can sometimes use published information from other studies.
- Like (estimated) reliabilities.

Typical Example

$$\begin{aligned} D_1 &= F_1 + e_1 \\ D_2 &= F_2 + e_2 \\ D_3 &= F_3 + e_3 \end{aligned}$$

$$\mathbf{\Phi} = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{12} & \phi_{22} & \phi_{23} \\ \phi_{13} & \phi_{23} & \phi_{33} \end{pmatrix} \qquad \mathbf{\Sigma} = \begin{pmatrix} \phi_{11}+\omega_1 & \phi_{12} & \phi_{13} \\ \phi_{12} & \phi_{22}+\omega_2 & \phi_{23} \\ \phi_{13} & \phi_{23} & \phi_{33}+\omega_3 \end{pmatrix}$$

What if you knew the reliabilities?

$$\rho_1^2 = \frac{\phi_{11}}{\phi_{11}+\omega_1} \qquad \rho_2^2 = \frac{\phi_{22}}{\phi_{22}+\omega_2} \qquad \rho_3^2 = \frac{\phi_{33}}{\phi_{33}+\omega_3}$$

Solve for $\mathbf{\Phi}$: Compute $\phi_{jj} = \sigma_{jj}\rho_j^2$

$$\mathbf{\Phi} = \left(\begin{array}{ccc} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{12} & \phi_{22} & \phi_{23} \\ \phi_{13} & \phi_{23} & \phi_{33} \end{array} \right) \qquad \mathbf{\Sigma} = \left(\begin{array}{ccc} \phi_{11}+\omega_1 & \phi_{12} & \phi_{13} \\ \phi_{12} & \phi_{22}+\omega_2 & \phi_{23} \\ \phi_{13} & \phi_{23} & \phi_{33}+\omega_3 \end{array} \right)$$

$$\rho_1^2 = \frac{\phi_{11}}{\phi_{11}+\omega_1} \quad \rho_2^2 = \frac{\phi_{22}}{\phi_{22}+\omega_2} \quad \rho_3^2 = \frac{\phi_{33}}{\phi_{33}+\omega_3}$$

- Do it using the estimates: $\widehat{\phi}_{jj} = \widehat{\sigma}_{jj}\widehat{\rho}_j^2$,
- Converting $\widehat{\mathbf{\Sigma}}$ to $\widehat{\mathbf{\Phi}}$.
- Operate on $\widehat{\mathbf{\Phi}}$ as if it's a $\widehat{\mathbf{\Sigma}}$ and the variables are measured without error.

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