SAS Example 3: Deliberately create numerical problems

$$\begin{aligned} Y &= \beta_1 X_1 + \beta_2 X_2 + \epsilon \\ V &= Y + e \end{aligned}$$

$$\boldsymbol{\theta} = (\beta_1, \beta_2, \phi_{11}, \phi_{12}, \phi_{22}, \psi, \omega)$$

	X_1	X_2	V
X_1	ϕ_{11}	ϕ_{12}	$\beta_1\phi_{11} + \beta_2\phi_{12}$
X_2		ϕ_{22}	$\beta_1\phi_{12} + \beta_2\phi_{22}$
V			$\beta_1^2\phi_{11} + \beta_2^2\phi_{22} + 2\beta_1\beta_2\phi_{12} + (\psi + \omega)$

Four experiments

1. Try to fit this model, failing the parameter count rule.

2. Set ϕ_{12}=0 to pass the parameter count rule, but still not identifiable.

3. Set β_1=β_2 instead -- still not identifiable.

4. Set β_1=β_2, but do not force ω>0.

```
/* calculus3.sas */
options linesize=79 pagesize=500 noovp formdlim='_' nodate;
title 'Calculus 3: Deliberately cause trouble with identifiability';
title2 'By adding measurement error to response variable';

data math;
     infile 'calculus.data'  firstobs=2;
     input id hscalc hsengl hsgpa test grade;

/* Exclude some output you really don't want to see.
   The (persist) option means keep doing it.      */
ods exclude
     Calis.ModelSpec.LINEQSEqInit                 (persist)
     Calis.ModelSpec.LINEQSVarExogInit            (persist)
     Calis.ModelSpec.LINEQSCovExogInit            (persist)
     Calis.ML.SqMultCorr                          (persist)
     Calis.StandardizedResults.LINEQSEqStd        (persist)
     Calis.StandardizedResults.LINEQSVarExogStd (persist)
     Calis.StandardizedResults.LINEQSCovExogStd (persist)
     ;

proc calis cov ; /* Analyze the covariance matrix (Default is corr) */
     title3 'Fails parameter count test';
     var grade hsgpa test; /* Declare observed vars */
     lineqs                /* Simultaneous equations, separated by commas */
          Fgrade = beta1 hsgpa + beta2 test + epsilon,
          grade = Fgrade + e;
     std                      /* Variances (not standard deviations) */
          hsgpa = phi11,
          test  = phi22,
          epsilon = psi,
          e = omega;
     cov                      /* Covariances */
          hsgpa test = phi12; /* Unmentioned pairs get covariance zero */
     bounds 0.0 < phi11,
            0.0 < phi22,
            0.0 < psi,
            0.0 < omega;


Log file says:

WARNING: The estimation problem is not identified: There are more parameters
         to estimate ( 7 ) than the total number of mean and covariance
         elements ( 6 ).
NOTE: Convergence criterion (ABSGCONV=0.00001) satisfied.
NOTE: The Moore-Penrose inverse is used in computing the covariance matrix for
      parameter estimates.
WARNING: Standard errors and t values might not be accurate with the use of
         the Moore-Penrose inverse.
WARNING: Critical N is not computable for df= -1.
```

The CALIS Procedure
Covariance Structure Analysis: Model and Initial Values

Modeling Information

Data Set	WORK.MATH
N Records Read	287
N Records Used	287
N Obs	287
Model Type	LINEQS
Analysis	Covariances

Variables in the Model

Endogenous	Manifest	grade
	Latent	Fgrade
Exogenous	Manifest	hsgpa test
	Latent	
	Error	e epsilon

Number of Endogenous Variables = 2
Number of Exogenous Variables = 4

The CALIS Procedure
Covariance Structure Analysis: Descriptive Statistics

Simple Statistics

Variable	Mean	Std Dev
grade	60.98955	17.73355
hsgpa	80.98293	5.97063
test	8.81533	3.56910

The CALIS Procedure
Covariance Structure Analysis: Optimization

Initial Estimation Methods

1 Observed Moments of Variables
2 McDonald Method
3 Two-Stage Least Squares

Optimization Start
Parameter Estimates

N	Parameter	Estimate	Gradient	Lower Bound	Upper Bound
1	beta1	1.46805	1.2905E-17	.	.
2	beta2	1.34956	1.404E-17	.	.
3	phi11	35.64841	1.0173E-18	0	.
4	phi22	12.73851	2.8469E-18	0	.
5	psi	185.72484	2.89877E-7	0	.
6	omega	0.01000	2.89877E-7	0	.
7	phi12	7.24928	-5.581E-18	.	.

