STA 347F2003 Assignment 7

Do this assignment in preparation for the quiz on Friday, Oct. 31st. It is not to be handed in.

1. Do exercises 1.1, 1.3, 1.4, 1.5, 1.10 starting on page 208.

2. Let $X_0, X_1, \ldots$ be a *regular* stationary Markov chain with state space $\{0, \ldots, N\}$, so that the limiting probabilities described on page 199 exist. Show that the row vector $\boldsymbol{\pi}$ satisfies $\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}$.

3. Let $X_0, X_1, \ldots$ be a *regular* stationary Markov chain with state space $\{0, \ldots, N\}$, and let the row vector $\mathbf{x}$ satisfy both $\mathbf{x} = \mathbf{xP}$ and $\sum_{k=0}^{N} x_k = 1$. Show $\mathbf{x} = \boldsymbol{\pi}$.

4. Let $X_0, X_1, \ldots$ be a *regular* stationary Markov chain with state space $\{0, \ldots, N\}$. Prove or disprove: $\lim_{n\to\infty} \mathbf{p}^{(n)} = \boldsymbol{\pi}$.

5. Let $X_0, X_1, \ldots$ be a stationary Markov chain with transition matrix

	0	1	2
0	γ_{00}	γ_{01}	γ_{02}
1	γ_{10}	γ_{11}	γ_{12}
2	0	0	1

, where $0 < \gamma_{ij} < 1$.

   (a) Is this Markov chain regular? Answer Yes or No, and prove it.
   (b) Using common sense, what is $\lim_{n\to\infty} \mathbf{P}^n$?
   (c) Find $\boldsymbol{\pi}$ the usual way.
   (d) What fact does this problem illustrate?

6. Let $X_0, X_1, \ldots$ be a regular stationary Markov chain with state space $\{0, \ldots, N\}$ and a transition matrix that is *doubly stochastic* — that is, $\sum_{i=0}^{N} P_{ij} = 1$ (the columns sum to one as well as the rows). Show that the limiting probability π_j equals $\frac{1}{N+1}$ for $j = 0, \ldots, N$.

7. Let $X_0, X_1, \ldots$ be a stationary Markov chain with transition matrix

	0	1	2	3
0	a_1	a_2	a_3	a_4
1	a_1	a_2	a_3	a_4
2	a_1	a_2	a_3	a_4
3	a_1	a_2	a_3	a_4

,

   where $0 < a_k < 1$ for $k = 1, 2, 3, 4$. What is $\boldsymbol{\pi}$?

8. Do Problems

- 1.1: Very easy if you see it.
- 1.2: Routine; answer is 1/32
- 1.3: It's interesting how the answer comes out in terms of $\sum_{k=1}^{6} k\,\alpha_k$, a kind of expected value
- 1.4: They want $\lim_{n\to\infty} Pr\{X_{n+1} = m, X_n = k\}$.
- 1.6:
  - (a) It's obvious, but try getting the answer formally by conditioning on X_n (using the Law of Total Probability).
  - (b) So the answer to Problem 1.4 does not depend on where you start.
  - (c) I don't see how you can avoid saying $P_{ik}^{(n-1)} \to \pi_k$, either "obviously," or by using the definition of a limit. This is perfectly okay; it's just that there does not seem to be any nice trick like the one suggested for part (a).
- 1.10: Easy if you see it.
- 1.13: You can get a backwards transition probability. Then, Theorem 1.0 from lecture will help. The answer is 0.171428.