STA 347F2003 Quiz 6

1. Let $X_0, X_1, \ldots$ be a stationary Markov chain with transition matrix

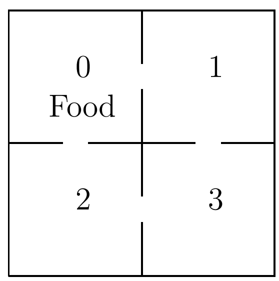
	0	1	2	3
0	1	0	0	0
1	0.1	0.4	0.1	0.4
2	0.2	0.1	0.6	0.1
3	0	0	0	1

.

   (a) (30 Points) Starting in state 1, determine the probability that the Markov chain ends in zero.

   (b) (30 Points) Determine the expected time to absorbtion starting from state 1.

2. (40 Points) A rat is placed into room 3 of the maze shown below.



The rat moves through the maze at random; it is always equally likely to choose any available door, independently of past choices. What is the probability that the rat finds the food without ever passing through room 1?

(Q6 Ans)

Jerry's answers to Quiz 6

① a) Letting T denote absorbtion time,

$$u_1 = P_R\{X_T = 0 \mid X_0 = 1\} = \sum_{k=0}^{3} P_R\{X_T = 0 \mid X_0 = 1, X_1 = k\} P_{1k}$$

$$= 1 \cdot P_{10} + u_1 P_{11} + u_2 P_{12} + 0$$

$$= .1 + u_1(.4) + u_2(.1)$$

$$\Rightarrow 10u_1 = 1 + 4u_1 + u_2$$

$$\Rightarrow u_2 = 6u_1 - 1$$

And

$$u_2 = P_R\{X_T = 0 \mid X_0 = 2\} = \sum_{k=0}^{3} P_R\{X_T = 0 \mid X_0 = 2, X_1 = k\} P_{2k}$$

$$= (1)(P_{20}) + u_1 P_{21} + u_2 P_{22} + 0$$

$$= \frac{2}{10} + \frac{1}{10}u_1 + \frac{6}{10}u_2$$

$$\Rightarrow 4u_2 = 2 + u_1$$ Substituting in first eq, get

$$4(6u_1 - 1) = 2 + u_1 \Leftrightarrow 24u_1 - 4 = 2 + u_1$$

$$\Rightarrow 23u_1 = 6 \Rightarrow u_1 = 6/23$$

Q6 Answer 2

(1b) $v_1 = E[T \mid X_0 = 1] = \sum_{k=0}^{3} E[T \mid X_0 = 1, X_1 = k]\, P_{1k}$

$= (1)P_{10} + (v_1 + 1)P_{11} + (v_2 + 1)P_{12} + (1)P_{13}$

$= 1 + \frac{4}{10} v_1 + \frac{1}{10} v_2$

$\Rightarrow 6 v_1 = 10 + v_2 \Rightarrow v_2 = 6 v_1 - 10$

and

$v_2 = 1 + \frac{1}{10} v_1 + \frac{6}{10} v_2$

$\Rightarrow 4 v_2 = 10 + v_1 \Rightarrow 4(6v_1 - 10) = 10 + v_1$

$\Rightarrow 23 v_1 - 40 = 10 \Rightarrow v_1 = \frac{50}{23}$

(2) Make 1 absorbing and

$$\mathbf{P} = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 3 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}$$

$u_2 = \Pr\{X_T = 0 \mid X_0 = 2\}$

$\Pr\{X_T = 0 \mid X_0 = 2, X_1 = 0\}\frac{1}{2}$

$+ \Pr\{X_T = 0 \mid X_0 = 2, X_1 = 3\}\frac{1}{2}$

$= \frac{1}{2} + \frac{1}{2} u_3$

And $u_3 = \Pr\{X_T = 0 \mid X_0 = 3, X_1 = 1\}(\frac{1}{2}) + \Pr\{X_T = 0 \mid X_0 = 3, X_1 = 2\}(\frac{1}{2})$

$\Rightarrow u_3 = 0 + \frac{1}{2} u_2$ substitute for u_2 using first equation,

$\Rightarrow u_3 = \frac{1}{2}(\frac{1}{2} + \frac{1}{2} u_3) = \frac{1}{4}(1 + u_3) \Rightarrow 4 u_3 = 1 + u_3$

$\Rightarrow u_3 = \frac{1}{3}$