Value of Objective Function = 0

Calculus 3: Deliberately cause trouble with identifiability 4
By adding measurement error to response variable
Fails parameter count test

The CALIS Procedure
Covariance Structure Analysis: Optimization

Levenberg-Marquardt Optimization

Scaling Update of More (1978)

Parameter Estimates	7
Functions (Observations)	6
Lower Bounds	4
Upper Bounds	0

Optimization Start

Active Constraints	0	Objective Function	0
Max Abs Gradient Element	2.8987665E-7	Radius	1

Optimization Results

Iterations	0	Function Calls	4
Jacobian Calls	1	Active Constraints	0
Objective Function	0	Max Abs Gradient Element	2.8987665E-7
Lambda	0	Actual Over Pred Change	0
Radius	1		

Convergence criterion (ABSGCONV=0.00001) satisfied.

NOTE: The Moore-Penrose inverse is used in computing the covariance matrix for parameter estimates.

WARNING: Standard errors and t values might not be accurate with the use of the Moore-Penrose inverse.

NOTE: Covariance matrix for the estimates is not full rank.

NOTE: The variance of some parameter estimates is zero or some parameter estimates are linearly related to other parameter estimates as shown in the following equations:

```
omega     =     -6407030    +       34497    *    psi
```

By adding measurement error to response variable

Fails parameter count test

The CALIS Procedure

Covariance Structure Analysis: Maximum Likelihood Estimation

Fit Summary

Modeling Info	N Observations	287
	N Variables	3
	N Moments	6
	N Parameters	7
	N Active Constraints	0
	Baseline Model Function Value	0.6496
	Baseline Model Chi-Square	185.7968
	Baseline Model Chi-Square DF	3
	Pr > Baseline Model Chi-Square	<.0001
Absolute Index	Fit Function	0.0000
	Chi-Square	0.0000
	Chi-Square DF	-1
	Pr > Chi-Square	.
	Z-Test of Wilson & Hilferty	.
	Hoelter Critical N	.
	Root Mean Square Residual (RMSR)	0.0041
	Standardized RMSR (SRMSR)	0.0000
	Goodness of Fit Index (GFI)	1.0000
Parsimony Index	Adjusted GFI (AGFI)	.
	Parsimonious GFI	-0.3333
	RMSEA Estimate	.
	Probability of Close Fit	.
	ECVI Estimate	0.0426
	ECVI Lower 90% Confidence Limit	.
	ECVI Upper 90% Confidence Limit	.
	Akaike Information Criterion	14.0000
	Bozdogan CAIC	46.6164
	Schwarz Bayesian Criterion	39.6164
	McDonald Centrality	0.9983
Incremental Index	Bentler Comparative Fit Index	0.9945
	Bentler-Bonett NFI	1.0000
	Bentler-Bonett Non-normed Index	.
	Bollen Normed Index Rho1	.
	Bollen Non-normed Index Delta2	0.9946
	James et al. Parsimonious NFI	-0.3333

WARNING: Indices for models with negative degrees of freedom may not be interpretable.

