STA 413F2011 Assignment 2

Do this assignment in preparation for Quiz Two in tutorial on Friday Sept 23d. The problems are practice for the quiz, and are not to be handed in. See the formula sheet; a copy will be supplied with the quiz.

1. State and prove Markov's inequality for a *discrete* random variable X.
2. Use Markov's inequality to prove the Weak Law of Large Numbers.
3. Let $A_1, A_2, \ldots$ be a sequence of sets with $A_1 \subseteq A_2 \subseteq \ldots$ and $A = \cup_{n=1}^{\infty} A_n$. Prove that $\lim_{n\rightarrow\infty} P(A_n) = P(A)$.
4. Use the Theorem of Question 3 (the Nested Sets Theorem) to establish the following: If $A_1, A_2, \ldots$ is a sequence of sets with $A_1 \supseteq A_2 \supseteq \ldots$ and $A = \cap_{n=1}^{\infty} A_n$, then $\lim_{n\rightarrow\infty} P(A_n) = P(A)$.
5. Let X be a random variable with cumulative distribution function $F(x)$.
   (a) Prove that $\lim_{x\rightarrow\infty} F(x) = 1$.
   (b) Prove that $\lim_{x\rightarrow-\infty} F(x) = 0$.
6. Do Exercise 4.1.3 from the text.
7. Let $X_1, \ldots, X_n$ be a random sample from a distribution with density
$$f(x;\theta) = \theta x^{\theta-1} I(0 < x < 1),$$
   where $\theta > 0$.
   (a) What is $E(X_i)$? Show your work.
   (b) What is $E(\overline{X}_n)$? There is no need to show your work.
   (c) Is $T = \frac{\overline{X}_n}{1-\overline{X}_n}$ an unbiased estimator of θ? Answer "Yes," "No," or "I don't think so." Explain your answer.
8. Please do Exercise 4.1.26 from the text. The problem refers to Theorem 3.3.1, which just says that if $X \sim \chi^2(r)$, then $E(X^k) = \frac{2^k\Gamma(r/2+k)}{\Gamma(r/2)}$. In fact, you proved a more general version of this in Problem 4 of Assignment 1. Anyway, another hint is that here, the statistic S is being adjusted so that it has the right expected value.
9. Read Section 5.1 in the text, and do Exercises 5.1.2, 5.1.3, 5.1.4 and 5.1.5.