Simple Double Measurement with `proc calis`

```
/* double.sas */
options linesize=79 noovp formdlim='_';
title 'Simple double measurement regression';

data double;
     infile 'double.data' firstobs=2; /* Skip first line */
     input W1 W2 Y;

     proc means;
          var W1 W2 Y;
     proc corr;
          var W1 W2 Y;

proc calis cov     /* Analyze covariance matrix - default is corr */
           pcorr;  /* Print observed and predicted covariance matrix */
     /* Name the observed variables */
     var W1 W2 Y;
     /* Now give simultaneous equations, separated by commas. Latent
        variables begin with F for factor. Error terms begin with
        E for error or D for disturbance. SAS is not case sensitive.
        You must name all the parameters. Optional starting values in
        parentheses may be given after the parameters. */
     lineqs
          Y  = beta1 FX + epsilon,
          W1 = FX + e1,
          W2 = FX + e2;
     std                      /* Variances (not standard deviations) */
          FX = phi, epsilon = psi, e1 = omega1, e2 = omega2;
     /* The cov statement would specify covariances between exogenous
        variables. */
        bounds 0.0 < phi psi omega1 omega2;  /* Variances should be positive */

proc calis cov pcorr;
     title2 'Try to fit model with a non-identifiable parameter';
     var W1 W2 Y;
     lineqs
          Y  = beta1 FX + epsilon,
          W1 = FX + e1,
          W2 = FX + e2;
     std                      /* Variances (not standard deviations) */
          FX = phi, epsilon = psi, e1 = omega1, e2 = omega2;
     /* The cov statement names covariances between exogenous
        variables. Covariances equal zero if not mentioned.
        This should be a killer. */
     cov  e1 e2 = omega12;
        bounds 0.0 < phi psi omega1 omega2;  /* Variances should be positive */
```

Simple double measurement regression 1

The MEANS Procedure

Variable	N	Mean	Std Dev	Minimum	Maximum
W1	200	5.0215500	1.5687056	0.7300000	8.8000000
W2	200	5.0862000	1.8693536	0.7100000	9.6900000
Y	200	5.0281000	1.5971464	0.0500000	9.8700000

Simple double measurement regression 2

The CORR Procedure

3 Variables: W1 W2 Y

Simple Statistics

Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
W1	200	5.02155	1.56871	1004	0.73000	8.80000
W2	200	5.08620	1.86935	1017	0.71000	9.69000
Y	200	5.02810	1.59715	1006	0.05000	9.87000

Pearson Correlation Coefficients, N = 200
Prob > |r| under H0: Rho=0

	W1	W2	Y
W1	1.00000	0.56680 <.0001	0.71152 <.0001
W2	0.56680 <.0001	1.00000	0.51584 <.0001
Y	0.71152 <.0001	0.51584 <.0001	1.00000

Simple double measurement regression 3

The CALIS Procedure
Covariance Structure Analysis: Pattern and Initial Values

LINEQS Model Statement

		Matrix	Rows	Columns	------Matrix Type-------	
Term 1	1	_SEL_	3	7	SELECTION	
	2	_BETA_	7	7	EQSBETA	IMINUSINV
	3	_GAMMA_	7	4	EQSGAMMA	
	4	_PHI_	4	4	SYMMETRIC	

The 3 Endogenous Variables

Manifest	W1	W2	Y
Latent			

The 4 Exogenous Variables

Manifest			
Latent	FX		
Error	epsilon	e1	e2

The CALIS Procedure
Covariance Structure Analysis: Pattern and Initial Values

Manifest Variable Equations with Initial Estimates

W1	=	1.0000	FX	+	1.0000 e1
W2	=	1.0000	FX	+	1.0000 e2
Y	=	.*	FX	+	1.0000 epsilon
		beta1			

Variances of Exogenous Variables

Variable	Parameter	Estimate
FX	phi	.
epsilon	psi	.
e1	omega1	.
e2	omega2	.

