STA 347F2000 Quiz 1

Print your name and student number *neatly* on the first sheet.

1. (20 points) The probability mass function of the Bernoulli is $p(x) = p^x(1-p)^{1-x}\mathbf{1}\{x = 0, 1\}$.
   (a) Denote the distribution function of the Bernoulli by $F(x)$. What is $F(-2)$?
   (b) What is $F(0)$?
   (c) What is $F(\frac{3}{4})$?
   (d) What is $F(17.4)$?
   (e) Derive the variance.

2. (20 points) A fraction $p = 0.05$ of the items coming off a production process are defective. The output of the process is sampled one by one, in a random manner. What is the probability that exactly four non-defective items are found before the second defective item is found? You do *not* need to simplify your answer. Just write it down. This is not a lot of work for the number of points.

3. (20 points) Let X and Y be continuous random variables that are *independent*. Either
   (a) Prove the claim that $E[XY] = E[X]E[Y]$ in general (*not* by giving an example), or
   (b) Prove the claim is false by giving an example of two continuous independent random variables with $E[XY] \neq E[X]E[Y]$.

   Whether you are proving or disproving the claim, write "Here is where I use independence," and draw an arrow to the place in your proof. Half marks off if you fail to do this. Also, you must clearly choose whether you are proving the claim true or false. If you do both, you get zero even if one of your answers is correct.

4. (20 points) A continuous random variable has density $f(x) = \frac{1}{2}e^{-|x|}$. Yes, that's an absolute value. Find $E[X]$. Hint: Notice the symmetry.

5. (20 points) Suppose X has a uniform distribution on the interval (0,1), and let $Y = -\frac{1}{2}\ln(1-X)$. Derive the density of Y. Whether you use indicators or not, the support is worth half marks; *please simplify the support*. If you do use indicators, it is helpful to write the uniform density as $f_X(x) = \mathbf{1}\{0 < x < 1\}$.