By adding measurement error to response variable
Fails parameter count test

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Linear Equations

```
Fgrade   =    1.4680*hsgpa   +  1.3496*test    +  1.0000 epsilon
Std Err       0.1435 beta1      0.2401 beta2
t Value      10.2280            5.6206
grade    =    1.0000 Fgrade  +  1.0000 e
```

Estimates for Variances of Exogenous Variables

Variable Type	Variable	Parameter	Estimate	Standard Error	t Value
Observed	hsgpa	phi11	35.64841	2.98107	11.95826
	test	phi22	12.73851	1.06525	11.95826
Disturbance	epsilon	psi	185.72484	7.76596	23.91523
Error	e	omega	0.01000	7.76596	0.00129

Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate	Standard Error	t Value
hsgpa	test	phi12	7.24928	1.33099	5.44653

Earlier output

Linear Equations

```
grade    =    1.4680*hsgpa   +  1.3496*test    +  1.0000 epsilon
Std Err       0.1435 beta1      0.2401 beta2
t Value      10.2283            5.6207
```

Estimates for Variances of Exogenous Variables

Variable Type	Variable	Parameter	Estimate	Standard Error	t Value
Observed	hsgpa	phi11	35.64841	2.98107	11.95826
	test	phi22	12.73851	1.06525	11.95826
Error	epsilon	psi	185.72484	15.53109	11.95826

Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate	Standard Error	t Value
hsgpa	test	phi12	7.24928	1.33099	5.44653

```
proc calis cov ; /* Analyze the covariance matrix (Default is corr) */
     title3 'Non-ident but passes parameter count test with phi12=0';
     var grade hsgpa test; /* Declare observed vars */
     lineqs                /* Simultaneous equations, separated by commas */
          Fgrade = beta1 hsgpa + beta2 test + epsilon,
          grade = Fgrade + e;
     std                      /* Variances (not standard deviations) */
          hsgpa = phi11,
          test  = phi22,
          epsilon = psi,
          e = omega;
     /* No covariance between expl vars */
     cov                      /* Covariances */
          hsgpa test = 0;
     bounds 0.0 < phi11,
            0.0 < phi22,
            0.0 < psi,
            0.0 < omega;
```

Log file says

```
NOTE: Convergence criterion (ABSGCONV=0.00001) satisfied.
NOTE: The Moore-Penrose inverse is used in computing the covariance matrix for
      parameter estimates.
WARNING: Standard errors and t values might not be accurate with the use of
         the Moore-Penrose inverse.
WARNING: Critical N is not computable for df= 0.
```

List file says

```
                              Optimization Start

Active Constraints                          0    Objective Function    0.1229884791
Max Abs Gradient Element    2.8987665E-7    Radius                            1


                             Optimization Results

Iterations                            0  Function Calls                       4
Jacobian Calls                        1  Active Constraints                   0
Objective Function         0.1229884791  Max Abs Gradient Element  2.8987665E-7
Lambda                                0  Actual Over Pred Change              0
Radius                                1

Convergence criterion (ABSGCONV=0.00001) satisfied.

NOTE: The Moore-Penrose inverse is used in computing the covariance matrix for
                             parameter estimates.


 WARNING: Standard errors and t values might not be accurate with the use of
                          the Moore-Penrose inverse.


NOTE: Covariance matrix for the estimates is not full rank.
```

```
NOTE: The variance of some parameter estimates is zero or some parameter
      estimates are linearly related to other parameter estimates as shown in
      the following equations:


   omega        =        -6407030     +           34497     *     psi
```

```
            By adding measurement error to response variable
          Non-ident but passes parameter count test with phi12=0

                          The CALIS Procedure
        Covariance Structure Analysis: Maximum Likelihood Estimation

                              Fit Summary

      Modeling Info         N Observations                                287
                            N Variables                                     3
                            N Moments                                       6
                            N Parameters                                    6
                            N Active Constraints                            0
                            Baseline Model Function Value              0.6496
                            Baseline Model Chi-Square                185.7968
                            Baseline Model Chi-Square DF                    3
                            Pr > Baseline Model Chi-Square             <.0001
      Absolute Index        Fit Function                               0.1230
                            Chi-Square                                35.1747
                            Chi-Square DF                                   0
                            Pr > Chi-Square                                 .
                            Z-Test of Wilson & Hilferty                     .
                            Hoelter Critical N                              .
                            Root Mean Square Residual (RMSR)          13.4541
                            Standardized RMSR (SRMSR)                  0.1637
                            Goodness of Fit Index (GFI)                0.9284
      Parsimony Index       Adjusted GFI (AGFI)                             .
                            Parsimonious GFI                           0.0000
                            RMSEA Estimate                                  .
                            Probability of Close Fit                        .
                            ECVI Estimate                              0.0426
                            ECVI Lower 90% Confidence Limit                 .
                            ECVI Upper 90% Confidence Limit                 .
                            Akaike Information Criterion              47.1747
                            Bozdogan CAIC                             75.1316
                            Schwarz Bayesian Criterion                69.1316
                            McDonald Centrality                        0.9406
      Incremental Index     Bentler Comparative Fit Index              0.8076
                            Bentler-Bonett NFI                         0.8107
                            Bentler-Bonett Non-normed Index                 .
                            Bollen Normed Index Rho1                        .
                            Bollen Non-normed Index Delta2             0.8107
                            James et al. Parsimonious NFI              0.0000
```

