STA 347F2000 Quiz 4

Print your name and student number *neatly* on the first sheet.

1. (15 Points) Let X, Y and Z be discrete random variables. Show that $E[g(Z)|X = x] = \sum_y E[g(Z)|X = x, Y = y]Pr(Y = y|X = x)$. As usual, you may exchange order of summation without comment.

2. (15 Points) A particle moves in a circle through points which have been labelled 1, 2, 3, 4 (in a clockwise order). At each step k, it has probability $k/4$ of moving to the right (clockwise), and probability $1 - k/4$ of moving to the left (counterclockwise). Give the transition probability matrix for this Markov chain.

3. Let $X_0, X_1, \ldots$ be a stationary Markov chain with transition matrix

	0	1	2
0	0.1	0.1	0.8
1	0.2	0.2	0.6
2	0.3	0.3	0.4

   (a) (10 Points) What is $Pr(X_{17} = 2|X_{15} = 0)$? Show your work, what there is of it.

   (b) (10 Points) Suppose $Pr(X_1 = 0) = 0.1$, $Pr(X_1 = 1) = 0.4$ and $Pr(X_1 = 2) = 0.5$. What is $Pr(X_1 = 0, X_2 = 1, X_3 = 2)$? Show your work.

   (c) (10 Points) Again, suppose $Pr(X_1 = 0) = 0.1$, $Pr(X_1 = 1) = 0.4$ and $Pr(X_1 = 2) = 0.5$. What is $Pr(X_2 = 1)$? Show some calcuations.

4. (20 Points) Let $X_0, X_1, \ldots$ be a stationary Markov chain. Use the Markov property and common rules of probability to show $Pr\{X_3 = j|X_0 = i_0, X_1 = i_1, X_2 = i_2\} = Pr\{X_3 = j|X_1 = i_1, X_2 = i_2\}$.

5. (20 Points) Debbie and Wanda are gambling. They toss a fair coin. If it is heads, Debbie wins one dollar from Wanda. If it is tails, Wanda wins one dollar from Debbie. Suppose Debbie starts with \$2 and Wanda starts with \$1. What is the expected number of coin tosses before one of them goes broke?