STA 347F2003 Quiz 2

1. Let $X_0, X_1, \ldots$ be a stationary Markov chain with transition matrix

	0	1	2
0	0.1	0.1	0.8
1	0.2	0.2	0.6
2	0.3	0.3	0.4

   (a) (5 Points) What is $Pr\{X_3 = 2|X_2 = 0\}$?

   (b) (10 Points) Suppose $Pr\{X_0 = 0\} = 0.1$, $Pr\{X_0 = 1\} = 0.4$ and $Pr\{X_0 = 2\} = 0.5$. What is $Pr\{X_0 = 0, X_1 = 1, X_2 = 2\}$? Show your work.

   (c) (15 Points) What is $Pr\{X_1 = 1, X_2 = 1|X_0 = 0\}$ Show your work.

   (d) (20 Points) What is $Pr\{X_2 = 1, X_3 = 1|X_0 = 2, X_1 = 0\}$ Show your work.

   (e) (25 Points) Suppose $Pr\{X_0 = 0\} = 0.3$, $Pr\{X_0 = 1\} = 0.3$ and $Pr\{X_0 = 2\} = 0.4$. What is $Pr\{X_1 = 0, X_2 = 1, X_3 = 0\}$? Show your work.

2. (25 Points) Let $\xi_0, \xi_1, \xi_2, \ldots$ be independent random variables with $Pr\{\xi_k = 1\} = \alpha$ and $Pr\{\xi_k = 0\} = 1 - \alpha$ for $k = 0, 1, \ldots$. Let $Y_n = \sum_{k=0}^{n} \xi_k$; finally, let $X_n = 1$ if Y_n is odd, and $X_n = 0$ if Y_n is even (zero is an even number). Clearly, $X_0, X_1, \ldots$ is a stationary Markov chain. Give its transition matrix.

Jerry's Answers to Quiz 2

Q2 Answ 1

(1) (a) .8

(b) $P_n\{X_0=0, X_1=1, X_2=2\}$

$= P_n\{X_0=0\} P_n\{X_1=1 \mid X_0=0\} P_n\{X_2=2 \mid X_0=0, X_1=1\}$

$= (.1)(.1)(.6) =$ .006

(c) $P_n\{X_1=1, X_2=1 \mid X_0=0\} = \dfrac{P_n\{X_0=0, X_1=1, X_2=1\}}{P_n\{X_0=0\}}$

$= \dfrac{P_n\{X_0=0\} P_n\{X_1=1 \mid X_0=0\} P_n\{X_2=1 \mid X_0=0, X_1=1\}}{P_n\{X_0=0\}}$

$= (.1)(.2) =$ .02

(d) $P_n\{X_2=1, X_3=1 \mid X_0=2, X_1=0\} = \dfrac{P_n\{X_0=2, X_1=0, X_2=1, X_3=1\}}{P_n\{X_0=2, X_1=0\}}$

$= \dfrac{P_n\{X_0=2\} P_n\{X_1=0 \mid X_0=2\} P_n\{X_2=1 \mid X_0=2, X_1=0\} P_n\{X_3=1 \mid X_0=2, X_1=0, X_2=1\}}{P_n\{X_0=2\} P_n\{X_1=0 \mid X_0=2\}}$

$= P_{01} P_{11} = (.1)(.2) =$ .02

Q2 Answ 2

(1e) Need $P_n\{X_1 = 0\}$

$= P_n\{X_1=0 \mid X_0=0\} P_n\{X_0=0\} + P_n\{X_1=0 \mid X_0=1\} P_n\{X_0=1\}$
$+ P_n\{X_1=0 \mid X_0=2\} P_n\{X_0=2\}$

$= (.1)(.3) + (.2)(.3) + (.3)(.4)$

$= .03 + .06 + .12 = .21$, and

$P_n\{X_1=0, X_2=1, X_3=0\}$

$= P_n\{X_1=0\} P_n\{X_2=1 \mid X_1=0\} P_n\{X_3=0 \mid X_1=0, X_2=1\}$

$= (.21)(.1)(.2) = .0042$

(2)

$\underset{\sim}{P} =$

	0	1
0	$1-\alpha$	α
1	α	$1-\alpha$