

Exploratory Factor Analysis on Simulated Data

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/* efac.sas */
options linesize=79 pagesize=200noovp formdlim='_';
title 'Exploratory factor analysis with simulated data';

data latent1; /* True factor structure is like Varimax */
  n = 500;

  /* True factor loadings (All communalities = .25) */
  L11 = .5; L12 = 0;
  L21 = .5; L22 = 0;
  L31 = .5; L32 = 0;
  L41 = .5; L42 = 0;
  L51 = 0; L52 = .5;
  L61 = 0; L62 = .5;
  L71 = 0; L72 = .5;
  L81 = 0; L82 = .5;

  /* Variances of error terms */
  v1 = 1 - L11**2 - L12**2;
  v2 = 1 - L21**2 - L22**2;
  v3 = 1 - L31**2 - L32**2;
  v4 = 1 - L41**2 - L42**2;
  v5 = 1 - L51**2 - L52**2;
  v6 = 1 - L61**2 - L62**2;
  v7 = 1 - L71**2 - L72**2;
  v8 = 1 - L81**2 - L82**2;

  do i=1 to n;
    /* Factors are independent standard normal */
    F1 = rannor(0); F2 = rannor(0);
    /* Observed variables all have variance one */
    X1 = L11*F1 + L12*F2 + sqrt(v1)*rannor(0);
    X2 = L21*F1 + L22*F2 + sqrt(v2)*rannor(0);
    X3 = L31*F1 + L32*F2 + sqrt(v3)*rannor(0);
    X4 = L41*F1 + L42*F2 + sqrt(v4)*rannor(0);
    X5 = L51*F1 + L52*F2 + sqrt(v5)*rannor(0);
    X6 = L61*F1 + L62*F2 + sqrt(v6)*rannor(0);
    X7 = L71*F1 + L72*F2 + sqrt(v7)*rannor(0);
    X8 = L81*F1 + L82*F2 + sqrt(v8)*rannor(0);
    output; /* Create a case */
  end;

proc factor method=ML rotate=varimax;
  title2 'True factor structure is like Varimax';
  var X1 - X8;
```

```

data latent2; /* Truth is not like Varimax */
  n = 500;

  /* True factor loadings (All communalities = .5^2 + .8^2 = 0.89) */
  L11 = .5; L12 = -.8;
  L21 = .5; L22 = -.8;
  L31 = .5; L32 = -.8;
  L41 = .5; L42 = -.8;
  L51 = .8; L52 = .5;
  L61 = .8; L62 = .5;
  L71 = .8; L72 = .5;
  L81 = .8; L82 = .5;

  /* Variances of error terms */
  v1 = 1 - L11**2 - L12**2;
  v2 = 1 - L21**2 - L22**2;
  v3 = 1 - L31**2 - L32**2;
  v4 = 1 - L41**2 - L42**2;
  v5 = 1 - L51**2 - L52**2;
  v6 = 1 - L61**2 - L62**2;
  v7 = 1 - L71**2 - L72**2;
  v8 = 1 - L81**2 - L82**2;

  do i=1 to n;
    /* Factors are independent standard normal */
    F1 = rannor(0); F2 = rannor(0);
    /* Observed variables all have variance one */
    X1 = L11*F1 + L12*F2 + sqrt(v1)*rannor(0);
    X2 = L21*F1 + L22*F2 + sqrt(v2)*rannor(0);
    X3 = L31*F1 + L32*F2 + sqrt(v3)*rannor(0);
    X4 = L41*F1 + L42*F2 + sqrt(v4)*rannor(0);
    X5 = L51*F1 + L52*F2 + sqrt(v5)*rannor(0);
    X6 = L61*F1 + L62*F2 + sqrt(v6)*rannor(0);
    X7 = L71*F1 + L72*F2 + sqrt(v7)*rannor(0);
    X8 = L81*F1 + L82*F2 + sqrt(v8)*rannor(0);
    output; /* Create a case */
  end;