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Observations	200	Model Terms	1
Variables	3	Model Matrices	4
Informations	6	Parameters	5

Variable	Mean	Std Dev
W1	5.02155	1.56871
W2	5.08620	1.86935
Y	5.02810	1.59715

Covariances

	W1	W2	Y
W1	2.460837284	1.662117980	1.782667281
W2	1.662117980	3.494482975	1.540115859
Y	1.782667281	1.540115859	2.550876774

Determinant 7.073358 Ln 1.956335

NOTE: Some initial estimates computed by instrumental variable method.

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Vector of Initial Estimates

	Parameter	Estimate	Type
1	beta1	0.92660	Matrix Entry: _GAMMA_[3:1]
2	phi	1.74483	Matrix Entry: _PHI_[1:1]
3	psi	1.05279	Matrix Entry: _PHI_[2:2]
4	omega1	0.71601	Matrix Entry: _PHI_[3:3]
5	omega2	1.74965	Matrix Entry: _PHI_[4:4]

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Levenberg-Marquardt Optimization

Scaling Update of More (1978)

Parameter Estimates	5
Functions (Observations)	6
Lower Bounds	4
Upper Bounds	0

Optimization Start

Active Constraints	0	Objective Function	0.0289151886
Max Abs Gradient Element	0.0803614766	Radius	1

Iter	Rest arts	Func Calls	Act Con	Objective Function	Obj Fun Change	Max Abs Gradient Element	Lambda	Actual Over Pred Change
1	0	2	0	0.00894	0.0200	0.0102	0	1.121
2	0	3	0	0.00881	0.000135	0.000823	0	1.078
3	0	4	0	0.00880	1.087E-6	0.000068	0	1.079
4	0	5	0	0.00880	9.515E-9	7.011E-6	0	1.079

Optimization Results

Iterations	4	Function Calls	6
Jacobian Calls	5	Active Constraints	0
Objective Function	0.0088042798	Max Abs Gradient Element	7.0106697E-6
Lambda	0	Actual Over Pred Change	1.0786339445
Radius	0.0004805198		

ABSGCONV convergence criterion satisfied.

Predicted Model Matrix

	W1	W2	Y
W1	2.381727438	1.746936038	1.724140915
W2	1.746936038	3.743228937	1.724140915
Y	1.724140915	1.724140915	2.550876757

Determinant 7.135909 Ln 1.965140

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Fit Function	0.0088
Goodness of Fit Index (GFI)	0.9942
GFI Adjusted for Degrees of Freedom (AGFI)	0.9651
Root Mean Square Residual (RMR)	0.1370
Standardized Root Mean Square Residual (SRMR)	0.0434
Parsimonious GFI (Mulaik, 1989)	0.3314
Chi-Square	1.7521
Chi-Square DF	1
Pr > Chi-Square	0.1856
Independence Model Chi-Square	225.23
Independence Model Chi-Square DF	3
RMSEA Estimate	0.0615
RMSEA 90% Lower Confidence Limit	.
RMSEA 90% Upper Confidence Limit	0.2104
ECVI Estimate	0.0601
ECVI 90% Lower Confidence Limit	.
ECVI 90% Upper Confidence Limit	0.1012
Probability of Close Fit	0.2894
Bentler's Comparative Fit Index	0.9966
Normal Theory Reweighted LS Chi-Square	1.7444
Akaike's Information Criterion	-0.2479
Bozdogan's (1987) CAIC	-4.5463
Schwarz's Bayesian Criterion	-3.5463
McDonald's (1989) Centrality	0.9981
Bentler & Bonett's (1980) Non-normed Index	0.9898
Bentler & Bonett's (1980) NFI	0.9922
James, Mulaik, & Brett (1982) Parsimonious NFI	0.3307
Z-Test of Wilson & Hilferty (1931)	0.9074
Bollen (1986) Normed Index Rho1	0.9767
Bollen (1988) Non-normed Index Delta2	0.9966
Hoelter's (1983) Critical N	438

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Manifest Variable Equations with Estimates