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Linear Equations

```
Fgrade  =    1.4680*hsgpa   +  1.3496*test    +  1.0000 epsilon
Std Err      0.1350 beta1      0.2258 beta2
t Value     10.8767            5.9771
grade   =    1.0000 Fgrade  +  1.0000 e
```

Estimates for Variances of Exogenous Variables

Variable Type	Variable	Parameter	Estimate	Standard Error	t Value
Observed	hsgpa	phi11	35.64841	2.98107	11.95826
	test	phi22	12.73851	1.06525	11.95826
Disturbance	epsilon	psi	185.72484	7.76596	23.91523
Error	e	omega	0.01000	7.76596	0.00129

Covariances Among Exogenous Variables

Var1	Var2	Estimate	Standard Error	t Value
hsgpa	test	0		

```
proc calis cov ; /* Analyze the covariance matrix (Default is corr) */
     title3 'beta1 = beta2 = beta but phi12 ne 0';
     var grade hsgpa test; /* Declare observed vars */
     lineqs                /* Simultaneous equations, separated by commas */
          Fgrade = beta hsgpa + beta test + epsilon,
          grade = Fgrade + e;
     std                      /* Variances (not standard deviations) */
          hsgpa = phi11,
          test  = phi22,
          epsilon = psi,
          e = omega;
     /* No covariance between expl vars */
     cov                      /* Covariances */
          hsgpa test = phi12;
     bounds 0.0 < phi11,
            0.0 < phi22,
            0.0 < psi,
            0.0 < omega;


proc calis cov ; /* Analyze the covariance matrix (Default is corr) */
     title3 'beta1 = beta2 = beta, phi12 ne 0, no bound on omega';
     var grade hsgpa test; /* Declare observed vars */
     lineqs                /* Simultaneous equations, separated by commas */
          Fgrade = beta hsgpa + beta test + epsilon,
          grade = Fgrade + e;
     std                      /* Variances (not standard deviations) */
          hsgpa = phi11,
          test  = phi22,
          epsilon = psi,
          e = omega;
     cov                      /* Covariances */
          hsgpa test = phi12;
     bounds 0.0 < phi11,
            0.0 < phi22,
            0.0 < psi;
```

Log file says

```
WARNING: There are 1 active boundary or linear inequality constraints at the
         solution. The standard errors and chi-square test statistic assume
         that the solution is located in the interior of the parameter space;
         hence, they do not apply if it is likely that some different set of
         inequality constraints could be active.
NOTE: The degrees of freedom are increased by the number of active
      constraints. The number of parameters used to calculate fit indices is
      decreased by the number of active constraints. To turn off the
      adjustment, use the NOADJDF option.
WARNING: The estimated variance of error variable e is zero or very close to
         zero.
WARNING: Although all predicted variances for the observed and latent
         variables are positive, the corresponding predicted covariance matrix
         is not positive definite. It has one zero eigenvalue.
```

Optimization Start

Active Constraints	0	Objective Function	0.0005028148
Max Abs Gradient Element	0.0052378258	Radius	1

Iter	Restarts	Func Calls	Act Con	Objective Function	Obj Fun Change	Max Abs Gradient Element	Lambda	Actual Over Pred Change
1*	0	4	1	0.0004825	0.000020	4.088E-7	111E-16	1.000

Optimization Results

Iterations	1	Function Calls	7
Jacobian Calls	3	Active Constraints	1
Objective Function	0.0004825473	Max Abs Gradient Element	4.08842E-7
Lambda	1.110223E-14	Actual Over Pred Change	0.9999215546
Radius	2		

Convergence criterion (ABSGCONV=0.00001) satisfied.

