STA 347F2003 Assignment 8

Do this assignment in preparation for the quiz on Friday, Nov. 14th. It is not to be handed in.

Read Sections 3 and 4 in Chapter 4. Then do the following exercises and problems. You will be given a copy of the formula sheet (see link on the class home page).

1. Let X have a geometric distribution; that is, $Pr\{X = k\} = (1-a)a^k$ for $k = 0, 1, \ldots$, where $0 < a < 1$. Find $E[X[$; show your work.
2. Let $i \to j$ and $j \to k$. Show $i \to k$.
3. Do Exercises 3.1, 3.2 and 3.3 starting on page 243.
4. Do Problem 3.1a; also prove recurrence. Skip Part b.
5. Do Problem 3.3
6. Do Exercise 4.1. Show $\pi_k = \frac{p^k(1-p)}{1-p^5}$ (equivalent to book's answer).
7. Do Exercise 4.3. Also, what is the period? Is it regular?
8. Let $X_0, X_1, \ldots$ be a stationary Markov chain with transition matrix

	0	1	2
0	$\frac{1}{2}$	$\frac{1}{2}$	0
1	0	0	1
2	1	0	0

.

   (a) What is the period of this Markov chain? Why?
   (b) Is it irreducible?
   (c) What are $f_{00}^{(1)}$, $f_{00}^{(2)}$, $f_{00}^{(3)}$?
   (d) What is $f_{00}^{(n)}$ for $n > 3$?
   (e) What is f_{00}?
   (f) Is state zero recurrent? Why?
   (g) Is state zero *positive* recurrent? Show your work.
   (h) Using your answer to the preceding question, what is π_0?
   (i) Is state 2 recurrent? Without any calculations, why?
   (j) What are $f_{11}^{(1)}$ and $f_{11}^{(2)}$?
   (k) What is $f_{11}^{(n)}$ for $n > 2$?
   (l) Show recurrence of State 1 from the definition.
   (m) Is State 1 positive recurrent? Show your work.

9. Consider a stationary Markov chain with transition probabilities $P_{i,i} = \frac{9}{10}$, $P_{i,i+1} = \frac{1}{10}$, and zero otherwise.

   (a) What is the period of this Markov chain? Why?

   (b) Is it irreducible?

   (c) What is $f_{00}^{(n)}$?

   (d) What is f_{00}?

   (e) Is state zero recurrent?

   (f) What is $P_{00}^{(n)}$?

   (g) Use your answer to the preceding item to show recurrence or transience.

   (h) Is the general state i transient, or is it recurrent?

10. Consider a stationary Markov chain with transition probabilities $P_{i,j} = (1-a)a^j$ for $i = 0, 1, \ldots$ and $j = 0, 1, \ldots$.

   (a) What is the period of this Markov chain? Why?

   (b) Is it irreducible?

   (c) What is $P_{00}^{(2)}$? Show your work.

   (d) What is $P_{00}^{(n)}$?

   (e) Is state zero transient, or is it recurrent? Why?

   (f) What is $P_{ij}^{(2)}$? Show your work. What is $P_{ij}^{(n)}$?

   (g) Is the general state i transient, or is it recurrent? Why?

   (h) What is $f_{00}^{(n)}$?

   (i) What is f_{00}?

   (j) Is state zero positive recurrent or null recurrent? Check.

   (k) Apply Theorem 4.2.

11. Consider a stationary Markov chain with transition probabilities $P_{i,0} = \frac{1}{i+2}$ and $P_{i,i+1} = \frac{i+1}{i+2}$.

   (a) What is the period of this Markov chain? Why?

   (b) Is it irreducible?

   (c) What are $f_{00}^{(1)}$, $f_{00}^{(2)}$, $f_{00}^{(3)}$?

   (d) What is $f_{00}^{(n)}$ in general?

   (e) What is f_{00}?

   (f) Is state zero recurrent? Why?

   (g) Is state zero positive recurrent? Show your work.

   (h) Does Theorem 4.2 apply? Answer Yes or No and say why.

   (i) Try to find the stationary distribution anyway. That is, find the solution to $\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}$. Is it a probability distribution?

(j) Use Theorem 4.1 to find π_0. Is this consistent with your answer to the last item?

12. Do Problem 4.1.

13. Do Problem 4.3, except let $p_i = q_i = \frac{1}{2}$. Try letting $N = 6$ at first. After setting up the equations, express all the other limiting probabilities in terms of π_0. You will see the pattern for general N.