W1	=	1.0000 FX	+	1.0000 e1
W2	=	1.0000 FX	+	1.0000 e2
Y	=	0.9870*FX	+	1.0000 epsilon
Std Err		0.0846 beta1		
t Value		11.6673		

Variances of Exogenous Variables

Variable	Parameter	Estimate	Standard Error	t Value
FX	phi	1.74694	0.23667	7.38
epsilon	psi	0.84923	0.15249	5.57
e1	omega1	0.63479	0.13456	4.72
e2	omega2	1.99629	0.23261	8.58

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Manifest Variable Equations with Standardized Estimates

```
      W1      =   0.8564 FX      +  0.5163 e1
      W2      =   0.6831 FX      +  0.7303 e2
      Y       =   0.8168*FX      +  0.5770 epsilon
                        beta1
```

Squared Multiple Correlations

	Variable	Error Variance	Total Variance	R-Square
1	W1	0.63479	2.38173	0.7335
2	W2	1.99629	3.74323	0.4667
3	Y	0.84923	2.55088	0.6671

The CALIS Procedure
Covariance Structure Analysis: Pattern and Initial Values

LINEQS Model Statement

		Matrix	Rows	Columns	------Matrix Type-------	
Term 1	1	_SEL_	3	7	SELECTION	
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	3	_GAMMA_	7	4	EQSGAMMA	
	4	_PHI_	4	4	SYMMETRIC	

The 3 Endogenous Variables

Manifest	W1	W2	Y
Latent			

The 4 Exogenous Variables

Manifest				
Latent	FX			
Error	epsilon	e1	e2	

The CALIS Procedure
Covariance Structure Analysis: Pattern and Initial Values

Manifest Variable Equations with Initial Estimates

W1	=	1.0000	FX	+	1.0000	e1
W2	=	1.0000	FX	+	1.0000	e2
Y	=	.*	FX	+	1.0000	epsilon
		beta1				

Variances of Exogenous Variables

Variable	Parameter	Estimate
FX	phi	.
epsilon	psi	.
e1	omega1	.
e2	omega2	.

Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate
e1	e2	omega12	.

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Observations	200	Model Terms	1
Variables	3	Model Matrices	4
Informations	6	Parameters	6

Variable	Mean	Std Dev
W1	5.02155	1.56871
W2	5.08620	1.86935
Y	5.02810	1.59715

Covariances

	W1	W2	Y
W1	2.460837284	1.662117980	1.782667281
W2	1.662117980	3.494482975	1.540115859
Y	1.782667281	1.540115859	2.550876774

Determinant 7.073358 Ln 1.956335

NOTE: Some initial estimates computed by instrumental variable method.

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Vector of Initial Estimates

	Parameter	Estimate	Type
1	beta1	0.92660	Matrix Entry: _GAMMA_[3:1]
2	phi	1.74483	Matrix Entry: _PHI_[1:1]
3	psi	1.05279	Matrix Entry: _PHI_[2:2]
4	omega1	0.71601	Matrix Entry: _PHI_[3:3]
5	omega12	-0.08271	Matrix Entry: _PHI_[4:3]
6	omega2	1.74965	Matrix Entry: _PHI_[4:4]

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Levenberg-Marquardt Optimization

Scaling Update of More (1978)

Parameter Estimates	6
Functions (Observations)	6
Lower Bounds	4
Upper Bounds	0

Optimization Start

Active Constraints	0	Objective Function	0.0265279744
Max Abs Gradient Element	0.1079966004	Radius	1

Iter	Rest arts	Func Calls	Act Con	Objective Function	Obj Fun Change	Max Abs Gradient Element	Lambda	Actual Over Pred Change
1*	0	2	0	0.00893	0.0176	0.00818	111E-16	1.164
2*	0	3	0	0.00881	0.000120	0.000794	111E-16	1.082
3*	0	4	0	0.00880	1.042E-6	0.000066	111E-16	1.080
4*	0	5	0	0.00880	9.126E-9	6.8E-6	111E-16	1.080

Optimization Results

Iterations	4	Function Calls	6
Jacobian Calls	5	Active Constraints	0
Objective Function	0.0088042798	Max Abs Gradient Element	6.8004169E-6
Lambda	1.110223E-14	Actual Over Pred Change	1.0802573811
Radius	24		

ABSGCONV convergence criterion satisfied.