Earlier results with beta1=beta2

Optimization Start

Active Constraints	0	Objective Function	0.0005028133
Max Abs Gradient Element	0.0052381077	Radius	1

Iter	Restarts	Func Calls	Act Con	Objective Function	Obj Fun Change	Max Abs Gradient Element	Lambda	Actual Over Pred Change
1	0	4	0	0.0004825	0.000020	1.091E-7	0	1.000

Optimization Results

Iterations	1	Function Calls	7
Jacobian Calls	3	Active Constraints	0
Objective Function	0.0004825446	Max Abs Gradient Element	1.0907832E-7
Lambda	0	Actual Over Pred Change	1
Radius	0.0127338107		

```
           Covariance Structure Analysis: Maximum Likelihood Estimation

                              Linear Equations

        Fgrade   =     1.4304*hsgpa   +  1.4304*test    +  1.0000 epsilon
        Std Err        0.1016 beta       0.1016 beta
        t Value       14.0720           14.0720
        grade    =     1.0000 Fgrade  +  1.0000 e

                Estimates for Variances of Exogenous Variables

Variable                                                       Standard
Type           Variable     Parameter      Estimate          Error         t Value

Observed       hsgpa        phi11          35.64841        2.98107        11.95826
               test         phi22          12.73851        1.06525        11.95826
Disturbance    epsilon      psi           185.82860       15.53977        11.95826
Error          e            omega                 0              0               .


                    Covariances Among Exogenous Variables

                                                         Standard
     Var1      Var2     Parameter      Estimate          Error        t Value

     hsgpa     test     phi12           7.24928        1.33099        5.44653
```

Earlier results with beta1=beta2

```
                    Reduced model with beta1 = beta2 = beta

                             The CALIS Procedure
           Covariance Structure Analysis: Maximum Likelihood Estimation

                              Linear Equations

        grade    =     1.4304*hsgpa   +  1.4304*test    +  1.0000 epsilon
        Std Err        0.1016 beta       0.1016 beta
        t Value       14.0724           14.0724


                Estimates for Variances of Exogenous Variables

Variable                                                       Standard
Type           Variable     Parameter      Estimate          Error         t Value

Observed       hsgpa        phi11          35.64841        2.98107        11.95826
               test         phi22          12.73851        1.06525        11.95826
Error          epsilon      psi           185.81825       15.53890        11.95826


                    Covariances Among Exogenous Variables

                                                         Standard
     Var1      Var2     Parameter      Estimate          Error        t Value

     hsgpa     test     phi12           7.24928        1.33099        5.44653
```

With omega unconstrained, log file says

```
NOTE: The Moore-Penrose inverse is used in computing the covariance matrix for
      parameter estimates.
WARNING: Standard errors and t values might not be accurate with the use of
         the Moore-Penrose inverse.
WARNING: The estimated variance of error variable e is negative.
WARNING: Although all predicted variances for the observed and latent
         variables are positive, the corresponding predicted covariance matrix
         is not positive definite. It has one negative eigenvalue.

WARNING: Critical N is not computable for df= 0.
```

List file is very similar, except ...

