# Likelihood Ratio Tests

$$D_1, \ldots, D_n \overset{i.i.d.}{\sim} P_\theta,\ \theta \in \Theta,$$
$$H_0 : \theta \in \Theta_0 \text{ v.s. } H_A : \theta \in \Theta \cap \Theta_0^c,$$

$$\begin{aligned} G &= -2\ln\left(\frac{\max_{\theta\in\Theta_0} L(\theta)}{\max_{\theta\in\Theta} L(\theta)}\right) \\ &= -2\ln\left(\frac{L(\widehat{\theta}_0)}{L(\widehat{\theta})}\right) \\ &= -2\ln L(\widehat{\theta}_0) - [-2\ln L(\widehat{\theta})] \end{aligned}$$

= Difference in chi-square tests for goodness of fit,
df = difference in df

# G = Difference in chi-square tests for goodness of fit

$$
\begin{aligned}
G &= -2\ln L(\widehat{\theta}_0) - [-2\ln L(\widehat{\theta})] \\
&= -2\ln L(\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}}_0)) - [-2\ln L(\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}}))]
\end{aligned}
$$

$$
\begin{aligned}
G_0 &= -2\ln L(\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}}_0)) - [-2\ln L(\widehat{\boldsymbol{\Sigma}})] \\
G_1 &= -2\ln L(\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}})) - [-2\ln L(\widehat{\boldsymbol{\Sigma}})]
\end{aligned}
$$

$$\text{And } G_0 - G_1 = G$$

$$L(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2}(2\pi)^{-nk/2}\exp -\frac{n}{2}\left\{tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\overline{\mathbf{x}} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\overline{\mathbf{x}} - \boldsymbol{\mu})\right\}$$

$$G = G_0 - G_1$$

Under $H_0$, G has an approximate chi-square distribution for large N. Degrees of freedom = number of (non-redundant, linear) equalities specified by $H_0$. Reject when G is large.