NOTE: Moore-Penrose inverse is used in covariance matrix.

NOTE: Covariance matrix for the estimates is not full rank.

NOTE: The variance of some parameter estimates is zero or some parameter estimates are linearly related to other parameter estimates as shown in the following equations:

```
omega12    =       0.240561   -       0.517970   *   beta1      +
                                1.000000   *   phi        +       0.893052
                               *   psi         -       1.000000   *   omega1
                               -       1.000000   *   omega2
```

Simple double measurement regression 16
Try to fit model with a non-identifiable parameter

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Predicted Model Matrix

	W1	W2	Y
W1	2.381727225	1.746936214	1.724140892
W2	1.746936214	3.743229502	1.724140892
Y	1.724140892	1.724140892	2.550876752

Determinant 7.135909 Ln 1.965140

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Fit Function	0.0088
Goodness of Fit Index (GFI)	0.9942
GFI Adjusted for Degrees of Freedom (AGFI)	.
Root Mean Square Residual (RMR)	0.1370
Standardized Root Mean Square Residual (SRMR)	0.0434
Parsimonious GFI (Mulaik, 1989)	0.0000
Chi-Square	1.7521
Chi-Square DF	0
Pr > Chi-Square	<.0001
Independence Model Chi-Square	225.23
Independence Model Chi-Square DF	3
RMSEA Estimate	0.0000
RMSEA 90% Lower Confidence Limit	.
RMSEA 90% Upper Confidence Limit	.
ECVI Estimate	0.0615
ECVI 90% Lower Confidence Limit	.
ECVI 90% Upper Confidence Limit	.
Probability of Close Fit	.
Bentler's Comparative Fit Index	0.9921
Normal Theory Reweighted LS Chi-Square	1.7444
Akaike's Information Criterion	1.7521
Bozdogan's (1987) CAIC	1.7521
Schwarz's Bayesian Criterion	1.7521
McDonald's (1989) Centrality	0.9956
Bentler & Bonett's (1980) Non-normed Index	.
Bentler & Bonett's (1980) NFI	0.9922
James, Mulaik, & Brett (1982) Parsimonious NFI	0.0000
Z-Test of Wilson & Hilferty (1931)	.
Bollen (1986) Normed Index Rho1	.
Bollen (1988) Non-normed Index Delta2	0.9922
Hoelter's (1983) Critical N	.

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Manifest Variable Equations with Estimates

W1	=	1.0000 FX	+	1.0000 e1
W2	=	1.0000 FX	+	1.0000 e2
Y	=	0.9450*FX	+	1.0000 epsilon
Std Err		0.0623 beta1		
t Value		15.1681		

Variances of Exogenous Variables

Variable	Parameter	Estimate	Standard Error	t Value
FX	phi	1.82446	0.18303	9.97
epsilon	psi	0.92154	0.11881	7.76
e1	omega1	0.55727	0.10822	5.15
e2	omega2	1.91877	0.19998	9.60

Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate	Standard Error	t Value
e1	e2	omega12	-0.07752	0.09311	-0.83

Try to fit model with a non-identifiable parameter

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Manifest Variable Equations with Standardized Estimates

```
W1      =   0.8752 FX      +  0.4837 e1
W2      =   0.6981 FX      +  0.7160 e2
Y       =   0.7992*FX      +  0.6011 epsilon
                beta1
```

Squared Multiple Correlations

	Variable	Error Variance	Total Variance	R-Square
1	W1	0.55727	2.38173	0.7660
2	W2	1.91877	3.74323	0.4874
3	Y	0.92154	2.55088	0.6387

Correlations Among Exogenous Variables

Var1	Var2	Parameter	Estimate
e1	e2	omega12	-0.07497