Estimates for Variances of Exogenous Variables

Variable Type	Variable	Parameter	Estimate	Standard Error	t Value
Observed	hsgpa	phi11	35.64841	2.98107	11.95826
	test	phi22	12.73851	1.06525	11.95826
Disturbance	epsilon	psi	185.82860	7.76945	23.91785
Error	e	omega	-0.01035	7.76945	-0.00133

Stay in the parameter space with different starting values

```
From calculus2 with   grade = beta hsgpa + beta test + epsilon,   had

                              Linear Equations

        grade   =    1.4304*hsgpa   +  1.4304*test    +  1.0000 epsilon
        Std Err      0.1016 beta       0.1016 beta
        t Value     14.0724           14.0724

                Estimates for Variances of Exogenous Variables

Variable                                                     Standard
Type           Variable    Parameter      Estimate            Error       t Value

Observed       hsgpa       phi11          35.64841          2.98107      11.95826
               test        phi22          12.73851          1.06525      11.95826
Error          epsilon     psi           185.81825         15.53890      11.95826


                    Covariances Among Exogenous Variables

                                                        Standard
     Var1     Var2    Parameter      Estimate            Error       t Value

     hsgpa    test    phi12           7.24928          1.33099       5.44653
```

```
/* calculus3b.sas */
options linesize=79 pagesize=500 noovp formdlim='_' ;
title 'Calculus 3b: Start at another point on the maximum surface';
title2 'Deeper in parameter space';

data math;
     infile 'calculus.data'  firstobs=2;
     input id hscalc hsengl hsgpa test grade;

/* Exclude less output this time */
ods exclude
     Calis.ML.SqMultCorr                        (persist)
     Calis.StandardizedResults.LINEQSEqStd      (persist)
     Calis.StandardizedResults.LINEQSVarExogStd (persist)
     Calis.StandardizedResults.LINEQSCovExogStd (persist)
     ;

/* Specify starting values to be final answer from calculus2, except
   split the variance of 185.81825 between psi and omega. */

proc calis cov ; /* Analyze the covariance matrix (Default is corr) */
     title3 'beta1 = beta2 = beta, no bound on variances';
     var grade hsgpa test; /* Declare observed vars */
     lineqs               /* Simultaneous equations, separated by commas */
          Fgrade = beta (1.4304) hsgpa + beta (1.4304) test + epsilon,
          grade = Fgrade + e;
     std                     /* Variances (not standard deviations) */
          hsgpa = phi11 (35.64841),
          test  = phi22 (12.73851),
          epsilon = psi (100),
          e = omega (85.81825);
     cov                     /* Covariances */
          hsgpa test = phi12 (7.24928);
```

Deeper in parameter space

beta1 = beta2 = beta, no bound on variances

The CALIS Procedure

Covariance Structure Analysis: Model and Initial Values

Modeling Information

Data Set	WORK.MATH
N Records Read	287
N Records Used	287
N Obs	287
Model Type	LINEQS
Analysis	Covariances

Variables in the Model

Endogenous	Manifest	grade
	Latent	Fgrade
Exogenous	Manifest	hsgpa test
	Latent	
	Error	e epsilon

Number of Endogenous Variables = 2

Number of Exogenous Variables = 4

Initial Estimates for Linear Equations

```
Fgrade =    1.4304*hsgpa   +  1.4304*test    +  1.0000 epsilon
                   beta               beta
grade  =    1.0000 Fgrade  +  1.0000 e
```

Initial Estimates for Variances of Exogenous Variables

Variable Type	Variable	Parameter	Estimate
Observed	hsgpa	phi11	35.64841
	test	phi22	12.73851
Disturbance	epsilon	psi	100.00000
Error	e	omega	85.81825

Initial Estimates for Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate
hsgpa	test	phi12	7.24928

Calculus 3b: Start at another point on the maximum surface 2
Deeper in parameter space
beta1 = beta2 = beta, no bound on variances

The CALIS Procedure
Covariance Structure Analysis: Descriptive Statistics

Simple Statistics

Variable	Mean	Std Dev
grade	60.98955	17.73355
hsgpa	80.98293	5.97063
test	8.81533	3.56910

Calculus 3b: Start at another point on the maximum surface 3
Deeper in parameter space
beta1 = beta2 = beta, no bound on variances

