STA 347F2000 Quiz 6

Print your name and student number *neatly* on the first sheet.

1. (10 Points) A radioactive source emits particles according to a Poisson process of rate $\lambda = 2$ per minute. What is the probability that the *second* particle emitted comes in the interval between 3 and 5 minutes?

2. (10 Points) Let $X(t)$ be a *non*homogeneous Poisson process with a rate function $\lambda(t) = t$. What is $Pr\{X(2) = 1\}$?

3. (20 Points) Let $X(t)$ be a homogeneous Poisson process with rate $\lambda > 0$, and assume that exactly one event has occurred in the interval $(0, t]$. Let Y denote the time that the event occurred. Derive the density of Y. Don't forget the support; it is worth 5 marks out of 20.

4. (30 Points) Let $X(t)$ be a homogeneous Poisson process with rate $\lambda > 0$, and let $0 < r < s$ and $0 \leq k \leq n$. Derive $Pr\{X(r) = k | X(s) = n\}$. In this question, please use only basic properties of the Poisson process (stationarity, independent increments, Poisson distribution for number of events in an interval) and ordinary rules of probability. Show all the steps.

5. (30 Points) For a stationary counting process with independent increments, let $N(a, b]$ denote the number of events in the interval $(a, b]$. Assume also that $Pr\{N(0, h] \geq 1\} = \lambda h + o(h)$ as $h \downarrow 0$, where λ is a positive constant. Prove $Pr\{X(t) = 0\} = e^{-\lambda t}$. You may assume that the derivative of the required probability exists.