proc factor method=ML rotate=varimax;
  title2 'Truth is not like Varimax';
  var X1 - X8;

```

Exploratory factor analysis with simulated data
 True factor structure is like Varimax

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The FACTOR Procedure
 Initial Factor Method: Maximum Likelihood

Prior Communalities Estimates: SMC

	X1	X2	X3	X4
	0.15896611	0.19179222	0.18988932	0.15876910
	X5	X6	X7	X8
	0.13435261	0.15335863	0.14831620	0.15189696

Preliminary Eigenvalues: Total = 1.53904092 Average = 0.19238011

	Eigenvalue	Difference	Proportion	Cumulative
1	1.34909241	0.31098963	0.8766	0.8766
2	1.03810279	1.02830964	0.6745	1.5511
3	0.00979314	0.04366770	0.0064	1.5575
4	-.03387455	0.10575369	-0.0220	1.5354
5	-.13962824	0.05421296	-0.0907	1.4447
6	-.19384120	0.01716791	-0.1259	1.3188
7	-.21100911	0.06858522	-0.1371	1.1817
8	-.27959432		-0.1817	1.0000

2 factors will be retained by the PROPORTION criterion.

Iteration	Criterion	Ridge	Change	Communalities			
1	0.0388682	0.0000	0.1529	0.25244	0.34472	0.33210	0.25354
				0.23131	0.27923	0.28150	0.26357
2	0.0388635	0.0000	0.0016	0.25286	0.34506	0.33189	0.25505
				0.22973	0.27799	0.28253	0.26450
3	0.0388634	0.0000	0.0002	0.25278	0.34516	0.33186	0.25506
				0.22957	0.27778	0.28271	0.26470

Convergence criterion satisfied.

Significance Tests Based on 500 Observations

Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors	28	393.9502	<.0001
HA: At least one common factor			
H0: 2 Factors are sufficient	13	19.2050	0.1169
HA: More factors are needed			

Chi-Square without Bartlett's Correction	19.392832
Akaike's Information Criterion	-6.607168
Schwarz's Bayesian Criterion	-61.397074
Tucker and Lewis's Reliability Coefficient	0.963480

Squared Canonical Correlations

Factor1	Factor2
0.63895620	0.57831146

Eigenvalues of the Weighted Reduced Correlation
 Matrix: Total = 3.14116548 Average = 0.39264569

	Eigenvalue	Difference	Proportion	Cumulative
1	1.76974707	0.39832858	0.5634	0.5634
2	1.37141849	1.20393915	0.4366	1.0000
3	0.16747934	0.05932794	0.0533	1.0533
4	0.10815140	0.09241129	0.0344	1.0877
5	0.01574011	0.05452570	0.0050	1.0928
6	-.03878559	0.04434192	-0.0123	1.0804
7	-.08312751	0.08633032	-0.0265	1.0539
8	-.16945783		-0.0539	1.0000

Factor Pattern

	Factor1	Factor2
X1	0.47672	0.15972
X2	0.54544	0.21829
X3	0.54343	0.19117
X4	0.47671	0.16676
X5	-0.13685	0.45916
X6	-0.24082	0.46879
X7	-0.24621	0.47128
X8	-0.23501	0.45770

Variance Explained by Each Factor

Factor	Weighted	Unweighted
Factor1	1.76974707	1.23990696
Factor2	1.37141849	0.99970068

Final Commuality Estimates and Variable Weights
 Total Commuality: Weighted = 3.141166 Unweighted = 2.239608

Variable	Commuality	Weight
X1	0.25277411	1.33828993
X2	0.34515767	1.52707894
X3	0.33186198	1.49669969
X4	0.25505982	1.34238297
X5	0.22955587	1.29798207
X6	0.27776158	1.38461238
X7	0.28272060	1.39413158
X8	0.26471602	1.35998793

Exploratory factor analysis with simulated data
 True factor structure is like Varimax

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The FACTOR Procedure
 Rotation Method: Varimax

Orthogonal Transformation Matrix

	1	2
1	0.92591	-0.37775
2	0.37775	0.92591

Rotated Factor Pattern

	Factor1	Factor2
X1	0.50173	-0.03220
X2	0.58749	-0.00393
X3	0.57538	-0.02827
X4	0.50438	-0.02567
X5	0.04674	0.47683
X6	-0.04589	0.52503
X7	-0.04994	0.52936
X8	-0.04470	0.51256

Variance Explained by Each Factor

Factor	Weighted	Unweighted
Factor1	1.71290709	1.19112537
Factor2	1.42825847	1.04848227

Final Communalities Estimates and Variable Weights
 Total Communality: Weighted = 3.141166 Unweighted = 2.239608

Variable	Communality	Weight
X1	0.25277411	1.33828993
X2	0.34515767	1.52707894
X3	0.33186198	1.49669969
