

STA 347F2003 Quiz 1

1. One jar contains blue balls numbered 1 and 2. Another jar contains red balls numbered 1, 2, and 3. A jar is chosen at random, and then a ball is chosen at random from that jar.
 - (a) (15 pts) What is $Pr\{2|\text{Red}\}$?
 - (b) (15 pts) What is $Pr\{\text{Blue}|3\}$?
 - (c) (25 pts) What is $Pr\{2\}$?
 - (d) (25 pts) What is $Pr\{\text{Blue}|2\}$?
2. (20 pts) Is it true that $Pr(A|B) + Pr(A^c|B) = 1$? If it is true, then prove it. If it is not true, give a simple counter-example. Begin your answer with the words “The statement is true,” or “The statement is false.” Hint: You might want to start with $Pr\{B\} = Pr\{B|A\}Pr\{A\} + Pr\{B|A^c\}Pr\{A^c\}$ (Law of Total Probability), which you need not prove.

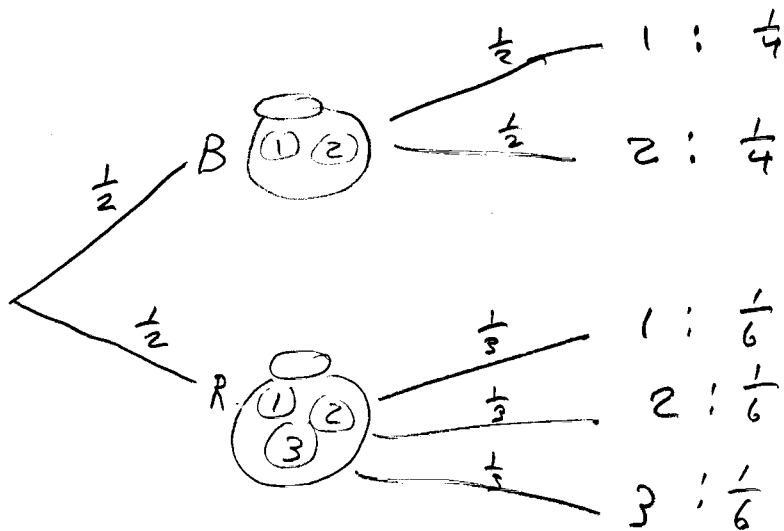
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Jerry's Answers to Quiz One

Q1 Answ
1

① It is OKAY to make a tree



a) $P_n \{2|R\} = \frac{1}{3}$

b) $P_n \{B|3\} = 0$

c) $P_n \{2\} = \frac{1}{4} + \frac{1}{6} = \frac{3}{12} + \frac{2}{12} = \frac{5}{12}$

Or, $P_n \{2\} = P_n \{2|B\}P_n\{B\} + P_n \{2|R\}P_n\{R\}$
 $= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

d) $P_n \{B|2\} = \frac{P_n \{B \cap 2\}}{P_n \{2\}} = \frac{1/4}{5/12} = \frac{3}{5}$

Or using Bayes' theorem explicitly is fine

② The statement is true. By the Law of Total Probability,

$$\begin{aligned}
 P\{B\} &= P\{B|A\}P\{A\} + P\{B|A^c\}P\{A^c\} \\
 &= \frac{P\{A \cap B\}}{\cancel{P\{A\}}} \cancel{P\{A\}} + \frac{P\{A^c \cap B\}}{\cancel{P\{A^c\}}} \cancel{P\{A^c\}}
 \end{aligned}$$

$$\Rightarrow 1 = \frac{P\{A \cap B\}}{P\{B\}} + \frac{P\{A^c \cap B\}}{P\{B\}} = P\{A|B\} + P\{A^c|B\}$$

done