STA 347F2003 Quiz 5

1. (30 Points) Consider a spare parts inventory model in which either 0, 1 or 2 repair parts are demanded in any period, with

$$Pr\{\xi_n = 0\} = 0.5, \;\; Pr\{\xi_n = 1\} = 0.3, \;\; Pr\{\xi_n = 2\} = 0.2$$

and suppose $s = 1$ and $S = 3$. Give the transition probability matrix for the Markov chain $\{X_n\}$, where X_n is defined as the quantity on hand at the end of period n.

2. Two jars A and B, contain a total of N marbles. At each step, a marble is selected at random (all N are equally likely), and moved to the other jar. Letting X_n represent the number of marbles in jar A at step n,

   (a) (5 Points) What is $P_{0,0}$?

   (b) (5 Points) What is $P_{N,N-1}$?

   (c) (10 Points) If $0 < i < N$, what is $P_{i,i-1}$?

   (d) (10 Points) If $0 < i < N$, what is $P_{i,i+1}$?

3. Consider a simple queueing system in which at most one customer arrives during a time period, and at most one customer is served during a time period. The probability that a new customer will arrive during any time period is a. Independently of customer arrivals, the probability that service will be completed during any time period is c. For the Markov chain in which X_n is the number of customers waiting for service or being served at the beginning of time period n, what is

   (a) (5 Points) What is $P_{0,0}$?

   (b) (5 Points) What is $P_{0,1}$?

   (c) (10 Points) If $i \geq 1$, what is $P_{i,i-1}$?

   (d) (10 Points) If $i \geq 1$, what is $P_{i,i+1}$?

   (e) (10 Points) If $i \geq 1$, what is $P_{i,i}$?

Jerry's Answers to Quiz 5

(1)

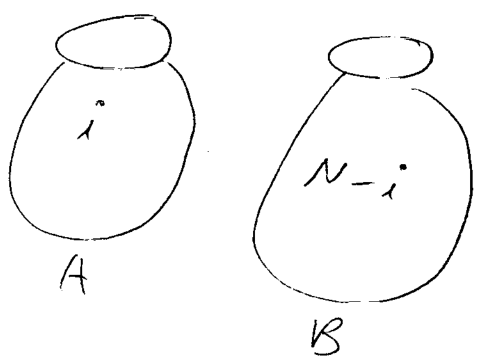
	0	1	2	3
0	0	.2	.3	.5
1	0	.2	.3	.5
2	.2	.3	.5	0
3	0	.2	.3	.5

(2) (a) $P_{00} = 0$

(b) $P_{N,N-1} = 1$

(c) $P_{i,i-1} = \frac{i}{N}$

(d) $P_{i,i+1} = \frac{N-i}{N}$



(3) (a) $P_{00} = 1 - a$

(b) $P_{01} = a$

(c) $P_{i,i-1} = (1-a)c$

(d) $P_{i,i+1} = a(1-c)$

(e) $P_{ii} = (1-a)(1-c) + ac$