X4	0.25505982	1.34238297
X5	0.22955587	1.29798207
X6	0.27776158	1.38461238
X7	0.28272060	1.39413158
X8	0.26471602	1.35998793

Exploratory factor analysis with simulated data
 Truth is not like Varimax

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The FACTOR Procedure
 Initial Factor Method: Maximum Likelihood

Prior Communality Estimates: SMC

X1	X2	X3	X4
0.84388116	0.84717600	0.84959658	0.84791201
X5	X6	X7	X8
0.85689298	0.86937749	0.87227084	0.84695200

Preliminary Eigenvalues: Total = 47.1791673 Average = 5.89739591

	Eigenvalue	Difference	Proportion	Cumulative
1	27.0704456	5.5407046	0.5738	0.5738
2	21.5297410	21.6806815	0.4563	1.0301
3	-0.1509405	0.0229821	-0.0032	1.0269
4	-0.1739226	0.0458219	-0.0037	1.0232
5	-0.2197445	0.0247911	-0.0047	1.0186
6	-0.2445357	0.0508435	-0.0052	1.0134
7	-0.2953792	0.0411176	-0.0063	1.0071
8	-0.3364968		-0.0071	1.0000

2 factors will be retained by the PROPORTION criterion.

Iteration	Criterion	Ridge	Change	Communalities			
1	0.0203369	0.0000	0.0369	0.87871	0.88407	0.88651	0.88447
				0.88840	0.90368	0.90615	0.87752
2	0.0203349	0.0000	0.0002	0.87872	0.88398	0.88646	0.88459
				0.88829	0.90367	0.90636	0.87742

Convergence criterion satisfied.

Significance Tests Based on 500 Observations

Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors	28	5263.1031	<.0001
HA: At least one common factor			
H0: 2 Factors are sufficient	13	10.0488	0.6899
HA: More factors are needed			

Chi-Square without Bartlett's Correction	10.147125
Akaike's Information Criterion	-15.852875
Schwarz's Bayesian Criterion	-70.642781
Tucker and Lewis's Reliability Coefficient	1.001214

Squared Canonical Correlations

Factor1	Factor2
0.97291189	0.96620418

Eigenvalues of the Weighted Reduced Correlation
 Matrix: Total = 64.50603 Average = 8.06325375

	Eigenvalue	Difference	Proportion	Cumulative
1	35.9165695	7.3271091	0.5568	0.5568
2	28.5894605	28.4773099	0.4432	1.0000
3	0.1121506	0.0337536	0.0017	1.0017
4	0.0783971	0.0576221	0.0012	1.0030
5	0.0207749	0.0342166	0.0003	1.0033
6	-0.0134417	0.0605386	-0.0002	1.0031
7	-0.0739803	0.0499203	-0.0011	1.0019
8	-0.1239006		-0.0019	1.0000

Factor Pattern

	Factor1	Factor2
X1	0.51001	0.78652
X2	0.48985	0.80251
X3	0.49799	0.79904
X4	0.49959	0.79686
X5	0.84107	-0.42532
X6	0.85006	-0.42552
X7	0.85035	-0.42810
X8	0.84329	-0.40776

Variance Explained by Each Factor

Factor	Weighted	Unweighted
Factor1	35.9165695	3.86187517
Factor2	28.5894605	3.24760431

Final Communalities Estimates and Variable Weights

Total Communalities: Weighted = 64.506030 Unweighted = 7.109479

Variable	Communalities	Weight
X1	0.87872066	8.2457066
X2	0.88398014	8.6190458
X3	0.88646106	8.8073129
X4	0.88458628	8.6646020
X5	0.88828997	8.9514229
X6	0.90366870	10.3805801
X7	0.90636477	10.6797209
X8	0.87740788	8.1576387

Exploratory factor analysis with simulated data
Truth is not like Varimax

4

The FACTOR Procedure
Rotation Method: Varimax

Orthogonal Transformation Matrix

	1	2
1	0.87215	0.48923
2	-0.48923	0.87215

Rotated Factor Pattern

	Factor1	Factor2
X1	0.06001	0.93548
X2	0.03461	0.93956
X3	0.04341	0.94052
X4	0.04587	0.93941
X5	0.94162	0.04053
X6	0.94956	0.04476
X7	0.95108	0.04264
X8	0.93497	0.05694

Repeat Rotated factor loadings. Compare them to the truth.

Rotated Factor Pattern

	Factor1	Factor2
X1	0.06001	0.93548
X2	0.03461	0.93956
X3	0.04341	0.94052
X4	0.04587	0.93941
X5	0.94162	0.04053
X6	0.94956	0.04476
X7	0.95108	0.04264
X8	0.93497	0.05694

The Truth

```
/* True factor loadings */
```

```
L11 = .5; L12 = -.8;  
L21 = .5; L22 = -.8;  
L31 = .5; L32 = -.8;  
L41 = .5; L42 = -.8;  
L51 = .8; L52 = .5;  
L61 = .8; L62 = .5;  
L71 = .8; L72 = .5;  
L81 = .8; L82 = .5;
```