The CALIS Procedure
Covariance Structure Analysis: Optimization

Initial Estimation Methods

1 User Specifications
2 Observed Moments of Variables

Optimization Start
Parameter Estimates

N	Parameter	Estimate	Gradient
1	beta	1.43040	7.73673E-6
2	phi11	35.64841	8.2795E-19
3	phi22	12.73851	2.9964E-18
4	psi	100.00000	1.0903E-7
5	omega	85.81825	1.0903E-7
6	phi12	7.24928	-5.421E-18

Value of Objective Function = 0.0004825446

Calculus 3b: Start at another point on the maximum surface 4
Deeper in parameter space
beta1 = beta2 = beta, no bound on variances

The CALIS Procedure
Covariance Structure Analysis: Optimization

Levenberg-Marquardt Optimization

Scaling Update of More (1978)

Parameter Estimates	6
Functions (Observations)	6

Optimization Start

Active Constraints	0	Objective Function	0.0004825446
Max Abs Gradient Element	7.7367309E-6	Radius	1

Optimization Results

Iterations	0	Function Calls	4
Jacobian Calls	1	Active Constraints	0
Objective Function	0.0004825446	Max Abs Gradient Element	7.7367309E-6
Lambda	0	Actual Over Pred Change	0
Radius	1		

Convergence criterion (ABSGCONV=0.00001) satisfied.

NOTE: The Moore-Penrose inverse is used in computing the covariance matrix for parameter estimates.

WARNING: Standard errors and t values might not be accurate with the use of the Moore-Penrose inverse.

NOTE: Covariance matrix for the estimates is not full rank.

NOTE: The variance of some parameter estimates is zero or some parameter estimates are linearly related to other parameter estimates as shown in the following equations:

```
omega        =        -3452756     +           34528     *     psi
```

Calculus 3b: Start at another point on the maximum surface 5
Deeper in parameter space
beta1 = beta2 = beta, no bound on variances

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Fit Summary

Modeling Info	N Observations	287
	N Variables	3
	N Moments	6
	N Parameters	6
	N Active Constraints	0
	Baseline Model Function Value	0.6496
	Baseline Model Chi-Square	185.7968
	Baseline Model Chi-Square DF	3
	Pr > Baseline Model Chi-Square	<.0001
Absolute Index	Fit Function	0.0005
	Chi-Square	0.1380
	Chi-Square DF	0
	Pr > Chi-Square	.
	Z-Test of Wilson & Hilferty	.
	Hoelter Critical N	.
	Root Mean Square Residual (RMSR)	0.4367
	Standardized RMSR (SRMSR)	0.0057
	Goodness of Fit Index (GFI)	0.9997
Parsimony Index	Adjusted GFI (AGFI)	.
	Parsimonious GFI	0.0000
	RMSEA Estimate	.
	Probability of Close Fit	.

Deeper in parameter space
beta1 = beta2 = beta, no bound on variances

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Linear Equations

```
Fgrade  =    1.4304*hsgpa   +  1.4304*test    +  1.0000 epsilon
Std Err      0.1016 beta       0.1016 beta
t Value     14.0725           14.0725
grade   =    1.0000 Fgrade  +  1.0000 e
```

Estimates for Variances of Exogenous Variables

Variable Type	Variable	Parameter	Estimate	Standard Error	t Value
Observed	hsgpa	phi11	35.64841	2.98107	11.95826
	test	phi22	12.73851	1.06525	11.95826
Disturbance	epsilon	psi	100.00000	7.76945	12.87092
Error	e	omega	85.81825	7.76945	11.04560

Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate	Standard Error	t Value
hsgpa	test	phi12	7.24928	1.33099	5.44653

Again, compare

Linear Equations

```
grade   =    1.4304*hsgpa   +  1.4304*test    +  1.0000 epsilon
Std Err      0.1016 beta       0.1016 beta
t Value     14.0724           14.0724
```

Estimates for Variances of Exogenous Variables

Variable Type	Variable	Parameter	Estimate	Standard Error	t Value
Observed	hsgpa	phi11	35.64841	2.98107	11.95826
	test	phi22	12.73851	1.06525	11.95826
Error	epsilon	psi	185.81825	15.53890	11.95826

Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate	Standard Error	t Value
hsgpa	test	phi12	7.24928	1.33099	5.44653