STA 413F2008 Assignment 5

Please read Section 4.4. You are not responsible for the proof of the Central Limit Theorem. But please pay attention to the continuity correction. In this class, we will use the continuity correction whenever we are approximating probabilities from a discrete distribution, but we will not bother with it when we are constructing confidence intervals.

When you approximate probabilities, don't be too proud! I suggest you draw a rough picture of the normal curve and shade in the probability you are calculating. This will make it easier to use our text's table of the normal distribution, which is reproduced for your convenience at the end of this document.

For this assignment, don't worry about Taylor's theorem or any applications, including the proof of the proof of the Delta Method. However, Example 4.4.4 is very interesting, and will be covered in lecture.

1. (a) Do Exercise 4.4.2.
   (b) Actually, an approximation is not necessary here, because the exact distribution of $\overline{X}$ is easy to find using moment-generating functions. Please do so; show your work. I used R to get the exact probability for this problem; it's 0.9547, so the approximation is very good.
2. (a) Let $Y_1, \ldots, Y_{72}$ be i.i.d. Bernoulli with $\theta = \frac{1}{3}$. Use moment-generating functions to find the distribution of $Y = \sum_{i=1}^{n} Y_i$.
   (b) Do Exercise 4.4.3.
3. Do Exercise 4.4.4. The value $n = 15$ violates the conventional rule of $n \geq 25$, but the Central Limit Theorem is all you have, so use it. By the way, I approximated the answer another way and got 0.8346, so the Central Limit Theorem really is working pretty well here.
4. Do Exercise 4.4.5. Again the sample size is small, but the Central Limit Theorem does a good job because the distribution of the data is symmetric. This example and the preceding one tell us that the $n \geq 25$ rule is not set in stone. Also, it is easy to construct examples with highly skewed distributions where $n > 500$, but the Central Limit Theorem performs terribly.
5. Do Exercise 4.4.7. What would happen if you forgot the continuity correction here?
6. Do Exercise 4.4.9. You are being asked for a binomial probability.
7. Let $X_1, \ldots, X_n$ be a random sample from a distribution with expected value μ and variance σ^2. The "Modified" Central Limit Theorem (see lecture notes and Formula Sheet) says that the usual Central Limit Theorem still holds if σ is replaced by any consistent estimator – call it $\widehat{\sigma}_n$. Using this result, *derive* an approximate $(1-\alpha)100\%$ confidence interval for μ. Show your work.

8. A random sample of size $n = 150$ yields a sample mean of $\overline{X}_n = 8.2$. Give a point estimate and an approximate 95% confidence interval

   (a) For λ, if $X_1, \ldots, X_n$ are from a Poisson distribution with parameter λ.

   (b) For θ, if $X_1, \ldots, X_n$ are from an Exponential distribution with parameter θ.

   (c) For μ, if $X_1, \ldots, X_n$ are from a Normal distribution with mean μ and variance one. (This confidence interval is exact, not an approximation.)

   (d) For θ, if $X_1, \ldots, X_n$ are from a Uniform distribution on $[0, \theta]$.

   (e) For θ, if $X_1, \ldots, X_n$ are from a Uniform distribution on $[\theta, \theta + 1]$.

   (f) For θ, if $X_1, \ldots, X_n$ are from a Geometric distribution with parameter θ.

   (g) For θ, if $X_1, \ldots, X_n$ are from a Binomial distribution with parameters 10 and θ.

   In each case, your answer is three numbers: the point estimate, the lower confidence limit and the upper confidence limit.

Table III
Normal Distribution

The following table presents the standard normal distribution. The probabilities tabled are

$$P(X \le x) = \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw.$$

Note that only the probabilities for $x \ge 0$ are tabled. To obtain the probabilities for $x < 0$, use the identity $\Phi(-x) = 1 - \Phi(x)$.

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998