STA 347F2000 Quiz 3

Print your name and student number *neatly* on the first sheet.

1. (15 points) Let X and Y be independent and discrete, with $Z = X+Y$. Derive a convolution formula for $p_Z(z)$.

2. (25 points) Using just expected value notation and double expectation, show $Var(X) = E[Var(X|Y)] + Var(E[X|Y])$. It's easiest to start working on the right hand side.

3. Let $X_1, \ldots, X_N$ be independent Normal(0,1) random variables, and let N be a Poisson random variable independent of $X_1, \ldots, X_N$, with parameter $\lambda = 5$. Recall this means that $E(N) = Var(N) = 5$. Let $S = \sum_{i=1}^{N} X_i$.

   (a) (5 Points) What is $E(S|N = n)$. No need to show any work unless you want to. Just write down the answer.

   (b) (5 Points) What is $Var(S|N = n)$. No need to show any work unless you want to. Just write down the answer.

   (c) (25 Points) What is $Var(S)$? Hint: Use Question 2. Show your work.

4. (25 points) Suppose that four contestants on a quiz show are each given the same question, and that each answers it independently of the others, with probability of a correct answer equal to P. But the question itself is randomly chosen from a large population of questions, so that the value P is a random variable. Suppose $f_P(p) = 2p\,\mathbf{1}\{0 < p < 1\}$. What is the probability that exactly three of the contestants answer the question correctly?