

Name \_\_\_\_\_

Student Number \_\_\_\_\_

Test 4  
STA 347s 1991  
Erindale College

1. Let  $\{N(t): t \geq 0\}$  be a Poisson process with rate  $\lambda$ . Given that exactly one event happened in  $(0, t]$ , find the conditional density of the time that the event occurred by answering the three questions below.

a) (5 pts) What is the support of the conditional density? (That is, give the interval where the value of the conditional density function will be non-zero.)

b) (20 pts) For  $s$  in the interval you specified in part (a), find the conditional distribution function of the arrival time, evaluated at  $s$ . That is, find  $P(T_1 \leq s \mid N(t)=1)$ .

c) (5 pts) Now use your answer to (b) to obtain the density  $f_{T_1 \mid N(t)}(s \mid 1)$ .

2. Radioactive particles arrive at a Geiger counter at a rate of six per minute. Each particle has probability  $2/3$  of being detected. Let  $X(t)$  denote the number of particles detected by time  $t$  minutes, and let  $Y(t)$  denote the number of undetected particles by that time.

a) (10 pts) What is  $P\{X(t)=0\}$ ?

b) (10 pts) What is  $E[Y(t)]$ ?

c) (5 pts) Are  $X(t)$  and  $Y(t)$  independent or dependent?  
(You don't have to prove anything; just answer the question.)

d) (10 pts) What is the probability that two particles pass by undetected before two particles are detected?

3. Refugees arrive at a refugee camp according to a Poisson process with rate  $\lambda$  per day (a day is 24 hours).

a) (15 pts) Given that only one refugee arrived on a particular day, what is the probability that she arrived during the first 12 hours?

b) (10 pts) Given that two refugees arrived on a particular day, what is the probability that they both arrived during the first 12 hours?

c) (10 pts) Given that  $n$  refugees arrived on a particular day, what is the probability that exactly  $k$  of them ( $0 \leq k \leq n$ ) arrived during the first 12 hours? (The space below is more space than you need; just write down the answer.)

**Total marks = 100 points**

(If you prepared examples 3f and 3g, you didn't waste your time. The quiz on Friday will cover that material.)